*ORIGINAL PAPER*

# ANOTHER LOOK AT THE TIME AS GEOMETRICAL PARAMETER IN THE KINEMATICS

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***Abstract.*** *In this paper we showed the contribution of time in kinematics by the different examples. Via these examples we have clearly showed that release time can be used as a geometrical parameter. We tried to select examples so that they would be more creative.*

***Keywords:*** *Kinematics – geometrical parameter – release time.*

## 1. INTRODUCTION

Our original motivation was to offer kinematics problems with particular solution so that they would be contained by geometrical conceptions. Some problems of mathematics especially geometry can be analysis by the physical conceptions and vice versa. For example a good collection of elementary mathematical theorems proven by novel mechanical methods by Uspenskii is found in [1].We know that kinematics is famous with freefall motion [2] then we tried to select vary examples in subject. Each of the examples is completely new and creative. Studying each of examples can require motivation of designing other problems.

One of the familiar examples is equi-time surface in kinematics. Consider the following ring so that it has different chords and all of these chords are equi-time directions for the bodies were released from at A.

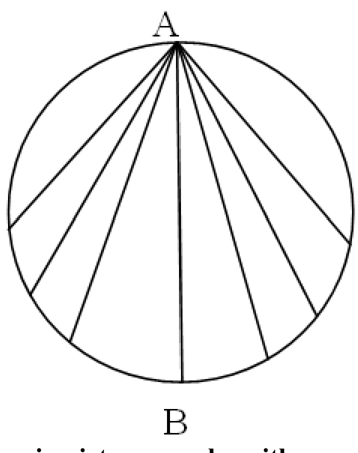


**Fig. 1. All of the routs that are shown in picture pass by with a same time by a body that is fallen down from A.**

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The gravity acceleration, g, is along of AB. So acceleration of other routs is component of g. Each of the routs close to AB, it has bigger acceleration and the length of path is bigger too. In general, the release time of all of the routs is $\sqrt{\frac{2AB}{g}}$ that $g$ is gravity acceleration.

Now we start our discussion by some examples.

## 2. METHODS

*EXAMPLE I: TIME AS GEOMETRICAL MEDIUM*

From the top of a tower, a body is thrown up. It reaches ground in $t_1$ s. A second body is thrown down with the same speed reaches the ground in $t_2$ s. A third body released from rest reaches the ground in $t$ s. then we can conclude $t$ is geometrical medium between $t_1$ and $t_2$.

When body is thrown vertically upwards from the top of tower of height $h$, then

$$h = \frac{1}{2}\, g t_1^2 - v_0 t_1 \tag{1}$$

where $v_0$ is initial speed of the body. When body is thrown vertically downwards from the top of tower, then

$$h = \frac{1}{2}\, g\, t_2^2 + v_0\, t_2 \tag{2}$$

When body is released from the top of tower, then

$$h = \frac{1}{2}\, g\, t^2 \tag{3}$$

From equation (1), we get

$$\frac{h}{t_1} = \frac{1}{2}\, g\, t_1 - v_0 \tag{4}$$

From equation (2), we get

$$\frac{h}{t_2} = \frac{1}{2}\, g t_2 + v_0 \tag{5}$$

Adding equations (4) and (5), we get

$$t = \sqrt{t_1 t_2} \tag{6}$$

*EXAMPLE II: THE RELEASE TIME IN RIGHT TRIANGLE*

In the right triangle below a bullet releases from A and passes through AC direction and another bullet with the similar specs release from the same location and passes through hypotenuse AD. Now we write the equations of both passes:

$$AC = \frac{1}{2}\, g\, cos\alpha\; t_1^2 \tag{7}$$

$$AD = \frac{1}{2}\, g \cos(\alpha+\beta) t_2^2 \tag{8}$$

By rewriting modified above equations

$$AB = \frac{1}{2}\, g \cos^2\alpha \; t_1^2 \tag{9}$$

$$AB = \frac{1}{2}\, g \cos^2(\alpha+\beta) t_2^2 \tag{10}$$

With mixing these two equations we can write:

$$\frac{t_1}{t_2} = \frac{AC}{AD} \tag{11}$$

Equation (11) shows that the ratio between time of releasing in both different paths is equal with the length of the routs.

According to the equality of speed in the points B and C we can write:

$$\frac{BC}{CD} = \frac{t_{BC}}{t_{DC}} \tag{12}$$

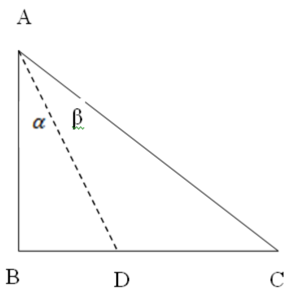


**Fig. 2. The time of releasing in the AC route is $t_1^s$ and in the AD route is $t_2^s$ . The angle between AD and AB is α , the angle between AC and AB is (α+β).**

*EXAMPLE III: THE RELEASE TIME ON THE RIGHT TRIANGLE3-4-5*

Consider an arbitrary right triangle and suppose two bullets so that released from A and pass through AC and AB. The angle between AB and AC is α. According to the following figure AC is hypotenuse of the triangle. We assume that ideally the triangle's right corner has an infinite small curve so that after collide bullet with horizontally side, it can continue its way.

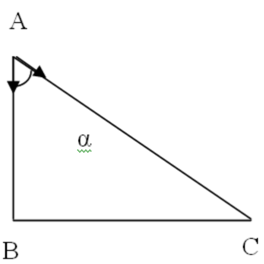


**Fig. 3. The bullet that is released from A and has passes through the ABC route, passes AB's route in $t_1^s$ and BC's route in $t_2^s$ and another bullet passes AC hypotenuse in $t^s$.**

We consider release time of AC is $t$ and AB's is $t_1$ and BC's is $t_2$.
So we have:

$$AC = \frac{1}{2} g \cos\alpha\, t^2 \tag{13}$$

$$AB = \frac{1}{2} g\, t_1^2 \tag{14}$$

$$BC = g\, t_1 t_2 \tag{15}$$

From equations (7) and (8), we get

$$t = \frac{t_1}{\cos\alpha} \tag{16}$$

From equations (7) and (9), we get

$$t_2 = \frac{\sin\alpha}{2}\, t \tag{17}$$

Then we distinguish the value of$t_1 + t_2$:

$$t_1 + t_2 = \left(\cos\alpha + \frac{\sin\alpha}{2}\right) t$$

or

$$t = \frac{t_1 + t_2}{\cos\alpha + \frac{\sin\alpha}{2}} \tag{18}$$

If $\cos\alpha + \frac{\sin\alpha}{2} = 1$, can be conclude$t = t_1 + t_2$. Then this condition is right if $\alpha = 53^o$ and our triangle is the right triangle 3-4-5.

Now a question may be asked that are there any other routes in triangle that their time is same as $t$ or $t_1 + t_2$.We analysis this problem with the proof by contradiction.

We think of an ADC route that AD's time is $t_1'$ and DC's time is $t_2'$ and $t_1' + t_2' = t$. Since $V_B = V_D$, time in the DC route is same for the both bullets that pass through ADC and ABD. So needful we have:

$$t_1 + (t_2 - t_2') = t_1' \tag{19}$$

It means that there is another right triangle 3-4-5 that and this is impossible. Therefore there isn't any other route.

*EXAMPLE IV: THE MINIMUM TIME OF RELEASING IN THE RIGHT TRIANGLE*

We consider a right triangle ABC, So that the angle of top of triangle is α. We want to find a route with minimum releasing time from A to C.

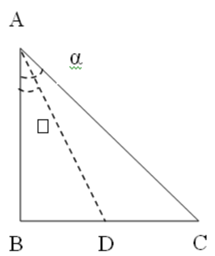


**Fig. 4. In this figure we assume that the route of ADC contains of minimum release time. AD make angle of ϴ with AB and α is the angle of top of the triangle.**

At first we calculate the releasing time of route of ADC that according to our assumption this path has the minimum releasing time. By the help of example II, we obtain the releasing time of AD as following

$$t_{AD} = \frac{t_1}{\cos\theta} \tag{20}$$

That $t_1$ is the releasing time of the route AB. Also by the example III, we obtain the time of passing BC and BD so that

$$t_{BC} = \frac{1}{2}t_1 \tan\alpha \tag{21}$$

$$t_{BD} = \frac{1}{2}t_1 \tan\theta \tag{22}$$

By the above equations we have

$$t_{DC} = \frac{1}{2}t_1 \tan\alpha - \frac{1}{2}t_1 \tan\theta \tag{23}$$

$$t_{ADC} = \frac{t_1}{\cos\theta} + \frac{1}{2}(\tan\alpha - \tan\theta)t_1 \tag{24}$$

For the minimize time in relation (24), we have

$$\frac{d\,t_{ADC}}{d\,\theta} = 0 \tag{25}$$

Then $\theta = \frac{\pi}{6}$.

Here we have two different questions. One of them is that the angle of $\alpha = \frac{\pi}{6}$, what is the minimum release time on the triangle?

The relation (24) doesn't have any root in $0 \le \theta \le \frac{\pi}{6}$ then we needful calculate the value of $t_{ADC}$ in boundary conditions, namely at $\theta = 0$ and $\theta = \frac{\pi}{6}$.

At the $\theta = 0$, the relation (25) concludes

$$t_{ABC} = t_1 + \frac{1}{2}t_1 \tan\frac{\pi}{6} \tag{26}$$

And at the $\theta = \frac{\pi}{6}$, the relation (24) concludes

$$t_{AC} = \frac{t_1}{\cos\frac{\pi}{6}} \tag{27}$$

Clearly by the comparison between (26) and (27) we have

$$t_{AC} < t_{ABC} \tag{28}$$

Finally we investigate the second question. Suppose that the angle of α is smaller than $\frac{\pi}{6}$, what is the minimum release time on the triangle?

At the first, we assume that α is an arbitrary angle so that continually it's smaller than $\frac{\pi}{6}$. Then the relations of (26) and (27), changes as following

$$t_{ABC} = t_1 + \frac{1}{2}t_1 \tan\alpha \tag{29}$$

$$t_{AC} = \frac{t_1}{\cos\alpha} \tag{30}$$

Now we need to contrast between the magnitude of $t_{ABC}$ and $t_{AC}$. After this comparison we obtain $t_{AC} < t_{ABC}$.

Final result of above discussion is that for $\alpha \leq \frac{\pi}{6}$ continually the minimum release time is hypotenuse of right triangle.

## CONCLUSIONS

Based on these examples and related discussions we showed that indeed the release time could have important role as geometrical parameter.

As a proposal the reader of our paper can follow the physical interpretations of obtained results.

## REFERENCES

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[2] Resnik, R., Halliday, D., Krane, K. S., *Physics*, Vol. 1, 4th Ed., Wiley, New York, 1992.