ORIGINAL PAPER

INTRODUCTION OF QUANTUM ENTANGLEMENT BY THE ESPECIALLY EXAMPLE

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Abstract. *We introduce and review the entanglement quantum. We will not attempt an in depth look at this approach as it would be impossible to treat it in such a short review. The emphasis is on understanding the vast meaning of the entanglement quantum. Also we showed the conception of entanglement by an example. Via this example we have clearly another conception such as pure state, mixed state and density matrix. Our motivation of this paper is to enable beginning students to start exploring the vast literature on this matter.*

Keywords: *entanglement quantum – pure state – mixed state – density matrix.*

1. INTRODUCTION

Entanglement was first used by Einstein, Podolski and Rosen (EPR) [1] to illustrate the conceptual differences between quantum and classical physics. In their seminal paper published in 1935, EPR argued that quantum mechanics is not a complete theory of nature, i.e. it does not include a full was not possible to ascribe definite element of reality. EPR defined an element of reality as a physical property, the value of which can be predicted with certainty before the actual property measurement. This condition is straightforwardly obeyed in the context of classical physics, but not in the context of quantum mechanics. The predictive Power of quantum mechanics is limited to, given a quantum state and an observable, the probabilities of the different measurement outcomes. This feature led EPR to deem quantum mechanics as incomplete. The incompleteness of quantum mechanics, as understood by EPR, was to plague physics for decades.

Quantum entanglement apparently leads to 'spooky' connections between subsystems that may be arbitrarily far apart in space. It is entanglement that forbids an explanation of the quantum randomness via hidden variables (Bell's inequalities [3]),[2] that allows some quantum algorithms to be much more efficient than their best classical counterparts (e.g. Shor's algorithm [4]), and that allows the possibility of quantum teleportation.

In the last 30 years, interest in quantum entanglement has risen sharply in various formerly disconnected scientific communities, bringing them together in unexpected ways. In the early 1980, the entanglement between quantum states with support both inside and outside of a black hole, arising for instance from particle pair creation near the event horizon, was suggested to be the basis for the properties of Hawking's radiation, in particular for the associated Beckenstein Hawking entropy. Technically, the idea is that in a pure, bi-partite

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state, an observer who can only measure one subsystem (e.g. outside the black hole) will perceive an effective mixed quantum state if there is entanglement with the rest of the system (e.g. inside the black hole). The corresponding entropy is the von Neumann entropy associated with the reduced density matrix – this is the entanglement entropy of a quantum subsystem. Although this idea does not provide the full explanation, it is nevertheless true that, like the Beckenstein-Hawking entropy, entanglement entropy, in many situation, grows like the area of the region separating the subsystems (in fact, it is certain quantum corrections to the black hole entropy that are given by the entanglement entropy). The idea that entanglement between subsystems of a pure state gives rise to effective mixed states is also used in the decoherence theory of quantum measurements.

Later on, in the 1990, the necessity of providing a quantitative measure of entanglement was understood in the science of quantum information, since, in this context, entanglement is an important resource. Although it is rather straightforward to determine whether entanglement between two subsystems exists, how do we quantify it? There are in fact many measures of entanglement that find applications in different situation.

2. CONCEPTION OF ENTANGLEMENT

We explains entanglement by below example. Two bothers and sisters named Ario (instead of Bob) and Utab (instead of Alis) are outside of earth into a ship. They are going to travel to Iran and United state. They know difference of time of these two countries is about 12 hours. That means if it is day time in Iran, that is night in United state and vice versa. We introduce dependent state ket to this system

$$|\psi\rangle = |\text{Iran day, USA night}\rangle - |\text{Iran night, USA day}\rangle \tag{1}$$

First Ario wants to go to Iran. It's probably clear if he faces to day time it will be $\frac{1}{2}$. Ario goes to Iran and gives message to Utab |Here is day>. At this moment Utab will get its night in United state and gives message |Here is night>. We call non-local sharing to this correlation. It's thoroughly clear that Ario Can't change night. So, it's impossible increasing or creat entanglement [5].

Now imagine another situation. Before Ario goes to Iran Utab goes to United state. Also he face to probably of $\frac{1}{2}$ to day time.

Now Ario is going to Iran, the probably that he face today time is equal to zero and face to night time is equal to one. So the probably of measurement on United state is effected on the probably measurement of Iran. We call entangled state to relation (1).

ENTANGLED STATE BY THE SYMMETRY

The system consist on two qubite. Those which have symmetry in arrangement of qubite are certainly entangled and those haven't Symmetry aren't entangled. Consider for example

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) \tag{2}$$

We can write

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) = |0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{3}$$

Then mixed state |ψ> is written like two separated qubits. Therefore |ψ> isn't entangled state. As first qubite is (|0>, |0>) and second qubite is (|0>, |1>) because they haven't symmetry aren't entangled. Pay attention to another example

$$|\psi> = \frac{1}{\sqrt{2}}(|01> -|10>) \qquad (4)$$

First qubite is (|0>, |1>) and second one is (|1>, |0>). As seen state of (4) is non factorizable so it's entangled state.

Conseqently whenever there is a symmetry among two qubits, that's entangled and very interesting matter is that without calculating can be reached to this result.

PURE STATE AND MIXED STATE:

When ever we know the physical state of an object, the object is said to be in a pure state. Suppose that the electron is in a pure state entering the apparatus, i.e. it's state is known to be

$$\psi = C_1\alpha + C_2\beta \qquad (5)$$

Where, for simplicity, we disregard the x, y, z degrees of freedom and concentrate on the spin. Then, if the detector haz been switched on, but before we look inside the black box, the electron exiting the detector must be either in state α, with probability $|C_1|^2$ or else in state β, with probability $|C_2|^2$.

This is an example of a mixed state or mixture. In general, if the state of an object is not know with certainly, but it is known that the object is in one of a number of possible state, together with probability of being in each state, then the object is said to be in a mixed state.

Ario, who was set up this experiment, has left the lab for the day. Utab, knowing Arios forgetful nature, goes to check that the detector inside the black box has been switched off, thereby conserving the very expensive electronics inside. To her consternation, she discovers that the box has already been locked by the janitor, who has the only key. Can she tell, without opening the box, whether the detector inside the box is on of off?

It is always possible to distinguish between a pure state and a mixture. Suppose Utab measures, on the beam of electrons emerging from the box, the value of the observable O. If the detector is switched off, then the particles emerging from the detector remaining the initial state ψ, so that

$$< O >_{pure} = <\psi|O|\psi> = |c_1|^2<\alpha|O|\alpha> + |c_2|^2<\beta|O|\beta> + C_1{}^*C_2<\alpha|O|\beta> + C_2{}^*C_1<\beta|O|\alpha> \qquad (6)$$

$$< O >_{mix} = \text{Prob. to find spin up} \times <\alpha|O|\alpha> + \text{Prob. to find spin down} \times = <\beta|O|\beta> = |C_1|^2<\alpha|O|\alpha> + |C_2|^2<\beta|O|\beta> \qquad (7)$$

The difference between the pure and mixed state result is called the interference term

$$< O >_{int} = < O >_{pure} - < O >_{mix} = C_1{}^*C_2<\alpha|O|\beta> + C_2{}^*C_1<\beta|O|\alpha> \qquad (8)$$

DENSITY MATRIX

In modern quantum mechanics, the density matrix or density operator is an essential tool for describing any quantum system.

A density matrix holds almost all the information about the observables of a system.

In the Dirac notation for quantum mechanics, it is natural to think of ket vectors as position of quantum mechanical states in a Hilbert space and that Bra vectors are a method of defining a basis in which to view the Hilbert space of states. When the density operator acts on the state vector (Ket) of a system it gives us an eigenstate of the system.

In a system with a state vector, we can define the density operatore for the system by the outer product $\rho(t) = |\psi, t\rangle\langle\psi, t|$

With ψ as the time-dependent wave function describing the system, We may notice the explicit time dependence of the operator. It is clear that any 1system may evolve in time, as the density operator contains information about the observables of a system it will be of the density operator–such equations are known as the master Equations of the system.

A density matrix or density operator for the ensemble of $|\psi\rangle$ is defined as

$$\rho = \Sigma_A |\psi_A\rangle\, p_A \langle\psi_A|$$

Such that

$$(1) \qquad \mathrm{Tr}(\rho) = 1,$$

for

$$\mathrm{Tr}(\rho) = \Sigma_n\langle n|\rho|n\rangle = \Sigma_n\langle n|\, \Sigma_A\{|\psi_A\rangle P_A\langle\psi_A|\}\, |n\rangle = \Sigma_n\, \Sigma_A\langle n|\psi_A\rangle P_A\langle\psi_A|n\rangle = \Sigma_A\, P_A\langle\psi_A|\psi_A\rangle = \Sigma_A\, P_A = 1 \qquad (9)$$

The final equality follows by imposing the normalization condition $\langle\psi_A|\psi_A\rangle = 1$

(2) ρ is positive semi-definite for any state $|A\rangle$, $\langle A|\rho|A\rangle = \Sigma_A P_A\, |\langle A|\psi_A\rangle|^2 \geq 0$

(3) If the ensemble of $|\psi_A\rangle$ has only one member, then $\rho = |\psi_A\rangle\langle\psi_A|$ is a pure state, with p being the probabilitistic weight of the ath state. Noting that the density matrix ρ is Hermitian, it can always be written as $\rho = \Sigma_i \lambda_i\ |i\rangle\langle i|$ Where λ are the eigen values of the density matrix and the $|i\rangle$ are number states. This describes a coherent quantum superposition of pure states.

The fact that ρ is Hermitian ensures that the eigen values are real and, hence, that the above statement is physically meaningful. The diagonalized matrix can then be given the standard interpretation with each eigen value being associated with the probability amplitude of the state with which it is linked (note that all the probabilities add up to1 and $\mathrm{Tr}\{\rho\} = 1$).

Importantly, when a measurement of a quantum system described by a density matrix ρ is performed, the expectation value of the observable is $\langle\hat{A}\rangle = \mathrm{Tr}(\hat{A}\rho)$ Indeed, as we expect to place a probabilistic physical interpretation on the density matrix, the following is also true $\mathrm{Tr}\{\rho^2\} \leq 1$ With equality only for a pure state. The expectation value of a quantum-mechanical operator is given by probabilistic average over the specific likelihood of the allowed states (p(a)). Hence, $\langle\hat{O}\rangle = \Sigma_A p_A \langle\psi_A|(\hat{O})|\psi_A\rangle$ Here, we can define the density operator to be $\rho = \Sigma_A P_A |\psi_A\rangle\langle\psi_A|$ Thus $\mathrm{Tr}(\rho^2) = \Sigma_{A,B} P_A P_B |\langle\psi_A|\psi_B\rangle|^2$ and since $|\langle\psi_A|\psi_B\rangle|^2 \leq 1$ and $\Sigma_B P_B = 1$ implying that $\Sigma_B P_B |\langle\psi_A|\psi_B\rangle|^2 \leq 1$, then $\mathrm{Tr}(\rho)^2 \leq \Sigma_A P_A = 1$.

CONTINUE OF DISCUSSION

Suppose that we have two particles with spin of $\frac{1}{2}$ and state of them

$$|\psi\rangle_{AB} = a\,|11\rangle + b\,|10\rangle + c\,|01\rangle + d\,|00\rangle \tag{10}$$

We ask what’s state of A?

That’s right that both of them are in one specified state, but cannot attribute specified state vector to A particle. In this case and all similar cases our quantum system is part of one bigger system and it’s state is defined by one density matrix. Generally assume that one system is consisted on two parts A and B. According to principles quantum system is attributed to this system Hilbert space H = $H_A \Box H_B$.

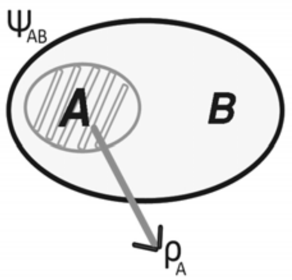

Fig. 1. This figure contain of two parts A and B. We ask what is state of A? state of A to be obtain by the trace of general density matrix of AB. $\rho_A = tr_B$ **(|ψ> <ψ|)**

Come back to our first example:

Ario and Utab know out of earth if Iran is day it’s night in united state and vice versa.

That means:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{Iran day, USA night}\rangle - |\text{Iran night, USA day}\rangle) \tag{11}$$

We know (11) is a pure state. Indeed they have enough information about whole system. But not only they don’t know if it’s day in Iran or night and but also for United state either. The density matrix of them information is

ρ = (|Iran day , USA night> - |Iran night , USA day>) (<USA night , Iran day| - <USA day , Iran night|) = |Iran day , USA night> <USA night , Iran day| - |Iran day , USA night> <USA day , Iran night| - |Iran night , USA day> <USA night , Iran day| + |Iran night , USA day> <USA day , Iran night| (12)

Up to now their information was about whole systems, now they want to say their information about part of system. For examples about measurement of Iran

$$\rho_{Iran} = \frac{1}{2}|\text{Iran day}\rangle\langle\text{Iran day}| + \frac{1}{2}|\text{Iran night}\rangle\langle\text{Iran night}| \tag{13}$$

His (her) information about Iran (part of system) is not exact. He (she) knows its probably day time in Iran 50% and 50% night there. This is just observer’s information but it’s not done any measurement yet. Now, measurement is doing

$$\rho_{Iran}\,|\text{Iran day}\rangle = \frac{1}{2}\,|\text{Iran day}\rangle \quad (14)$$

$$\rho_{Iran}\,|\text{Iran night}\rangle = \frac{1}{2}\,|\text{Iran night}\rangle \quad (15)$$

Information about whole is more careful from detail. To exact information about details, Ario goes to Iran and Utab to United state information which are exchanged by them will be entangled.

CONCLUSIONS

In this paper we showed that the conception of entanglement by the especially example. According to the our opinion to read this paper enable for student is effective.

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