ORIGINAL PAPER

CONVOLUTION STRUCTURE FOR TWO VERSION OF FRACTIONAL LAPLACE TRANSFORM

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Abstract. *Two versions of fractional Laplace transform are found in the literature. One by Sharma [4], which is a special case of linear canonical transform with representative matrix* $[i\cos\alpha, i\sin\alpha, i\sin\alpha, -i\cos\alpha]$ *and other by Torre [5] with representative matrix* $[\cos\alpha, i\sin\alpha, i\sin\alpha, \cos\alpha]$. *This paper is devoted to study the convolution structure for both versions of fractional Laplace transform. We have also proved convolution theorem for them.*

Keywords: *Linear canonical transform, Fractional Laplace transform, convolution.*

AMS Subject code: *44 A 10, 44 A 35.*

1. INTRODUCTION

The fractional Laplace transform plays an important role in many fields, including signal processing, wireless communication etc. The fractional Laplace transform L^α is a special case of complex linear canonical transform and depends on parameter α. A convolution structure for the fractional Fourier transform is discussed by Almeida [1] and Zayed [7]. Gudadhe [3] had defined convolution for fractional Mellin transform. Convolution suggested in these papers preserves the convolution theorem as in the Fourier transform. The definition of Laplace type convolution [8] does not seem as nice in fractional Laplace transform, because convolution theorem which states that the convolution of two functions is the product of their convolution is not satisfied. For the fractional Laplace transform C type here we have also proposed a new convolution structure, as in [6] that is different from those introduced in [7].

2. PRELIMINARIES

In this section we have given three definitions.

2.A. THE LINEAR CANONICAL TRANSFORM:

The linear canonical transform (LCT) of a signal *f(t)*, with a parameter *(a,b,c,d)*, is defined as:

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$$LCT_{(a,b,c,d)}\left[f(t)\right](u)=\begin{cases}\int_{-\infty}^{\infty} f(t)K_{(a,b,c,d)}(u,t)dt, & b\neq 0\\ \sqrt{d}\,e\dfrac{icd}{2}u^2 f\left(du^2\right), & b=0\end{cases} \tag{1}$$

where $K_{(a,b,c,d)}(u,t)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2b}\left(at^2+du^2-2tu\right)}$ and *a, b, c, d* are real numbers satisfying $ad-bc=1$.

The 2 x 2 matrix *(a, b, c, d)* is called representative matrix.

The inverse of linear canonical transform is:

$$f(t)=\int_{-\infty}^{\infty} LCT_{(a,b,c,d)}(u)K^*_{(a,b,c,d)}(u,t)du, \qquad b\neq 0 \tag{2}$$

where $K^*_{(a,b,c,d)}(u,t)=\sqrt{b}e^{-\frac{i}{2b}\left(at^2+du^2-2tu\right)}$.

If $b = 0$, linear canonical transform is just a chirp multiplication.

2.B. FRACTIONAL LAPLACE TRANSFORM:

Sharma [4] has defined fractional Laplace transform as,

$$L^{\alpha}\left[f(t)\right](u)=\sqrt{\frac{1}{-2\pi\sin\alpha}}e^{\frac{-iu^2\cot\alpha}{2}}\int_{-\infty}^{\infty} f(t)e^{\frac{-it^2\cot\alpha}{2}}e^{-ut\csc\alpha}dt$$

where $u=\sigma+iw$ denote the complex fractional Laplace transform variable.

$$\therefore \qquad L^{\alpha}\left[f(t)\right](u)=\frac{c(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{ia(\alpha)\left[t^2-u^2+2ib(\alpha)tu\right]}dt \tag{3}$$

where $C(\alpha)=\sqrt{1-i\cot\alpha}$, $a(\alpha)=\frac{1}{2}\cot\alpha$, $b(\alpha)=\sec\alpha$.

2.C. FRACTIONAL LAPLACE TRANSFORM (ANOTHER VERSION):

For the square integrable function *f* (*t*), fractional Laplace transform given by Torre [5] is defined in terms of a kernel is:

$$L^{\alpha}(u)=\int_{-\infty}^{\infty} f(t)K_{\alpha}(t,u)dt \tag{4}$$

where $K_{\alpha}(t,u)$ of the fractional Laplace transform is given by Torre A [5]

$$K_{\alpha}(t,u)=\sqrt{\frac{1-i\cot\alpha}{2\pi i}}e\frac{t^2}{2}\cot\alpha+\frac{u^2}{2}\cot\alpha-tu\cos ec\alpha, \qquad \alpha \text{ is not multiple of } \pi$$
$$=\delta(t-u) \qquad , \qquad \alpha \text{ is multiple of } \pi$$

Fractional Laplace transform given in B) and C) both are special cases of complex Linear canonical transform (where *a*, *b*, *c* and *d* are complex numbers).

3. CONVOLUTION STRUCTURE FOR (B)-TYPE FRACTIONAL LAPLACE TRANSFORM

Next we define convolution for B-type fractional Laplace transform as per Zayed [7].

3.1. DEFINITION:

For any function *f(t)*, let us define functions $\bar{f}(t)$ and $\bar{\bar{f}}(t)$ as $\bar{f}(t)=f(t)e^{ia(\alpha)t^2}$ and $\bar{\bar{f}}(t)=f(t)e^{-ia(\alpha)t^2}$.

For any two functions f and g, we define the convolution operator "*" by

$$h(t)=(f\odot g)(t)=\frac{C(\alpha)}{\sqrt{2\pi}}e^{-ia(\alpha)t^2}\left(\bar{f}*\bar{g}\right)(t) \tag{5}$$

where the convolution operator "*" for the Laplace transform is defined as

$$h(t)=(f*g)(t)=\int_{-\infty}^{\infty}f(u)g(t-u)du$$

Similarly we define convolution operator Θ by

$$(f\Theta g)(t)=\frac{C(\alpha)}{\sqrt{2\pi}}e^{ia(\alpha)t^2}\left(\bar{\bar{f}}*\bar{\bar{g}}\right)(t) \tag{6}$$

3.2. CONVOLUTION THEOREM:

Let $f,\ g,\ h$ be fractional Laplace transformable functions and

$$h(t)=(f\odot g)(t).$$

$L^{\alpha}(u), G^{\alpha}(u)$ and $H^{\alpha}(u)$ denote the fractional Laplace transform of f, g and h respectively then

$$H^{\alpha}(u)=e^{iau^2}L^{\alpha}(u)\cdot G^{\alpha}(u) \tag{7}$$

Proof: From the definition of fractional Laplace transform

$$H^{\alpha}(u)=\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}h(t)e^{ia\left[t^2-u^2+2ibtu\right]}dt$$

$$=\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(f*g)(t)e^{ia\left[t^2-u^2+2ibtu\right]}dt$$

$$=\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\frac{C(\alpha)}{\sqrt{2\pi}}e^{-iat^2}\left(\overline{f}*\overline{g}\right)(t)\right]e^{ia\left[t^2-u^2+2ibtu\right]}dt$$

$$=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty}e^{ia\left[t^2-u^2+2ibtu\right]}e^{-iat^2}dt\int_{-\infty}^{\infty}\left(f(x)e^{iax^2}\right)\left(g(t-x)e^{ia(t-x)^2}\right)dx$$

$$=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x)g(t-x)e^{ia\left(t^2-u^2+2ibtu+2x^2-2tx\right)}dxdt$$

$$H^{\alpha}(u)=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x)g(\nu)e^{ia\left[(\nu+x)^2-u^2+2ib(\nu+x)u+2x^2-2(\nu+x)x\right]}dxd\nu,\ \{\because (t-x)=\nu$$

$$=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x)g(\nu)e^{ia\left[\nu^2+x^2-u^2+2ib\nu u+2ibux\right]}dxd\nu$$

$$=e^{iau^2}\left\{\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{ia\left[x^2-u^2+2ibxu\right]}dx\right\}\left\{\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}g(\nu)e^{ia\left[\nu^2-u^2+2ib\nu u\right]}d\nu\right\}$$

$$=e^{iau^2}L^{\alpha}(u)G^{\alpha}(\nu)$$

3.3. PRODUCT THEOREM:

Let $l(x)=(f\Theta g)(t)$ and $\left[L^{\alpha}\mathbf{f}\right](u)$, $\left[L^{\alpha}\mathbf{g}\right](u)$ and $\left[L^{\alpha}l\right](u)$ denote the fractional Laplace transform of *f*, *g* and *l* respectively, then

$$\left[L^{\alpha}l\right](u)=e^{iau^2}\left[L^{\alpha}g\right](u)\left[L^{\alpha}f(x)e^{4ia\nu x}\right](u) \tag{8}$$

Proof: Using the definition of fractional Laplace transform [4]

$$\left[L^{\alpha}l\right](u)=\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}l(t)e^{\frac{i}{2}\cot\alpha\left(l^2-u^2+2ibtu\right)}dt$$

$$L^{\alpha}(u)=\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}l(t)e^{ia(\alpha)\left[t^2-u^2+2ibtu\right]}dt$$

$$=\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(f\Theta g)(t)e^{ia(\alpha)\left[t^2-u^2+2ibtu\right]}dt$$

$$=\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\frac{C(\alpha)e^{iat^2}}{\sqrt{2\pi}}\left(\overline{\overline{f}}*\overline{\overline{g}}\right)(t)\right]e^{ia(\alpha)\left[t^2-u^2+2ibtu\right]}dt$$

$$=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty} e^{ia(\alpha)\left[\left(t^2-u^2+2ibtu\right)+t^2\right]}\left(\overline{\overline{f}}*\overline{\overline{g}}\right)(t)\,dt$$

$$=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty} e^{ia(\alpha)\left[2t^2-u^2+2ibtu\right]}\int_{-\infty}^{\infty}\left(f(x)e^{-iax^2}\right)\left(g(t-x)e^{-ia(t-x)^2}\right)dxdt$$

$$=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)g(t-x)e^{ia(\alpha)\left[2t^2-u^2+2ibtu-x^2-t^2+2tx-x^2\right]}dxdt$$

$$=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)g(t-x)e^{ia(\alpha)\left[t^2-u^2+2ibtu-2x^2+2tx\right]}dxdt.\ \text{Put } (t-x)=v$$

$$=\frac{C^2(\alpha)}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)g(v)e^{ia(\alpha)\left[(v+x)^2-u^2+2ib(v+x)u-2x^2+2(v+x)x\right]}dvdx$$

$$=\left[\frac{C(\alpha)}{2\pi}\int_{-\infty}^{\infty} g(v)e^{ia(\alpha)\left[v^2-u^2+2ibuv\right]}dv\right]e^{iau^2}\left[\frac{C(\alpha)}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(f(x)e^{4iavx}\right)e^{ia\left[x^2-u^2+2ibxu\right]}dx\right]$$

$$=e^{iau^2}\left[L^{\alpha}\mathbf{g}\right](u)\left(L^{\alpha} fe^{4iavx}\right)(u)$$

4. NEW CONVOLUTION STRUCTURE FOR (C)-TYPE FRACTIONAL LAPLACE TRANSFORM

The definition of convolution applied in section 3 for the fractional Laplace transform of B-type is not suitable for the C-type fractional Laplace transform. Here we apply a new convolution structure as in [6]. It is different from those introduced in [2]. For this, first we define generalized translation and general framework of convolution.The τ − generalized translation of signal $f(t)$ defined as,

$$f(t\Theta\tau)=\int \rho(u)L^{\alpha}(u)K(\tau,u)K^*(t,u)\,du \tag{9}$$

where Θ is the argument of function, $f(t\Theta\tau)$ is the generalized delayed operator for generalized translation. $K(u,t)$ is the kernel of fractional Laplace transform, $K^*(u,t)$ is the conjugate of $K(u,t)$, $\rho(u)$ is the weight function.

4.1. τ -GENERALIZED TRANSLATION OF f(t) IN FRACTIONAL LAPLACE DOMAIN

$$f(t\Theta\tau)=\int_{-\infty}^{\infty}\rho(u)L^{\alpha}(u)K(\tau,u)K^{*}(t,u)du$$

$$=\int_{-\infty}^{\infty}L^{\alpha}(u)\left[\sqrt{\frac{1-i\cot\alpha}{2\pi i}}e^{\frac{\cot\alpha}{2}\left(\tau^{2}+u^{2}-2\sec\alpha.\tau u\right)}\right]\left[\sqrt{\frac{1+i\cot\alpha}{2\pi i}}e^{-\frac{\cot\alpha}{2}\left(t^{2}+u^{2}-2\sec\alpha.tu\right)}\right]du$$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\sqrt{\frac{1+i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}L^{\alpha}(u)e^{\frac{\cot\alpha}{2}\left(\tau^{2}+u^{2}-2\sec\alpha.\tau u\right)}e^{-\frac{\cot\alpha}{2}\left(t^{2}+u^{2}-2\sec\alpha.tu\right)}du$$

$$=\sqrt{\frac{\sin\alpha-i\cos\alpha}{2\pi i\sin\alpha}}\sqrt{\frac{\sin\alpha+i\cos\alpha}{2\pi i\sin\alpha}}\int_{-\infty}^{\infty}L^{\alpha}(u)e^{-\frac{\cot\alpha}{2}\left(t^{2}-\tau^{2}\right)+tu\csc\alpha-\tau u\csc\alpha}du$$

$$=\sqrt{\frac{-i(\cos\alpha+i\sin\alpha)}{2\pi i\sin\alpha}}\sqrt{\frac{i(\cos\alpha-i\sin\alpha)}{2\pi i\sin\alpha}}e^{-\frac{\cot\alpha}{2}\left(t^{2}-\tau^{2}\right)}\int_{-\infty}^{\infty}L^{\alpha}(u)e^{(t-\tau)\csc\alpha.u}du$$

$$=\sqrt{\frac{e^{i\alpha}e^{-i\alpha}}{2\pi i\sin\alpha}}\sqrt{\frac{1}{i\sin\alpha}}\frac{1}{\sqrt{2\pi}}e^{-\frac{\cot\alpha}{2}\left(t^{2}-\tau^{2}\right)}\int_{-\infty}^{\infty}L^{\alpha}(u)e^{(t-\tau)\csc\alpha.u}du$$

$$=\sqrt{\frac{1}{2\pi i\sin\alpha}}\sqrt{\frac{1}{i\sin\alpha}}\frac{1}{\sqrt{2\pi}}e^{-\frac{\cot\alpha}{2}\left(t^{2}-\tau^{2}\right)}\int_{-\infty}^{\infty}L^{\alpha}(u)e^{(t-\tau)\csc\alpha.u}du$$

$$=Be^{-\frac{\cot\alpha}{2}\left(t^{2}-\tau^{2}\right)}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}L^{\alpha}(u)e^{(t-\tau)\csc\alpha.u}du \tag{10}$$

$$=Be^{-\frac{\cot\alpha}{2}\left(t^{2}-\tau^{2}\right)}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}L^{\alpha}_{(\cos\alpha,i\sin\alpha,i\sin\alpha,\cos\alpha)}(u)e^{\cos ec\alpha(t-\tau)u}du$$

where $B=\sqrt{\frac{1}{2\pi i\sin\alpha}}\sqrt{\frac{1}{i\sin\alpha}}$.

Hence, the τ - generalized translation of a signal $f(t)$ denoted by $f(t\Theta\tau)$.

Now we define the convolution operation $\oplus$ for the C type fractional Laplace transform by:

$$z(t)=(f\oplus g)(t)=\int_{-\infty}^{\infty}f(\tau)g(t\Theta\tau)d\tau \tag{11}$$

4.2. CONVOLUTION THEOREM:

Let $Z(t)=(f\oplus g)(t)$ and $F^{\alpha}(u)$, $G^{\alpha}(u)$ and $Z^{\alpha}(u)$ denote the fractional Laplace transform of f, g and z respectively then,

$$Z^{\alpha}(u)=F^{\alpha}(u)G^{\alpha}(u) \tag{12}$$

Similary,

$$L^{\alpha}\left[f(t)g(t)\right](u)=\left(L^{\alpha}\oplus G^{\alpha}\right)(u) \tag{13}$$

Proof: Generalized translation in the fractional Laplace domain as obtained in equation (10) is

$$g(t\Theta\tau)=Be^{-\frac{\cot\alpha}{2}\left(t^2-\tau^2\right)}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}G^{\alpha}(u)e^{\cos ec\alpha(t-\tau)u}du$$

where $B=\sqrt{\dfrac{1}{2\pi i\sin\alpha}}\sqrt{\dfrac{1}{i\sin\alpha}}$.

Consider, $L^{\alpha}\left[z(t)\right]=L^{\alpha}\left[(f\oplus g)\right](u)$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}\left[f(t)g(t\Theta\tau)\right]e^{\frac{\cot\alpha}{2}\left(t^2+u^2-2\sec\alpha.tu\right)}dt$$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}f(\tau)\left[Be^{-\frac{\cot\alpha}{2}\left(t^2-\tau^2\right)}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}G^{\alpha}(u)e^{\cos ec\alpha(t-\tau)u}du\right]d\tau\right\}e^{\frac{\cot\alpha}{2}\left(t^2-u^2-2\sec\alpha.tu\right)}dt$$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}f(\tau)\left[Be^{-\frac{\cot\alpha}{2}\left(t^2-\tau^2\right)}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}G^{\alpha}(u)e^{(t-\tau)\csc\alpha.u}du\right]d\tau\right\}e^{\frac{\cot\alpha}{2}\left(t^2-u^2-2\sec\alpha.tu\right)}dt$$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\frac{B}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}f(\tau)e^{-\frac{\cot\alpha}{2}\left(t^2-\tau^2\right)+\frac{\cot\alpha}{2}\left(t^2+u^2-2\sec\alpha tu\right)}d\tau\int_{-\infty}^{\infty}G^{\alpha}(u)e^{(t-\tau)\csc\alpha.u}dudt\right\}$$

$$=\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\left[\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\sqrt{\frac{1+i\cot\alpha}{2\pi i}}\right]\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}f(\tau)e^{\frac{\cot\alpha\left(\tau^2+u^2-2\sec\alpha tu\right)}{2}}d\tau\int_{-\infty}^{\infty}G^{\alpha}(u)dudt\right\}$$

$$=\left\{\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}f(\tau)e^{a\left(\tau^2+u^2-2b\tau u\right)}d\tau\right\}\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\sqrt{\frac{1+i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}G^{\alpha}(u)dudt$$

$$=L^{\alpha}\left[f(\tau)\right](u)\sqrt{\frac{1-i\cot\alpha}{2\pi i}}\sqrt{\frac{1+i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}G^{\alpha}(u)e^{\frac{\cot\alpha}{2}\left(t^2+u^2-2\sec\alpha.tu\right)}du\cdot e^{-\frac{\cot\alpha}{2}\left(t^2+u^2-2\sec\alpha.tu\right)}dt$$

$$= L^{\alpha}\left[f(\tau)\right](u)\sqrt{\frac{1+i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}G^{\alpha}(u)\sqrt{\frac{1-i\cot\alpha}{2\pi i}}e^{\frac{\cot\alpha}{2}\left(t^2+u^2-2\sec\alpha.tu\right)}du\right]\cdot e^{-\frac{\cot\alpha}{2}\left(t^2+u^2-2\sec\alpha.tu\right)}dt$$

$$= L^{\alpha}\left[f(\tau)\right](u)\sqrt{\frac{1+i\cot\alpha}{2\pi i}}\int_{-\infty}^{\infty}g(t)e^{-\frac{\cot\alpha}{2}\left(t^2+u^2-2\sec\alpha.tu\right)}dt$$

$$= L^{\alpha}\left[f(\tau)\right](u)L^{\alpha}\left[g(t)\right](u) \qquad \because \left\{f(t)=\int\rho(u)L^{\alpha}(u)K_{\alpha}(t,u)du\right.$$

$$= F^{\alpha}(u)G^{\alpha}(u) \qquad \left\{L^{\alpha}(u)=\int\rho(t)f(t)K^{*}(t,u)dt\right.$$

$$\therefore\; Z^{\alpha}(u)=F^{\alpha}(u)\cdot G^{\alpha}(u)$$

Similarly, we can prove:

$$L^{\alpha}\left[f(t)g(t)\right]=\left(L^{\alpha}\oplus G^{\alpha}\right)(u)$$

The proof of this equation is similar to the proof of theorem 4.2 and is omitted.

5. CONCLUSION

We have introduced new convolution structure for two versions of the fractional Laplace transform and proved the convolution theorem for both of them. Product theorem is also proved for B-type fractional Laplace transform. The above results are more important practically in filtering.

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