THE RELATIONSHIP BETWEEN MATHEMATICS – AN EXACT SCIENCE – AND THE MUSICAL PHENOMENON

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Manuscript received: 17.07.2011. Accepted paper: 10.02.2011.

Published online: 15.05.2011.

Abstract. *A complex approach, an objective approach of the issue of an artistic field presumes the knowledge of its own scientific data and an understanding of their essence in a multidisciplinarity with connecting areas. The organization and logic belonging to the real science have always been a research support for the sound phenomenon, reaching a moment of maximum implication in the XXth century, with the forming of the mathematical musical linguistics. Conducting a diachronic analisis of the relationship between the two fields, we conclude that mathematics has constituted a methodological basis both in the musical theoretical educational sphere and in the aplicational one of creation. In the current research we proposed ourselves to evaluate the mathematics' involvement in the study of music, to understand the role of the exact science artistic musical education. The current material is to enriched with data from other researches of certain specialists from the field of exact science, because beyond the general objective – previously mentioned – the research has to find a finality in a special objective, such as the setting of the involvement of mathematics in the musical phenomenon through a conscious, directed education that would lead to a maximum efficiency in the forming of future musicians (teachers, interpreters, musicologists, compositiors).*

Keywords: *music mathematization, mathematical musical linguistics, electronical music, dodecaphonic serialism, golden section, stochastic music*

1. INTRODUCTION

In the current century, the forming of specialists for different scientific or artistic-scientific ares is achieved through a training that appeals to the data of that field, presented in a multidisciplinary interrelationship. The building of the elite intelectual future, is achieved at the same time through a conscious educations, efficiently planned since a young age, through the improvement of personality with the setting of certain high ethical-moral values and through the development of supperior cognitive and affectionate pshichological processes.

In the current material, we proposed to analyse the involvement of exact science – specially mathematics and acoustic physics – in the study of music, in orther to, in following research, to understand the role of exact science in the artistic education in general and in musical education in particular.

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We considered necessary the treatment of this issue, since, in the musical didactic activity, any former of young men has to be permanently preoccupied with the achievement of maximum efficiency in the musical training and forming the personality of the subjects, as future pedagogues, interpreters, musicologists.

The set objective presumes to complete the data of this material with information from previous research, eventually from teamwork, with specialists from the field of exact science. In the current research, we started from the analysis of mathematics in the area of science and art of music. For this, it was necessary to investigate the interrelationship of the two, from a diachronic point of view. We followed up in the presentation the evaluation of the involvement of mathematical data in the theoretical sphere of didactic music, as well as in the application sphere of creation. With the setting of the role of the exact science in the organization of musical data, we maintain the necessity of active implication of the mathematical means in the art of sounds. In the presentation of this material, for an edificatory analysis, we used bibliographical sourses belonging to specialists from different fields such as: music, mathematics, acoustics, psycho-pedagogy, sociology.

2. CONTENT

"Music is an act of the human mind that sets order in the world of the sounds" – Igor Stravinski.

The sound constitutes a complex phenomenon. Its knowing presumes the deciphering of the relationships between the art and the science of music. Analyzing the musical phenomena on solid scientific basis, implicit it is required to report to acoustics, and with it we report to the exact science: physics and mathematics. A complete and objective image is given through the relationship of these fields of science. The acoustics considers sounds in particular, while the relationship and the effects of resonating on an emotional plan are analyzed by looking at the musical opera.

Dem Urmă (in the work "Acoustics and music") affirms that sounds are onjly the physical support of the esthetic functions given by the musical art [1]. Of course, in the process of creation and interpretation of art, the transformation of the raw sound matterail start mainly from the quantity elements of sounds, given by the science of acoustics physics and mathematics. Victor Giuleanu (in "Treaty of the Theory of music") pointed out three aspects in the manifestation of the sound phenomena: the primordial one – the physical process; the sensory one - by watching the physiological and the non less important one, the psychological aspect [2].

As it was previously stated, in the analysis of the transformation of the raw sound material, in the art work it is treated initially the physical aspect. This treats the sound phenomena with the moment of its occurrence in nature. The physiological nature refers to the sensorial effect that the receiving of the artwork causes over man. The psychological aspect of the sound phenomena is more subtle- it is deciphered based on the physical data and physiological information about the sound matter, supplied by the exact science.

Since medieval times it was attempted to give a scientific explanation of the artistic musical phenomena. 600 hundred years before our era, the theoretical school of Pythagoras (represented by the famous philosopher and Greek mathematician Pythagoras of Samos, as well as by his followers) elaborated the main mathematical-physical elements that today's music is based on. Thus, Pythagoras (592-520 B.C.) discovered with the help of the monochord – the intimate report between numbers and sounds, setting the mathematical acoustics value of the perfect quint through the fraction 3/2. Subsequent, this interval was set

at the basis of the forming of the entire musical sound system. Pythagoras stated: "In the curve of the cords there is geometry".

The Greek astronomer and mathematician Claudius Ptolemy (90-168 B.C.) presented in the work "Harmonica" the principles of planetary harmony. In Antiquity, the masters of lyre such as Damon, Lassos, Pythoclide, Agathocles and others didn't study only the ability of the fingers or procedures of accompaniment, but also, the fundamental relationships that gave their art its mathematical significance. The investigations of the famous mathematician Pythagoras, edited afterwards by the famous Ptolemy, were carried on and developed in the Middle Ages by the school of acousticians represented by the venetian Gioseffo Zarlino (1517-1590). In the Middle Ages music was considered discipline close to mathematics, because they both used numbers.

During the Renaissance, translating the music of spheres through particular numerical reports, fractions: 2/1, 3/2 şi 4/3, we find correspondences of the perfect harmonical reports of the octave and quart. Theoretician musicians of the Renaissance, carryng on the principles of the planetary harmony exposed by Ptolemy, used regulated polygons setting concordances between these geometrical figures (representing planetary configurations) and musical intervals. In these conditions there were discovered equivalences of the right, of the equilateral triangle, of the rectangle and hexagon with musical intervals: octave, quint, quart and small third. In the work ,,Compendium Muzicae (1618) Rene Descartes (Renatus Cartesius – at the origins) stated the idea that music responds to a logical and mathematical ideal: "The beauty of music resides within the laws of mathematical proportions" [3].

Jean Philippe Rameau (1683-1764), in his harmony treaty, supports the same idea noting: "Music is a science that follows stable rules, deriving from an obvious principle that cannot be explained and understood without the help of mathematics" [4].

The periods that followed Renaissance were based on the theories formed by the exact science and applied in music, or innovated more thoroughly in parameters the metrics and rhythmic.

The tight collaboration between music and mathematics on the first ones realm, wasn't reduced in the Modern Age, on the contrary actually. With the progress of positive and human sciences came the psycho-physiological school of interpretation and treatment of the musical phenomena. To the previously mentioned representatives are added some of the Modern times, also personalities with training in the exact science. Among them we mention Hermann Helmholtz (1821-1894) and Carl Stumpf (1848-1936) make a connection between their previous rich time in considerations of psychics – mathematics and the following period dominated by the psycho – physiological thinking (continued of course until the end of the XIX - th century, throughout the XX – th century and at the beginning of the XXI – st century). Works such as "The psychology of sound" by Carl Stumpf and ,,The Concept about auditory sensations as a physiological basis for the theory of music" by Herman Helmholtz, had an important role in the path of the musical science and in its reports with other sciences, starting the basis of the modern psycho – physiological school of interpretation of the musical phenomena.

Of course, as it was mentioned before, the musical phenomena has to be seen from a triple point of view : physical, physiological and psychological and with an increasing attention to the direction of psycho- physiology, also called psycho – acoustic (at the end of the XIX th century and beginning of the XX th century) we cannot state that the physic – mathematical report has lost ground in the sound phenomena, on the contrary, it completed its theorization, it objectified itself, gaining a positive sense in interpretation.

In the beginning of the XX th century, in the second decade, with the development on a melodic composition sense of the direction of atonal dodecaphonic serial concept, which' basis were set by Arnold Schonberg (observation: we don't refer exclusively to the serial

dodecaphonic writings of Schonberg but at the same time to the condensed series – see Anton Webern and Alban Berg), it came back intensely not only in a theoretical sense as much as in a creation sense to the direction of the exact sciences. Serial Atonalism presumed the organizing of the sound material through mathematical means. The theoretician Joseph Mathias Hauer proved mathematically that if on a melodically sense there are used all the serial possibilities you get 479.001.600 solutions, resulting from the mathematical relationship of the twelve sounds of the chromatic step [2]. On an harmonic plane we can get only in the octave a large number of chords, and by overcoming the framework of the octave the composer would have at his disposal millions of possibilities to combine the sounds in opera. Mathia Hauer, with another follower of the serial system Herbert Eimert, suggested the super organizing of the sound material through mathematical means.

Initially the serial technique conceived by A.Schonberg referred only to the sound heights, the other parameters (length, intensity, timbre) being used according to traditional rules. The followers of Schonberg (Olivier Messiaen, Pierre Boulez, Luigi Nono, Karlheinz Stockhausen and others) extended the serial principal to all the compositional parameters reaching to the absolute serializing, a total serializing of the means of expressing – obtaining " pure musical structures" (see V.Giuleanu) [2]. The super serializing of the sound material replaced the creating fantasy of the composer with combining procedures with a quasi mathematical basis. Super serial exacerbations and their constraints led to the distruction of the system, limiting the originality of the expression, to the emotiveness of the artistic images, to a philosophical depth of the message, neglecting for a short time the psychological direction of the musical phenomena.

In the second half of the XXth century, another important representative music, Olivier Messiaen elaborated the theory of the modes with limited transposition. Those systems are organizations constituted out of simetrical groups of intra Octavian sounds, having each at their basis a model with an intervallic structure symmetrically reported to the semi tone – so mathematical.

Towards the end of the XXth century mathematical sciences entered all the fields – especially the artistic ones – imposing its principles of work. As we previously showed, in the musical field the concerns appeared long before our time – through Pytagoras who set the intimate report between numbers and sounds. Thus, the mathematization of music, initially towards the theoretical direction and later on into the practical one – of composition – started a long time ago.

In the last century, the mathematical means were initially used in the musical research of works of art (already elaborated) and then they were involved in the creation process, the beginning in practice being made by the non-tuned serial technique of composition. As a consequence of the post serial composition school, it was later noticed a neo positive conception towards the appreciation of music, arrived from the field of exact science.

Music, has always interfered with the mathematical forms of thinking. However, about the mathematization of music as a science, respectively the mathematical musical linguistics, was not spoken until the XXth century. The field in question studies the phenomena of the language of sound using mathematical means. The solving of artistic problems through mathematical means implies along with the study, work means that look into the elements of expression of music and research of the relationship between these elements. Through the mathematization of music, the composition elements and the relationships between them are encoded. Thus, the creation act has turned into a chain of mathematical operations using as a problematic: the theory of sets; the Fibonacci string or the golden section; automated processing of data; probabilistic calculus; combining analysis and others. The space of this material does not allow for a more detailed explanation of the mentioned means.

We mention that the principle of the golden section was utilized in other fields of the culture such as: architecture, plastic arts, poetic arts, ever since the old times, although it was later formulated (and transposed from distances into a string of numbers by Leonardo Fibonacci in the XIII th century). In music, in was found that, since a previous time of our era, people were attracted to songs based on a series of sounds from the Fibonacci string (proto pentatonic systems). The Hungarian researcher Erno Lendvai explained this fact through a logarithmic structure of cochlea (V.Giuleanu- Treaty on the theory of music) [2].

In the second half of the XXth century, using sinusoidal sounds artificially made in the acoustics laboratory, Herbert Eimert initiated the electronic music. The materialization of this technique was done through the use of electronic machines of emitting sounds, such as: the synthesizer, the trautoniom (that makes sounds with accurate frequencies, mathematically differentiated through smaller intervals than 1/3, ¼ sau 1/6 of tone). As a consequence of the appearance of new orientations, it was necessary to use a new musical grammar based on mathematical calculations. A series of composers such as: E. Varese, P. Boulez, K. Stockhausen, L. Berio, L. Nono, G. Ligeti, appealing in their creation to the electronic concept – expressed through combinations with traditional instruments or exclusively electronic – had as organizing principles the laws of partial and integral serials, as well as other procedures pure mathematical. The currents of the electro-acoustic music previously mentioned (of the concrete creation and electronic) as well as the random ones, are characterized through freedom, detrimental to a strict organization of the acoustics. This determined the musicologists of the time to affirm the need to organize the time and space of sound (C. Stockhausen) [4].

In an attempt to realize musical structure rigorously calculated, in the seventh decade of the XXth century, the musician an mathematician Yannis Xenakis, presented in the work " Formal musics" a series of work methods made on mathematical principles. This, he formulated the *stochastic* music based on the mathematical calculation of probability, the *symbolic* music with the implication of mathematical logics, the *strategic* music based on the theory of games. In fact, *the mathematization of music* appeared as a result of the search opened by integral serialism and electronic music. All these modern directions of creation, with an appeal to mathematical procedures of organizing the musical language, showed interest for a series of Romanian compositors at the end of the XXth century, as well as at the beginning of the XXI st century, of whom we mention: Anatol Vieru, Alexandru Paşcanu, Liviu Glodeanu, Mihai Brediceanu, Ştefan Niculescu, Adrian Iorgulescu, Liana Alexandru, Şerban Nichifor, Horaţiu Rădulescu, Mihnea Brumariu ş.a.

Regarding the electronic computer and the automated processing of data, of course the advantages are both in the musical analysis and in composition. The computer allows structural and statistic analysis: classifications, stylistic analysis; transcriptions of Listen and other old noting systems. In composition, the machine generates all the combinations of heights and accords. The composition may be done assisted by the computer or realized by it on the basis of some rules characteristic to a gender, style, compositor. Of course, the perspectives given by the computer are enormous, however, in the musical creation it is important that the language maintains the semantic content, and all the means be in the service of the artistic expression, in order to transmit a rich emotional content. For this reason, the user of the computer in musical opera has to have experience, artistic sensitivity, talent.

Through the use in music of electronics and computers, the relationships belonging to the three fields – musical art, science, technology – get in their manifestation an inter disciplinary character, involving the sciences from which we started at in our presentation: mathematics, acoustic physics and at the same time music – science and art. Herbert Brun wrote in the paper "Technology and the compositor": "I imagine an edifice in which the arts, technology and sciences would meet on a ground that is common. All the components of

these fields research, create and exploit systems… And their goals come to complete each other's differences".

3. FINAL CONSIDERATIONS

Analyzing in a diachronic view the relationship of the field of exact science with the one of art and musical science, you can conclude that mathematics has had and has powerful implications in the direction of the theoretical sphere of the educational music, as well as in the application one of creation. Thus, all the subjects that study the musical field – regardless that the purpose in informative or formative – appeals in the study of music to rich mathematical data, more precise in the fields: Theory of music, Harmony, Counter point, Orchestration, Musical Forms. In the same measure, the creators from arts – architecture, plastic, music – appealed according to some artistic currents to a multitude of models and procedures specific mathematical. In fact, the preoccupations for the mathematization of the sciences and arts constituted a specific phenomenon for the end of the last century.

Over the ages, the involvement of mathematics was realized in different manners: from the setting of reports between sounds and numbers, through association of fractions to some intervals or accords in the theoretical fields; at the same relationships in interpretive practice (example – digitations); or in composition creative activity, through association of geometrical figures of some musical intervals and organization of the sound material trough mathematical means in serial, or appealing to different procedures that imply the theory of sets, the Fibonacci string (golden section), probabilistic calculus (in stochastic music), combinatory music. With the revolution of technique from the XXth century, it was intensively to the electronically music creation, realized through an automated processing of data, and in musicology, the computer allowed structural and statistic analysis of high performance.

The current study shows the role held – over the ages – by the exact sciences, in identifying, study and ordering of the music data and sustains the need for involving mathematical means, on theoretical – practical coordinates of the music phenomena.

Since in personal research I evaluated the involvement of music in the development of thought, memory and affectivity (see – doctorate thesis with the theme "music and superior psychic processes") [5], we propose for future research to include the analysis of the involvement of exact science in the forming and development of some cognitive processes. Also, we will be able to analyze the way in which, through the involvement of exact sciences in music, we may obtain efficiency in musical education, as well as an increased proficiency in the development of human personality obtained through inter relationships in education of the two sciences.

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