ORIGINAL PAPER

UNSTEADY MHD FREE CONVECTION VISCOUS DISSIPATIVE FLOW PAST AN INFINITE VERTICAL PLATE WITH CONSTANT SUCTION AND HEAT SOURCE/SINK

MURALI GUNDAGANI[1]

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Abstract. *The objective of this study is to find numerical solution of unsteady magnetohydrodynamic free convective flow of viscous incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and heat source/sink has been made. The problem is governed by coupled non – linear partial differential equations. Dimensionless equations of the problem have been solved numerically by finite element method. The effects of flow parameters, viz. thermal Grashof number, Hartmann number, Prandtl number, Heat source/sink parameter and Eckert number on velocity and temperature fields are investigated through graphs. Skin – friction and Rate of heat transfer coefficients are derived, discussed numerically and presented in tabular forms.*

Keywords: *MHD, Free convection, Heat source/sink, Constant suction, viscous dissipation, Finite element method.*

1. INTRODUCTION

Free convection flow involving coupled heat and mass transfer occurs frequently in nature and in industrial processes. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution. Hydromagnetic flows and heat transfer have become more important in recent years because of its varied applications in agricultural engineering and petroleum industries. Recently, considerable attention has also been focused on new applications of magnetohydrodynamics (MHD) and heat transfer such as metallurgical processing. Melt refining involves magnetic field applications to control excessive heat transfer rate. Other applications of MHD heat transfer include MHD generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized – geothermal energy systems. Pop and Soundalgekar [1] have investigated the free convection flow past an accelerated infinite plate. Singh and Soundalgekar [2] have investigated the problem of transient free convection in cold water past an infinite vertical porous plate. An excellent summary of applications can be found in Hughes and Young [3]. Takar et al. [4] analyzed the radiation effects on MHD free convection flow past a semi – infinite vertical plate using Runge – Kutta – Merson quadrature. The natural convection flow of a conducting visco – elastic liquid between two

[1] Gandhi Institute of Technology, Basic Sciences Department, Rangareddy District, Telanagana, India.
E-mail: saihanuman79@yahoo.com.

heated vertical plates under the influence of transverse magnetic field has been studied by Sreehari Reddy et al. [5].

In all these investigations, the viscous dissipation is neglected. The viscous dissipation heat in the natural convective flow is important, when the flow field is of extreme size or at low temperature or in high gravitational field. Such effects are also important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Whenever the temperature of surrounding fluid is high, the radiation effects play an important role and this situation does exist in space technology. In such cases one has to take into account the effects of radiation and free convection. A number of authors have considered viscous heating effects on Newtonian flows. Israel – Cookey et al. [6] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Zueco Jordan [7] used network simulation method (NSM) to study the effects of viscous dissipation and radiation on unsteady MHD free convection flow past a vertical porous plate. Recently Suneetha et al. [8] studied the effects of thermal radiation on the natural convective heat and mass transfer of a viscous incompressible gray absorbing – emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Very recently Hitesh Kumar [9] has studied the boundary layer steady flow and radiative heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field.

The study of heat source/sink effects (generation/absorption) in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electric chips and semiconductor wafers. Patil and Kulkarni [10] studied the effects of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation. Radiation effects on an unsteady MHD convective heat and mass transfer flow past a semi – infinite vertical permeable moving plate embedded in a porous medium was studied by Ramachandra Prasad et al. [11]. Ramana Reddy et al. [12] have studied the mass transfer and radiation effects of unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption. In spite of all these studies, the unsteady MHD free convection heat and mass transfer flow past a moving semi – infinite titled porous plate with variable temperature in the presence of chemical reaction has received a little attention. MHD boundary layer flow over a heated stretching sheet with variable viscosity was studied by Samad et al. [13]. Satyanarayana and Venkataramana [14] studied Hall current effect on magneto hydrodynamics free convection flow past a semi – infinite vertical porous plate with mass transfer. Seddeek [15] studied the effects of chemical reaction, thermophoresis and variable viscosity on steady hydromagnetic flow with heat and mass transfer over a flat plate in the presence of heat generation/absorption. Sharma and Singh [16] investigates the flow of a viscous incompressible electrically conducting fluid along a porous vertical isothermal non – conducting plate with variable suction and exponentially decaying heat generation in the presence of transverse magnetic field. The governing equations of motion and energy are transformed into ordinary differential equations using time dependent similarity parameter. The ordinary differential equations are then solved numerically using Runge – Kutta method along with shooting technique. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana [17]. In spite of all these studies, the unsteady MHD free convection heat and mass transfer for a heat generating fluid with radiation absorption has received little attention. Venkateswalu et al. [18] deal with free convention flow along a vertical wall in a porous medium with magnetic field is considered. The horizontal walls are adiabatic. The magnetic field applied perpendicular to the rectangular channel. The flow problem axis described by means of parabolic partial differential equations and solutions are obtained by an implicit finite difference technique. Zueco [19] considered the transient magnetohydrodynamic natural convection boundary layer flow with suction, viscous dissipation and thermal radiation effects.

Hence, the main objective of the present investigation is to study the effect of Heat source/sink and viscous dissipation free convective fluid flow past a infinite vertical plate subjected to constant suction. The equations of continuity, linear momentum and energy, which govern the flow field, are solved numerically by using finite element method which is more economical from computational view point. The behavior of the velocity, temperature, skin – friction and Nusselt number has been discussed for variations in the governing parameters.

2. MATHEMATICAL FORMULATION

Let *x'* – axis be taken in the vertically upward direction along the infinite vertical plate and *y'* – axis normal to it. Neglecting the induced magnetic field and applying Boussinesq's approximation, the equation of the flow can be written as:

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v'_o \quad \text{(Constant)} \tag{1}$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + v\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o^2}{\rho}u' \tag{2}$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \kappa\frac{\partial^2 T'}{\partial y'^2} + \frac{v}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + S'(T' - T'_\infty) \tag{3}$$

The boundary conditions of the problem are:

$$\left.\begin{array}{l} t' \le 0: \;\; u' = 0,\; v' = 0,\;\; T' = T'_\infty \quad for\ all\ \ y' \\ t' > 0: \begin{cases} u' = 0,\, v' = -v'_o,\, T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{i\omega' t'}\ at\ \ y' = 0 \\ u' \to 0,\, T' \to T'_\infty\ \ as\ \ y' \to \infty \end{cases} \end{array}\right\} \tag{4}$$

Introducing the following non – dimensional variables and parameters,

$$\left.\begin{array}{l} y=\frac{y'v'_o}{v},\ t=\frac{t'v_o'^2}{4v},\ \omega=\frac{4v\omega'}{v_o'^2},\ u=\frac{u'}{v'_o},\ M=\left(\frac{\sigma B_o^2}{\rho}\right)\frac{v}{v_o'^2},\ T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ \Pr=\frac{v}{\kappa},Gr=\frac{vg\beta\left(T'-T'_\infty\right)}{v_o'^3}, \\ S=\frac{4S'v}{v_o'^2},\ Ec=\frac{v_o'^2}{C_p\left(T'_w-T'_\infty\right)} \end{array}\right\} \tag{5}$$

Substituting (5) in equations (2) and (3) under boundary conditions (4), we get:

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\left(Gr\right)T+\frac{\partial^2 u}{\partial y^2}-Mu \tag{6}$$

$$\frac{1}{4}\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}+\frac{1}{4}QT+(Ec)\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} u=0,\ T=1+\varepsilon e^{i\omega t}\quad at\ y=0 \\ u\to 0,\ T\to 0\qquad as\ y\to\infty \end{array}\right\} \tag{8}$$

3. METHOD OF SOLUTION

By applying Galerkin finite element method for equation (6) over the element (e), ($y_j \le y \le y_k$) is:

$$\int_{y_j}^{y_k}\left\{N^T\left[4\frac{\partial^2 u^{(e)}}{\partial y^2}-\frac{\partial u^{(e)}}{\partial t}+4\frac{\partial u^{(e)}}{\partial y}-4Mu^{(e)}+P\right]\right\}dy=0 \tag{9}$$

where $P=4(Gr)T$;

Integrating the first term in equation (9) by parts one obtains

$$N^{(e)T}\left\{4\frac{\partial u^{(e)}}{\partial y}\right\}_{y_j}^{y_k}-\int_{y_j}^{y_k}\left\{4\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}+N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t}-4\frac{\partial u^{(e)}}{\partial y}+4Mu^{(e)}-P\right)\right\}dy=0 \tag{10}$$

Neglecting the first term in equation (10), one gets:

$$\int_{y_j}^{y_k}\left\{4\frac{\partial N^{(e)^T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}+N^{(e)^T}\left(\frac{\partial u^{(e)}}{\partial t}-4\frac{\partial u^{(e)}}{\partial y}+4Mu^{(e)}-P\right)\right\}dy=0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ be the linear piecewise approximation solution over the element (e) ($y_j \le y \le y_k$), where $N^{(e)} = \begin{bmatrix} N_j & N_k \end{bmatrix}, \phi^{(e)} = \begin{bmatrix} u_j & u_k \end{bmatrix}^T$ and $N_j = \frac{y_k - y}{y_k - y_j}, N_k = \frac{y - y_j}{y_k - y_j}$ are the basis functions. One obtains:

$$\int_{y_j}^{y_k} \left\{ 4 \begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy + \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \right\} dy - 4 \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j' & N_j N_k' \\ N_j' N_k & N_k' N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy$$

$$+4M \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy = P \int_{y_j}^{y_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} dy$$

Simplifying we get

$$\frac{4}{l^{(e)^2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{1}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} - \frac{4}{2l^{(e)}} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{4M}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Where prime and dot denotes differentiation w.r.t 'y' and time 't' respectively. Assembling the element equations for two consecutive elements $y_{i-1} \le y \le y_i$ and $y_i \le y \le y_{i+1}$ following is obtained:

$$\frac{4}{l^{(e)^2}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} - \frac{4}{2l^{(e)}} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix}$$

$$+ \frac{4M}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{11}$$

Now put row corresponding to the node 'i' to zero, from equation (11) the difference schemes with $l^{(e)} = h$ is:

$$\frac{4}{h^2} \left[-u_{i-1} + 2u_i - u_{i+1} \right] + \frac{1}{6} \left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1} \right] - \frac{4}{2h} \left[-u_{i-1} + u_{i+1} \right] + \frac{4M}{6} \left[u_{i-1} + 4u_i + u_{i+1} \right] = P \tag{12}$$

Applying the trapezoidal rule, following system of equations in Crank – Nicholson method are obtained:

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^n + A_5 u_i^n + A_6 u_{i+1}^n + 12Pk \tag{13}$$

Now from equation (7) following equation is obtained:

$$G_1 T_{i-1}^{n+1} + G_2 T_i^{n+1} + G_3 T_{i+1}^{n+1} = G_4 T_{i-1}^{n} + G_5 T_i^{n} + G_6 T_{i+1}^{n} + 12Qk \tag{14}$$

where:
A_1 = 2 + 4Ak + 12rk – 24r; A_2 = 16Ak + 48r + 8; A_3 = 2 + 4Ak – 12rh – 24r;
A_4 = 2 – 4Ak – 12rh + 24r; A_5 = 8 – 16Ak – 48r; A_6 = 2 – 4Ak + 12rh + 24r;
G_1 = 2(Pr) + 12rh(Pr) – S(Pr)k – 24r; G_2 = 8(Pr) + 48r – 4S(Pr)k;
G_3 = 2(Pr) – 12hr(Pr) – 24r – S(Pr)k; G_4 = 2(Pr) – 12rh(Pr) + 24r + S(Pr)k;
G_5 = 8(Pr) – 48r + 4S(Pr)k; G_6 = 2(Pr) + 12rh(Pr) + 24r + S(Pr)k;

$$P = 4(Gr)T_i^j\,;\ Q = 4(\mathrm{Pr})(Ec)k\left(\frac{\partial u_i^j}{\partial y}\right)^2$$

Here $r = \frac{k}{h^2}$ and h, k are mesh sizes along y – direction and time – direction respectively. Index 'i' refers to space and 'j' refers to the time. In the equations (13) and (14) taking i = 1(1) n and using boundary conditions (8), then the following system of equations are obtained:

$$A_i X_i = B_i,\ i = 1(1)n \tag{15}$$

where A_i's are matrices of order n and X_i, B_i's are column matrices having n – components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity and temperature. Also, numerical solutions for these equations are obtained by C – programme. In order to prove the convergence and stability of Galerkin finite element method, the same C – programme was run with smaller values of h and k and no significant change was observed in the values of u and T. Hence the Galerkin finite element method is stable and convergent.

Skin – friction and Rate of heat transfer (Nusselt number):

The skin – friction and Rate of heat transfer (Nusselt number) are important physical parameters for this type of boundary layer flow.

The skin – friction at the plate, which in the non – dimensional form is given by

$$\tau = \frac{\tau'_w}{\rho v'_o v} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{16}$$

The rate of heat transfer coefficient, which in the non – dimensional form in terms of the Nusselt number is given by

$$Nu = -x\frac{\left(\frac{\partial T'}{\partial y'}\right)_{y'=0}}{T'_w - T'_\infty} \Rightarrow Nu\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \tag{17}$$

where $\mathrm{Re} = \frac{v_o' x}{\nu}$ is the local Reynolds number.

4. RESULTS AND DISCUSSION

The profiles of velocity and temperature are shown in the figs 1-3. Fig. 1 exhibits the effects of Hartmann number, Prandtl number and sink – strength on the velocity profiles with other parameters are fixed. The effect of the Hartmann number (M) is shown in fig. 1. It is observed that the velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the velocity as the Hartmann number (M) increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in fig. 1.

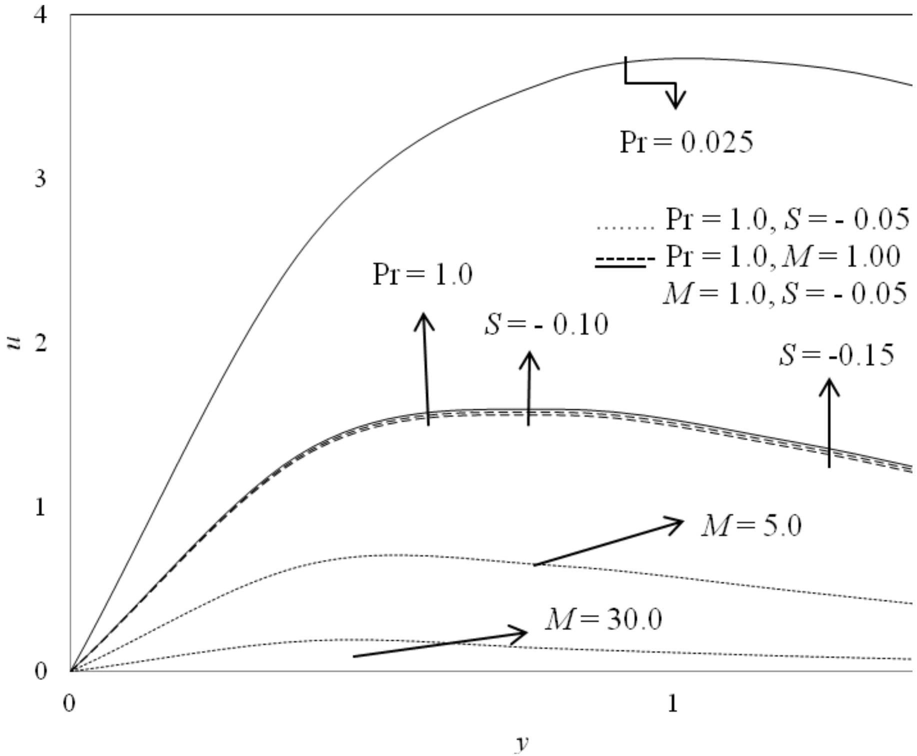

Figure 1. Effects of Pr, *M* and *S* on velocity for *Gr* = 5.0, *Ec* = 0.001, *ω* = 5.0, *ε* = 0.2 and *ωt* = π/2.

And from fig. 1, it is observed that the velocity is greater for mercury (P_r = 0.025) than that of electrolytic solution (P_r = 1.0). Also from fig. 1 shows the effect of heat sink (*S*) in the case of cooling plate (G_r>0), i.e., the free convection currents convey heat away from the plate into the boundary layer. With an increase in Sfrom -0.15 to -0.10 there is a clear increase in the velocity, i.e., the flow is accelerated. When heat is absorbed, the buoyancy force decreases, which retard the flow rate and thereby giving, rise to the increase in the velocity profiles.

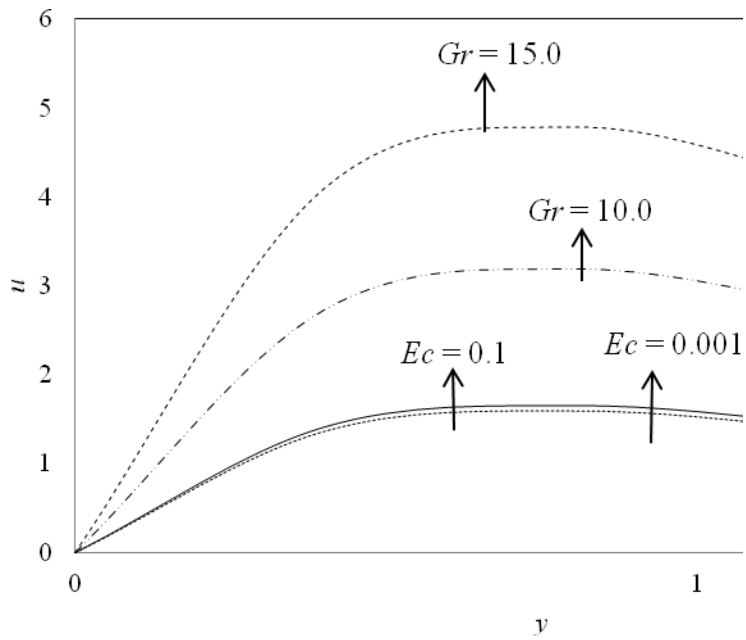

Figure 2. Effects of *Gr* and *Ec* on velocity for Pr = 1.0, *M* = 1.0, *ω* = 5.0, *ε* = 0.2 and *ωt* = *π*/2.

For various values of Grashof number (G_r) and Eckert number (E_c) the velocity profiles 'u' are plotted in fig. 2. The Grashof number G_r signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as G_r increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. Fig. 2 illustrates the effect of Eckert number E_c on velocity field under the influence of the Prandtl number P_r and the Hartmann number M. We observed an increase in Ec from 0.001 to 0.1 and that with P_r fixed either at 1.0 or at 0.025 increases in the velocity profile. However, a rise in E_c from 0.001 to 0.1 with constant P_r provides increase in the velocity profile. There is no back flow in the boundary layer for any combination of E_c and P_r.

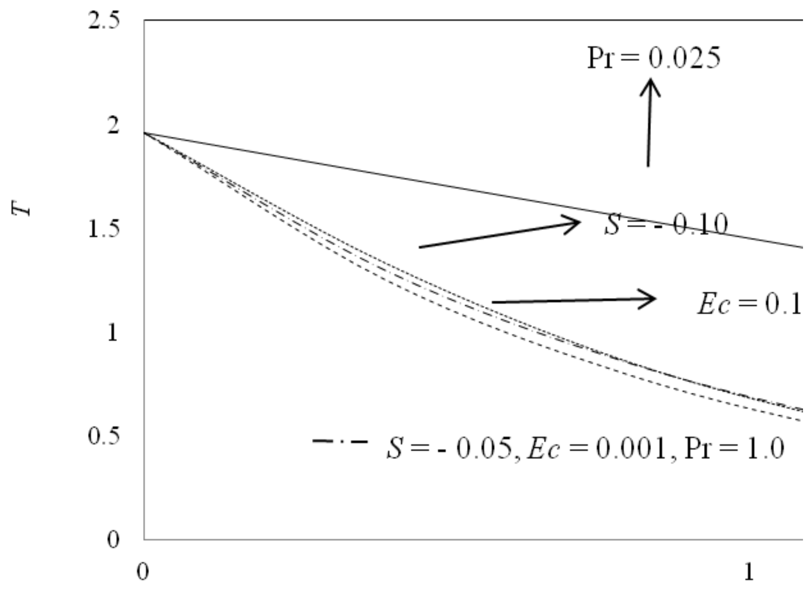

Figure 3. Effects of Pr, *Ec* and *S* on temperature for *Gr* = 5.0, *M* = 1.0, *ω* = 5.0, *ε* = 0.2 and *ωt* = *π*/2.

The temperature profiles Tare depicted in fig. 3 for different values of heat sink parameter S, Prandtl number P_r and the Eckert number E_c. The fluid temperature attains its maximum value at the plate surface, and decreases gradually to free stream zero value far away from the plate. It is seen that the fluid temperature increases with a rise in E_c. In the present study, we restrict our attention to the positive values of E_c, which corresponds to plate cooling, i.e., loss of heat from the plate to the fluid. Also, we note that increasing E_c causes an increase in Joule heating as the magnetic field adds energy to the fluid boundary layer due to the work done in dragging the fluid. Therefore, the fluid temperature is noticeably enhanced with an increase in S from -0.10 to -0.05. This increase in the temperature profiles is accompanied by the simultaneous increase in the thermal boundary layer thickness.

In fig. 3 we depict the effect of Prandtl number (P_r) on the temperature field. It is observed that an increase in the Prandtl number leads to decrease in the temperature field. Also, temperature field falls more rapidly for electrolytic solution and the temperature curve is exactly linear for mercury, which is more sensible towards change in temperature. From this observation it is conclude that mercury is most effective for maintaining temperature differences and can be used efficiently in the laboratory. Electrolytic solution can replace mercury, the effectiveness of maintaining temperature changes are much less than mercury. However, electrolytic solution can be better and cheap replacement for industrial purpose.This is because, either increase of kinematic viscosity or decrease of thermal conductivity leads to increase in the value of Prandtl number (P_r). Hence temperature decreases with increasing of Prandtl number (P_r).

Table 1. Values of skin friction (τ) for Gr = 5.0, Ec = 0.001, ω = 5.0, ε = 0.2 and $\omega t = \pi/2$

P_r	M	S	τ
Mercury (P_r = 0.025)	1.0	– 0.05	7.6548
	5.0	– 0.05	2.7271
	5.0	– 0.10	2.7045
Electrolytic Solution (P_r = 1.0)	1.0	– 0.05	3.0426
	5.0	– 0.05	1.6584
	5.0	– 0.10	1.5458

Table 2. Values of rate of heat transfer (Nu) for Gr = 5.0, Ec = 0.001, ω = 5.0, ε = 0.2 and $\omega t = \pi/2$

P_r	M	S	Nu
Mercury (P_r = 0.025)	1.0	– 0.05	– 0.0336
	5.0	– 0.05	– 0.0341
	5.0	– 0.10	– 0.0404
Electrolytic Solution (P_r = 1.0)	1.0	– 0.05	– 0.9330
	5.0	– 0.05	– 1.0089
	5.0	– 0.10	– 1.0209

The skin friction for mercury and electrolytic solution are given in the table 1. It is noticed that the increase in magnetic field strength decreases the skin friction for both mercury and electrolytic solution. The rate of heat transfer for mercury and electrolytic solution are given in the table 2. It is observed the rate of heat transfer decreases with increase in magnetic field strength or sink strength both mercury and electrolytic solution.

5. CONCLUSIONS

We summarize below the following results of physical interest on the velocity and temperature distribution of the flow field and also on the skin – friction and rate of heat transfer at the wall.

1. A growing Hartmann number or Prandtl number retards the velocity of the flow field at all points.
2. The effect of increasing Grashof number or heat source parameter or Eckert number is to accelerate velocity of the flow field at all points.
3. A growing Eckert number or heat source parameter increases temperature of the flow field at all points.
4. The Prandtl number decreases the temperature of the flow field at all points.
5. A growing Hartmann number decreases the skin friction while increasing the heat source parameter increases the skin friction for both mercury and electrolytic solution.
6. The rate of heat transfer for both mercury and electrolytic solution is decreasing with increasing of Hartmann number and increases with increasing of heat source parameter.

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