ORIGINAL PAPER

ANCIENT INDIAN SUBCONTINENT & MATHEMATICAL ADVANCEMENTS

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Abstract. *This paper deals with the advancements of Mathematics achieved by Indian Civilization in ancient period. It is noteworthy that many branches of mathematics were parallelly developed. Their applications in many areas highlight the profound understanding of the subject developed.*

Keywords: *History, Indian Civilization, Applications.*

1. INTRODUCTION

The word Indian Subcontinent, in general, geographically means the territory over which ancient united India was spread. As per modern state, it would cover: India, Pakistan, Bangladesh, Nepal, and few more neighbouring countries. The exact territorial extent of this word is debatable as there are differences of opinions amongst Historians; but all of them agree on one thing that Indian Subcontinent surely means what can be called as the ancient 'Greater India'.

From modern geographical point of view, one can resort to the terminology of *South Asia* to be quite synonymous with Indian Subcontinent. But since the research area of this paper focuses on development of Mathematics which has more to do with contribution of the culture as a whole, we would remain stuck to terminology of Indian Subcontinent.

The subject of Mathematics has been studied by all of the world's most advanced ancient civilizations like Sumerian, Babylonian, Egyptian, Greek, Chinese and Indian Civilization is no exception to that. Mathematics has been quite advanced in ancient India.

2. INDIAN NUMERAL SYSTEM

The human development in understanding the world around in a better way started with comparison and then consequently counting. Needless to say, positive integers were the first tools used by human race in this endeavor.

The two simplest yet all-time greatest mathematical contributions of Indian civilization to the world are the invention of zero as a number and the property-rich decimal numeral system.

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The Indian number system has multiple advantages over most of the other systems [3] like:

- ✓ Natural base 10
- ✓ Use of zero
- ✓ Limited number of symbols for digits, viz., ten
- ✓ Place value characters
- ✓ Positional characters
- ✓ Unidirectionality (Left to right)
- ✓ Capacity to represent numbers as large as needed without limit
- ✓ Availability of decimal point to accommodate rationals and irrationals

3. ARITHMETIC & THE LARGE NUMBERS

The greed for consideration of larger and larger positive integers can never end as they are infinite in number as we know very well today. But efforts of ancient cultures to consider very high magnitude numbers and name them properly have been marvelous. Indians stand in forefront in this domain also.

Arithmetic was quite rich in its development in India. There are *Shlokas* in classic Sanskrit literature like *Védas* which talk about very large numbers, particularly in powers of 10 with precise names to them. A table follows:

Table 1: Large Numbers with Names in One System

Sr. No.	Number Name	Number Value	Number as Ten Power	Modern International English Name
1	Dasha	10	10^1	Ten
2	Śata	100	10^2	Hundred
3	Sahasra	1,000	10^3	Thousand
4	Ayuta	10,000	10^4	Ten Thousand
5	Niyuta	100,000	10^5	Lakh
6	Prayuta	1,000,000	10^6	Ten Lakh / Million
7	Arbuda	10,000,000	10^7	Crore
8	Nyarbuda	100,000,000	10^8	Ten Crore
9	Samudra	1,000,000,000	10^9	Hundred Crore / Billion
10	Madhya	10,000,000,000	10^{10}	Ten Billion
11	Anta	100,000,000,000	10^{11}	Hundred Billion
12	Parārdha	1,000,000,000,000	10^{12}	Trillion

This is example of one naming system of large numbers. There have been various other systems. Many times we see conflicts between them as same name has been used to denote different numbers in different systems. But then this is drawback not limited to ancient Indian measurement nomenclature only. The well recognized British nomenclature of numbers, particularly large numbers, has clear differences in old classic and modern versions. Like a billion used to stand for 10^{12} in old British system which now is 10^9, a trillion meant 10^{18} in old fashion but now is taken to be 10^{12}. Also in ancient Greek alphabetical numeral system, the use of myriad symbol M was with two different meanings. β stood for 2 in that system and M for 10000. But $\overset{\beta}{M}$ meant sometimes $2 \times 10000 = 20000$ and at other times $10000^2 = 100000000$.

There are tens of such conflicting examples there also, but we have mentioned only those which are well known due to frequent appearance in routine use.

What has fascinated historians as well as mathematicians both alike is that if Indians invented numbers as large as 10^{12} and beyond, they must have felt the need of using them and any culture that is in requirement of such huge numbers must have been advanced on many fronts. Indians had done a deep study of astronomy where they used these huge numbers to estimate the distances between celestial objects.

Numbers very large in magnitude were classified as enumerable, innumerable, and infinite which were further classified in subcategories.

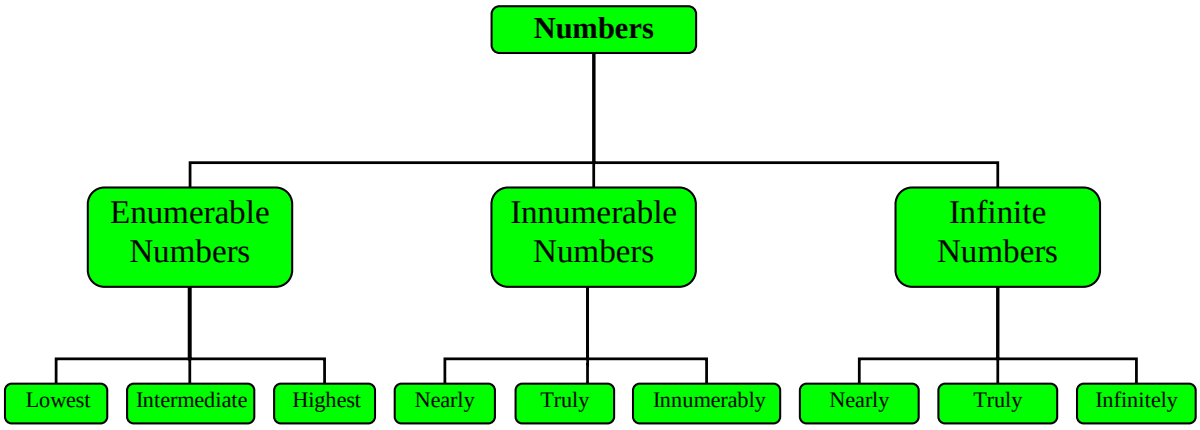

Fig. 1. Types of Numbers Identified and Classified by Indians.

For the infinite numbers, two categories were clearly distinguished, viz., *asaṃkhyāta* and *ananta* to mean countless and endless, respectively. Such a remarkable treatment of infinities is not found in any culture.

4. ŚULBA SŪTRAS

The *Śulba sutras* are regarded as one of the appendices to the great *Védas*. They are ancient with their origin in Vedic period. As such they are directly concerned with performing rituals and prima-facie talk about construction of various fire-altars [4]. Choice of different geometrical shapes of fire-altars was directly associated with the desire of concerned gift from the Gods.

In addition to many interesting properties of geometrical figures, the *Śulba sutras* also give some Pythagorean triplets and imply inherent study of irrational numbers up to certain degree of precision.

Many historians-cum-mathematicians have critically pointed out that there needs to be an in-depth study of these *Śulba sutras* to further explore their implications.

5. MATHEMATICS FOR AND THROUGH OTHER DISCIPLINES

The specialty of ancient Indian culture lies in the fact that it has applied mathematics and science after inventing them. This comes in different ways.

5.1. GRAMMAR

Today it is well recognized that grammar of any language – let it be any natural human language or a technical machine language like computer programming language – is purely a logical object. Indians have Sanskrit as their foremost language. It is classically called the divine language of Gods. Syntactically, logically, grammatically it is so perfect a language that even the Computer Scientists have never stopped praising it like anything. In fact, Sanskrit is considered as the future medium of best human-machine interface.

This ancient classical language Sanskrit of Indian culture is rich in many mathematical features like its logic, context free grammar, in particular Boolean logic, null operator, etc. [2]

A context-free grammar is simple and mathematically precise mechanism which gives methods by which phrases in language are built from smaller blocks, capturing the "block structure" of sentences in a natural way. It's where formalism is applied which is base for mathematical study.

This wouldn't have been possible without implicit and explicit knowledge and application of mathematical tools.

5.2. MUSIC

Another specialty of Indian culture is of rich musical tradition that it not found in any other contemporary civilizations, however advanced they might have been as per claims of western historians. Amongst the four *Védas*, the *Samvéda* specially deals with music [5]. *Nātya Śāstra* further contributed to its development. Pingala is considered to be musical theorist whose *Chhandas Śāstra* or *Chhandaḥ-sūtra* is notable work on music. All these have been with concrete mathematical base. The syllabic combinations in any musical note are purely mathematical construction. Importantly, this was fully realized by Indians in those days and mathematical techniques like modeling were induced for formation of different *Rāgas*.

5.3. MEMORIZATION TECHNIQUES

There are many things for which ancient Indian culture is well-known, of which one such is memorization. The rich cultural heritage of literature was preserved through *shruti* and *smruti*. *Shruti* means listening and *smruti* means memorizing. When the art of writing was yet not invented, gigantic text treatises like *Védas*, *Upanishadas*, *Puranas* etc. were transmitted from generations to generations through cycles of *shruti* and *smruti*, respectively. The thousands of verses in hundreds of epics couldn't have been remembered without precise mathematical techniques for memorization which were very profoundly developed. The

rhythms in *shlokas*, recitations of *mantras*, enchanting intonations were all inherently mathematical in nature. They are today studied as smart application techniques of mathematics with modern memorization methods heavily relying on these as base.

Today it has been confirmed beyond doubt that the development and application of mathematics, that the ancient Indian culture has been involved in, was unparalleled. What completeness means was very clear to ancient Indian philosophers-cum-mathematicians as reflected in brilliant verse [1]:

ॐ पूर्णमदः पूर्णमिदं पूर्णात्पूर्णमुदच्यते ।
पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते ॥

This inspires and is inspired by the concepts of two of the fundamental mathematical constants, viz., zero and infinity, of which the later one still continues to enjoy mathematically a mysterious status; the former one having no less potential.

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