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SOME REASONS TO FUZZY APPROACH OF THE CHOICE FUNCTIONS

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Abstract. *The human preferences and the choice represent a significant problem in many domains as the decision theory, economics or social life. In the real life there are a many choice function that are not rationalizable. The specialized literature gives as procedures which imbedded the non-rational functions in to one rational. A full of advantages method that treats the non-rational choice functions is the utilization to fuzzy theory in the choice problems.*

1. **Introduction.** A choice function are designed for describe a choice behaviour and it selects an object from a finite set X= $\{x_1,...x_I\}$ of I objects.

Definition: Let P(X) a collection of A, B,… nonempty subsets of X. A single-valued **choice function c** on P(X) is

$$c: P(X) \to X$$

with c(A)$\in$A for every A$\in$P(X)

Definition: For previous function, a **preference relation** $\succ$ is said to **rationalize c** if and only if

$$c(A)=x,\ x\in A \text{ and } x\succ y \text{ for every } y\in A,\ y\neq x$$

In these conditions the function c is named **rational choice function.**
The rational choice functions have the following property (see [1]):

Property: If A, B$\in$P(X), $A\subseteq B$ and c(B) $\in$A, then c(A)=c(B).

Also,
A choice function c is rationalizable $\Leftrightarrow$ c satisfies the previous property.

Observations:
A binary relation on X, $\succ$ is preference relation if is irreflexive, transitive and total. A preference relation rationalizes a choice function c when it chooses the most preferred object from a set A.

But in the real life there are a many choice function that are not rationalizable.

2. Reasons for fuzzy approach.

We start with a classical example. This show how fragile is the rationality in classical meaning.
A classical example:
Suppose that a person must to choose, for example, a piece of cheese from the set X where the objects are ordered by size

$$x_1 > x_2 > \ldots > x_I$$

If the preference relation $\succ$ is gave by the size then:

$$x_1 \succ x_2 \succ \ldots \succ x_I$$

If I=3 then X=$\{x_1, x_2, x_3\}$, P(X)=$\{\{x_1\},\{x_2\},\{x_3\},\{x_1,x_2\},\{x_1,x_3\},\{x_2,x_3\},\{x_1,x_2,x_3\}\}$
The choice function c is

A$\in$P(X)	$\{x_1\}$	$\{x_2\}$	$\{x_3\}$	$\{x_1,x_2\}$	$\{x_1,x_3\}$	$\{x_2,x_3\}$	$\{x_1,x_2,x_3\}$
c(A)	x_1	x_2	x_3	x_1	x_1	x_2	x_1

And it is rationalizable.

If the person choice results from some social reasons such cultural environment or loyalty to a person or group and the choice is the second preference, then function c is:

A$\in$P(X)	$\{x_1\}$	$\{x_2\}$	$\{x_3\}$	$\{x_1,x_2\}$	$\{x_1,x_3\}$	$\{x_2,x_3\}$	$\{x_1,x_2,x_3\}$
c(A)	x_1	x_2	x_3	x_2	x_3	x_3	x_2

It is obvious that A=$\{x_2, x_3\}$ $\subseteq$B=$\{x_1, x_2, x_3\}$ and c(B)$\in$A but c(A)$\neq$c(B), so choice function c is not rationalizable.

There is in the specialized literature a procedure which adds an supplementary dimension to the objects and embed the non rationalizable function in to a new rationalizable one. In this way the set X is transformed in set X$\times$W where W={ $w_1, w_2, \ldots w_I$ }and the preference relation $\succ$ is redefined on X$\times$W.

The existence of a large number of non rationalizable practical choice functions and the vague character of human preference lids to usages of the fuzzy theory in the choice problem.

Definition: A fuzzy binary relation on X is a function $r_{\prec} : X \times X \to [0,1]$ with

$$r_{\prec}(x, y) \in (0,1] \text{ if} (0,1] \in \prec$$

$$\text{And } r_{\prec} = 0 \text{ if } (x, y) \notin \prec .$$

In the specialized literature is considered the matrix representation of fuzzy binary relation which describes the preference relation:

$$M\left(r_{\prec}\right)=\begin{pmatrix} r_{\prec}\left(x_1,x_1\right) & r_{\prec}\left(x_1,x_2\right) & \dots & r_{\prec}\left(x_1,x_I\right) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ r_{\prec}\left(x_I,x_1\right) & r_{\prec}\left(x_I,x_2\right) & \dots & r_{\prec}\left(x_I,x_I\right) \end{pmatrix}$$

And

The sum-fuzzy rational choice function is defined as:

$$\text{c(X)=}\left\{ x\in X \,\middle|\, \sum_{z\in X} r_{\prec}\left(x,z\right) \geq \sum_{z\in X} r_{\prec}\left(y,z\right) \quad \textit{for} \quad \textit{all} \quad y\in X \right\}$$

Conclusions

The fuzzy theory must be taken in consideration when a choice problem is studied. There are many advantages including the rationability of the choice function. Also, this approach is more facile and uses small size of computations.

References

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