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THE POSITION VECTOR OF A POINT EXERCISES

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The vectorial method is is applicable for studying a large class of properties of the euclidian space (coliniariy, coplanarity, parallelism, perpendicularity, calculation of angles, distances, volumes, etc)

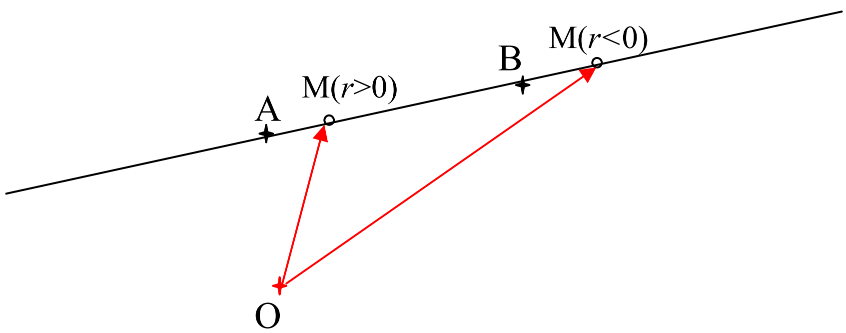
There are some exercices in geometry in which the vectorial method is more direct and eloquent. It is good to know several methods. But more important is to know to choose, adapt and use the best suited method.

There will be presented several geometry problems solved by using the vectorial method.

SENTENCE: Points A, B, M, where $M \neq B$ and $r \in R - \{-1\}$, $\overrightarrow{AM} = r \cdot \overrightarrow{MB}$. Then, for any point $O \in P$ we have

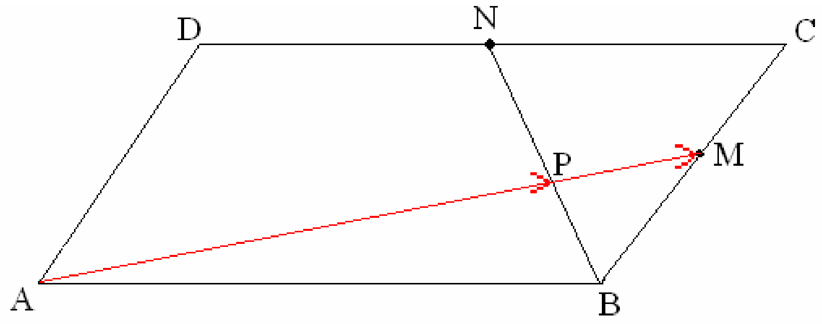
$$\overrightarrow{OM} = \frac{\overrightarrow{OA} + r \cdot \overrightarrow{OB}}{1 + r}$$

and reciprocal.



A1. M, N, middle points of BC, CD of the ABCD parallelogram and P intersection of AM, BN. Calculate $\frac{BP}{BN}$.

Solution



En general,

$$\frac{BM}{MC} = p \quad \text{and} \quad \frac{CN}{ND} = q$$

$$\frac{AP}{PM} = x \quad \text{and} \quad \frac{BP}{PN} = y$$

We will write that the position vectors of P and M raported with a point from the plan (point A) are collinear.

From $\frac{BP}{PN} = y$

results the position vector of point P:

$$\overline{AP} = \frac{\overline{AB} + y \cdot \overline{AN}}{1 + y} \quad (1)$$

From $\frac{CN}{ND} = q$

we obtain:

$$\overline{AN} = \overline{AD} + \frac{1}{q+1} \cdot \overline{AB} \quad (2)$$

Replacing (2) in (1) we obtain:

$$\overline{AP} = \frac{1+q+y}{(1+q)(1+y)} \cdot \overline{AB} + \frac{y}{1+y} \cdot \overline{AD} \quad (3)$$

We will calculate $\overrightarrow{AM}$ reported to $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

From $\frac{BM}{MC} = p$

we obtain the position vector of point M:

$$\overline{AM} = \frac{\overline{AB} + p \cdot \overline{AC}}{1 + \mathrm{p}}$$

or

$$\overline{AM} = \overline{AB} + \frac{p}{1+p}\cdot\overline{AD} \qquad (4)$$

Vectors $\overrightarrow{AP}$ and $\overrightarrow{AM}$ are collinear if exists $k \in R^*$

$$\overline{AP} = k\cdot\overline{AM}$$

From (3) and (4) we obtain:

$$\frac{1+q+y}{(1+q)(1+y)}\cdot\overline{AB} + \frac{y}{1+y}\cdot\overline{AD} = \overline{AB} + \frac{p}{1+p}\cdot\overline{AD}$$

Vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are not collinear, thus:

$$\frac{1+q+y}{(1+q)(1+y)} = 1 \quad \text{and} \quad \frac{y}{1+y} = \frac{p}{1+p}$$

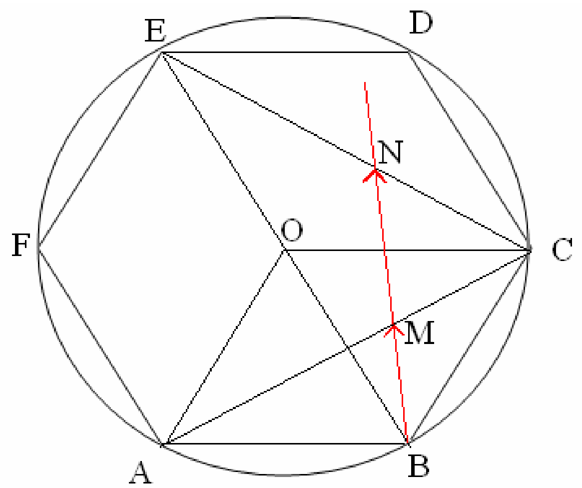
Which leads to:

$$y = \frac{BP}{PN} = \frac{p+pq}{1+q+pq}$$

A2. ABCDEF a hexagon and $M\in(AC)$, $N\in(CE)$, $\frac{\mathbf{AM}}{\mathbf{AC}} = \frac{\mathbf{CN}}{\mathbf{CE}} = \alpha$.

Calculate α knowing that B, M, N are collinear.

Solution



From $\frac{AM}{AC} = \alpha$ results $\frac{AM}{MC} = \frac{\alpha}{1-\alpha}$.

The position vector of point M is:

$$\overline{BM} = \frac{\overline{BA} + \frac{\alpha}{1-\alpha}\cdot\overline{BC}}{1+\frac{\alpha}{1-\alpha}} = (1-\alpha)\cdot\overline{BA} + \alpha\cdot\overline{BC} \qquad (1)$$

From $\frac{CN}{CE} = \alpha$ results $\frac{CN}{NE} = \frac{\alpha}{1-\alpha}$.

The position vector of point N is:

$$\overline{BN} = \frac{\overline{BC} + \frac{\alpha}{1-\alpha}\cdot\overline{BE}}{1+\frac{\alpha}{1-\alpha}} = (1-\alpha)\cdot\overline{BC} + \alpha\cdot\overline{BE} \qquad (2)$$

meaning

$$\overline{BN} = 2\alpha\overline{BA} + (1+\alpha)\overline{BC}$$

Points B, M, N are collinear if and only if $m \in R^*$ exists, and

$$\overrightarrow{BM} = m\cdot\overrightarrow{BN}$$

or

$$(1-\alpha)\overline{BA} + \alpha\overline{BC} = 2\alpha m\overline{BA} + m(1+\alpha)\overline{BC}$$

Vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ not being collinear, results

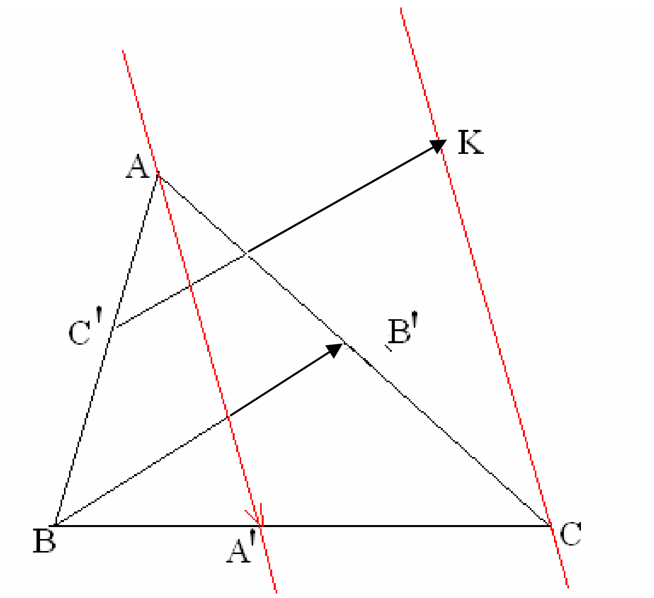
$1-\alpha = 2\alpha m$ şi $\alpha = m(1+\alpha)$

Results

$$\alpha = \frac{1}{\sqrt{3}}$$

A3. ABC a triangle, A', B', C' the middles of [BC], [CA], [AB] and a point K from the plan, $\overrightarrow{\mathbf{C'K}} = \overrightarrow{\mathbf{BB'}}$. Show that CK and AA' are parallel.

Solution



A' is the middle of [BC]. Results that the position vector of A' is

$$\overline{AA'} = \frac{\overline{AB} + \overline{AC}}{2} \qquad (1)$$

We have

$$\overline{KC} = \overline{B'C} - \overline{B'K} = \frac{1}{2}\overline{AC} - \frac{1}{2}\overline{BA} =$$

$$= \frac{1}{2}\overline{AC} + \frac{1}{2}\overline{AB} = \frac{\overline{AC} + \overline{AB}}{2} = \overline{AA'}$$

So, $\overrightarrow{\mathrm{KC}} = \overrightarrow{\mathrm{AA'}}$ equivalent to KC || AA'

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THE VECTOR ε ACCELERATION ALGORITHM

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Abstract: *The purpose of this article is to make a short introduction to the vector ε acceleration algorithm and give an example of how it can be used to approximate the solution of a linear system of equations.*

1. Section 1.

In this section is given the formula of the vector ε algorithm and a theorem related to the application of this algorithm to a sequence which satisfies a linear recursive equation.

Definition 1. *For any* $\mathbf{a} \in \mathbf{R}^p$, $\{0\}$ *we will denote by* $\mathbf{a}^{-1}$ *the following expression*

$$\mathbf{a}^{-1} = \frac{\mathbf{a}}{< \mathbf{a}, \mathbf{a} >}$$

It is easy to prove that the above definition for the inverse of a vector satisfies the following properties.

Proposition 2. *For any* $\mathbf{a} \in \mathbf{R}^p$, $\{0\}$ *we have*

1. $(\mathbf{a}^{-1})^{-1} = \mathbf{a}$
2. $< \mathbf{a}, \mathbf{a}^{-1} > = 1$

Definition 3. *Now we consider a sequence* $\{\mathbf{x}_n\}_{n \in N}$ *of vectors in* R^p *and define a double indexed sequence* $\varepsilon_k^{(n)}$ *by*

$$(1) \quad \begin{cases} \varepsilon_{-1}^{(n)} = 0_{R^p}, \varepsilon_0^{(n)} = x_n, n \in N \\ \varepsilon_{k+1}^{(n)} = \varepsilon_{k-1}^{(n+1)} + (\varepsilon_k^{(n+1)} - \varepsilon_k^{(n)})^{-1}, n, k \in N \end{cases}$$