MATHEMATICS AND MENTAL CALCULUS

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Abstract: *This paper presents some great skills of both mathematicians and nonmathematicians in mental calculus. These data are important because the personalities we spoke about were very significant for their times and their work meant a lot for the progress of the mathematical science.*
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1. INTRODUCTION

A lot of great mathematicians proved amazing mental abilities. The one we are going to speak about in this paper are not all with this kind of powers and also we intend to speak about some, who, although with no academic education were able to do great things that astounded their contemporaries and today still astound us.

For the beginning we will speak about John Wallis. He lived between 1616 and 1703 and he was an English mathematician. Wallis occupied himself in finding mentally the integral part of the square root of $3 \cdot 10^{40}$ and several hours afterwards wrote down the result from memory. This fact having attracted notice, two months later he was challenged to extract the square root of a number of 53 digits; then he performed mentally, and a month later he dictated the answer which he had not meantime committed to writing.

These remarkable performances were special because Wallis wasn't very young when he did describe these things; he was in fact 53 years old. Most of the ones are going to speak about their showed powers and very young often around 10 years of age.

We have to mention von Neumann. There exist a lot of incredible stories about this great man and a lot of them are true for sure. He was able to read just once a book or an article to quote it back verbatim. Moreover he could do it years later without any hesitation. He could also translate it with no diminution of speed from its original language which was Hungarian into English. Von Neumann's abilities to make mental calculus in arithmetic are the source of a large number of stories and it is difficult now to decide between fiction and true facts. However it is certain that multiplying two eight digit numbers in his head was a task which he could accomplish with no effort.

Other mathematicians exhibited great powers in mental arithmetic such as: Ampere, Hamilton and Gauss. Only one of them described in detail how he was able to perform incredible feats of memory and calculating and he is A.C. Aitken. Before speaking about him we will mention some special calculating prodigies who had no mathematical training.

First we will speak about Zerah Colburn who was born in Vermont USA in 1804 and visited Europe in 1812 when he was only 8 years old. His powers were described like this:

He could instantly give the product of two numbers each of four digits but hesitated if both numbers exceeded 10,000. Among questions asked him at this time were to raise 8 to the 16th power; in a few seconds he gave the answer 281,474,976,710,656 which is correct. ... he worked less quickly when asked to raise numbers of two digits like 37 or 59 to high powers. ... Asked for the factors of 247,483 he replied 941 and 263; asked for the factors of 171,395 he gave 5, 7, 59 and 83, asked for the factors of 36,083 he said there were none. He, however, found it difficult to answer questions about numbers higher than 1,000,000.

Colburn was special for a number of reasons. First of all because of him Hamilton started to study mathematics, secondly that he exhibited a common characteristic of most uneducated calculating prodigies that his abilities diminished when he underwent education.

This it could be caused by the fact that such calculating abilities require practice for many hours every day and education occupies too much time to allow this to go on. His powers were described like this:

Asked for the square of 4,395 he hesitated but on the question being repeated he gave the correct answer, namely 19,316,025. Questioned as to the cause of his hesitation, he said he did not like to multiply four figures by four figures, but he said 'I found out another way; I multiplied 293 by 293 and then multiplied this product twice by 15'. On another occasion when asked for the product of 21,734 by 543 he immediately replied 11,801,562; and being questioned explained that he had arrived at this by multiplying 65,202 by 181.

George Parker Bidder was born in 1806 in Devonshire, England. He did not loose his abilities when educated and also other of his family members showed incredibile mental powers. One of his brothers knew the Bible by heart, another brother who was an actuary had all his books destroied in a fire. This could be a problem if he was an ordinary man but he was able in a period of 6 months to rewrite them from his memory. One of Bidder`s sons was able to multiply two numbers of 15 digits but he was slow compared to his father. Bidder explained that the sound of numbers was much important for him then their visual representation:

... if I endeavour to get any number of figures that are represented on paper fixed in my memory, it takes me a much longer time and a very great deal more exertion than when they are expressed or enumerated verbally. ... if required to find the product of two numbers each of nine digits which were read to me, I should not require this to be done more than once; but if they were represented in the usual way, and put into my hands, it would probably take me four times to peruse them before it would be in my power to repeat them, and after all they would not be impressed so vividly on my imagination.

The last non mathematician to speak about is Dase and his talents were investigated by Gauss, Encke and other mathematicians. As an example he calculated $79532853 \cdot 93758479 = 7456879327810587$ and he needed for that 54 seconds. He multiplied two 20 digits numbers in 6 minutes, two 40 digit numbers in 40 minutes, two 100 digit numbers in 8 hours and 45 minutes. Gauss commented that in the last situation using a pencil someone skilled in calculation have done it in about half time.

Although Dase had no mathematical education he offered to use his powers to help mathematics. He was taught to use the formula:

$$(\pi/4) = \tan^{-1}(1/2) + \tan^{-1}(1/5) + \tan^{-1}(1/8)$$

and using this he calculated pi correctly to 200 places over a period of about two months. In his spare time, between 1844 and 1847, he calculated the natural logarithms of the first 1005000 numbers to 7 decimal places.

We will now speak about A.C.Aitken. He did not exhibit his talent at a young age and he explined:

Only at the age of 15 did I feel I might develop a real power and for some years about that time, without telling anyone, I practised mental calculation from memory like a Brahmin Yogi, a little extra here, a little extra there, until gradually what had been difficult at first became easier and easier...

Aitken became an excellent mathematical professor in Edinburgh and gave many examples of his extraordinary mental abilities he posessed like:

When he examined a new number of a mathematical journal he had only to scan it page by page, turning the pages over at a rate which the ordinary reader would record only half dozen lines or so. Subsequent discussion made it clear that he registered all the material. And, as he said, he never forgot what he had once seen.

Aitken`s life was described by himself but he was also sutudied by psihologists. One of those studying Aitken was Hunter who described Aitken reciting the first 1000 digits of pi to him:

Sitting relaxed and still, he speaks the first 500 digits without error or hesitation. He then pauses, almost literally for breath. The total time taken is 150 sec. The rhythm and tempo of speech is obvious; about five digits per second separated by a pause of about (1/2)sec. The temporal regularity is almost mechanical; to illustrate, each successive block of fifty digits is spoken in exactly 15 sec.

Then Aitken recited correctly the second 500 digits of π Here however Hunter reports that Aitken hesitated and sometimes corrected himself. When asked why he found the second 500 much harder than the first 500 Aitken had an interesting answer. Partly he said it was due to tiredness since recall requires a great effort. More interesting however was his other reason:-

Before the days of computing machines there was a kind of competition (human, I mean) in seeing how far they could calculate π. In 1873, Shanks carried this to 707 decimals; but it was not until 1948 that it was discovered that the last 180 of these were wrong. Now, in 1927 I had memorised those 707 digits for an informal demonstration to a students society, and naturally I was rather chagrined, in 1948, to find that I had memorised something erroneous. When π was calculated to 1000 and indeed more decimals, I re-memorised it. But I had to suppress my earlier memory of those erroneous digits, 180 of them...

So Aitken's problem was that he could not forget the incorrect 180 digits!

Here is the expansion of πarranged in groups of 50 digits as Aitken recited it.

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3.14159265358979323846264338327950288419716939937510
  58209749445923078164062862089986280348253421170679
  82148086513282306647093844609550582231725359408128
  48111745028410270193852110555964462294895493038196
  44288109756659334461284756482337867831652712019091
  45648566923460348610454326648213393607260249141273
  72458700660631558817488152092096282925409171536436
  78925903600113305305488204665213841469519415116094
  33057270365759591953092186117381932611793105118548
  07446237996274956735188575272489122793818301194912
  98336733624406566430860213949463952247371907021798
  60943702770539217176293176752384674818467669405132
  00056812714526356082778577134275778960917363717872
  14684409012249534301465495853710507922796892589235
  42019956112129021960864034418159813629774771309960
  51870721134999999837297804995105973173281609631859
  50244594553469083026425223082533446850352619311881
  71010003137838752886587533208381420617177669147303
  59825349042875546873115956286388235378759375195778
  18577805321712268066130019278766111959092164201989
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Numbers filled Aitken world. He said:

If I go for a walk and if a motor car passes and it has the registration number 731, I cannot but observe it is 17 times 43. ... When I see a bus conductor with a number on his lapel, I square it ... this isn't deliberate, I just can't help it. ... [Given a number] is it a prime of the form 4n+1, and so expressible as the sum of two squares in one way only? Is it the numerator of a Bernoullian number, or one occurring in some continued fraction? And so on. Sometimes a number has almost no properties at all, like 811, and sometimes a number, like 41, is deeply involved in many theorems that you know.

Some feats of memory are certainly visual in nature. The person can "see" the numbers of objects which have been commited to memory and in a way read them as if they had been

written on paper. Aitken said that his memory could work that way but it slowed him down. However when asked to recite the digits of pi backwards Aitken had no other option but to bring the numbers into a visual form and read them off backwards from his visual image. He was slower but still impressive in speed. The last 50 digits of the expansion required 18 seconds for him to recite them forwards and 34 seconds to recite them backwards.

Aitken`s powers were tested by the Psychology Department from Edinburgh. In 1930`s Aitken was tested and one of those test involved 25 random words which were selected from a dictionary. These words were read to Aitken and he was able to repeat them. In 1961 Hunter asked him if he remembered that test and Aitken was able to recite those 25 words again nearly 30 years later.

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