ORIGINAL PAPER

TEACHERS ASSESS STUDENT'S MATHEMATICAL CREATIVITY COMPETENCE IN HIGH SCHOOL

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Abstract. *Assessment is one of the most powerful educational tools for promoting effective learning. But it must be used in the right way. We need to be on helping teachers use assessment as part of teaching and learning, in ways that will raise pupil's achievement. This paper proposes some problems to assess student's mathematical creative competency in grade 10 in Vietnam.*

Keywords: *mathematics competency, creative competency, mathematics teaching methods.*

1. INTRODUCTION

Assessment is one of the most powerful educational tools for promoting effective learning. But it must be used in the right way. We need to be on helping teachers use assessment as part of teaching and learning, in ways that will raise pupils achievement.

Before teaching, the teachers need to determine educational aims and appropriate teaching methods for each pupil, Therefore, teachers need to have the initial assessment of the competence of each pupil about attitudes, ability to acquire knowledge, they use knowledge in practice, circumstances, etc., factors that impacts pupils educating process. Therefore, in teaching mathematics, the teachers should also have diagnostic competence, assessment competence of comprehensive. Currently assessment competency development of mathematics teachers are not good in Vietnam.

In this paper, we suggest some techniques for pupils process learning.

2. RESULTS AND DISCUSSION

We give out some examples in [3] that help pupils develope their creativity while teaching *quadratic equation* lesson in Mathematics grade 10 in Vietnam.

Given $f(x) = ax^2 + bx + c, a \neq 0, \Delta = b^2 - 4ac.$

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Proposition 1. Suppose x_1, x_2 are two solutions of $f(x) = 0.$ We get:

$$\begin{cases} f(x) = a(x - x_1)(x - x_2) \\ x_1 + x_2 = -\dfrac{b}{a} \\ x_1 x_2 = \dfrac{c}{a}. \end{cases}$$

Proposition 2. Suppose $f(x) \in R[x]$ and $\Delta = b^2 - 4ac.$ We get:

i) $f(x) > 0$ for all x if and only if { $a > 0$ and $\Delta < 0$ }.

ii) $f(x) \geq 0$ for all x if and only if { $a > 0$ and $\Delta \leq 0$ }.

iii) $f(x) < 0$ for all x if and only if { $a < 0$ and $\Delta < 0$ }.

iv) $f(x) \leq 0$ for all x if and only if { $a < 0$ and $\Delta \leq 0$ }.

v) $f(x) = 0$ have two solutions x_1, x_2 and real numbers $x_1 < \alpha < x_2$ if and only if $af(\alpha) < 0.$

Proposition 3. Suppose finite sequence of real numbers $(a_i),(b_i),(t_i)$ such that $0 < a \leq a_i \leq A, 0 < b \leq b_i \leq B$ and $t_i \geq 0$ where for all $i = 1, \ldots, n.$ We have

i) **[Polya]** $\dfrac{1}{4}(\sqrt{\dfrac{ab}{AB}} + \sqrt{\dfrac{AB}{ab}})^2 \geq \dfrac{\sum_{i=1}^{n} a_i^2 . \sum_{i=1}^{n} b_i^2}{(\sum_{i=1}^{n} a_i b_i)^2}.$

ii) **[Cantorovic]** $\dfrac{(a + A)^2}{4aA}(\sum_{i=1}^{n} t_i)^2 \geq (\sum_{i=1}^{n} t_i a_i)(\sum_{i=1}^{n} \dfrac{t_i}{a_i}).$

Proof. i) By $f(x) = x^2 - (\dfrac{b}{A} + \dfrac{B}{a})x + \dfrac{Bb}{Aa}$ have two solutions $\dfrac{b}{A}, \dfrac{B}{a}.$

Since $\dfrac{b}{A} \leq \dfrac{b_i}{a_i} \leq \dfrac{B}{a}$ we deduce the inequality $\dfrac{b_i^2}{a_i^2} - (\dfrac{b}{A} + \dfrac{B}{a})\dfrac{b_i}{a_i} + \dfrac{Bb}{Aa} \leq 0$ or $b_i^2 - (\dfrac{b}{A} + \dfrac{B}{a}) b_i a_i + \dfrac{Bb}{Aa} a_i^2 \leq 0$ where $i = 1, \ldots, n.$

Suming up we get:

$$(\frac{b}{A} + \frac{B}{a}) \sum_{i=1}^{n} b_i a_i \geq \sum_{i=1}^{n} b_i^2 + \frac{Bb}{Aa} \sum_{i=1}^{n} a_i^2 \geq 2\sqrt{(\sum_{i=1}^{n} b_i^2)(\frac{Bb}{Aa} \sum_{i=1}^{n} a_i^2)}.$$

Hence $\frac{1}{4}(\sqrt{\frac{ab}{AB}}+\sqrt{\frac{AB}{ab}})^2=\frac{1}{4}(\frac{b}{A}+\frac{B}{a})^2\frac{Aa}{Bb}\geq\frac{\sum_{i=1}^{n}a_i^2.\sum_{i=1}^{n}b_i^2}{(\sum_{i=1}^{n}a_ib_i)^2}$.

ii) So $0<a\leq a_i\leq A$ and $t_i\geq 0$ deduce $t_ia_i+\frac{t_iaA}{a_i}\leq(a+A)t_i$ where $i=1,\ldots,n$, deduce we have $\sum_{i=1}^{n}t_ia_i+\sum_{i=1}^{n}\frac{t_iaA}{a_i}\leq(a+A)\sum_{i=1}^{n}t_i$. Applying the Cauchy's Inequality, we get $(a+A)(\sum_{i=1}^{n}t_i)\geq 2\sqrt{aA(\sum_{i=1}^{n}t_ia_i)(\sum_{i=1}^{n}\frac{t_i}{a_i})}$ or the Inequality $\frac{(a+A)^2}{4aA}(\sum_{i=1}^{n}t_i)^2\geq(\sum_{i=1}^{n}t_ia_i)(\sum_{i=1}^{n}\frac{t_i}{a_i})$.

The first technique. Teachers help pupils to recognize the relation between equations and theory they learned.

Example 1. Given real numbers a_1,a_2,a_3,b_1,b_2,b_3 such that $a_1^2-a_2^2-a_3^2>0$. We have

$$(a_1^2-a_2^2-a_3^2)(b_1^2-b_2^2-b_3^2)\leq(a_1b_1-a_2b_2-a_3b_3)^2.$$

Proof. Considering the function $f(x)=(a_1^2-a_2^2-a_3^2)x^2-2(a_1b_1-a_2b_2-a_3b_3)x+(b_1^2-b_2^2-b_3^2)=(a_1x-b_1)^2-(a_2x-b_2)^2-(a_3x-b_3)^2$. From the supposition we deduce $a_1\neq 0$ and $f(\frac{b_1}{a_1})\leq 0$. Applying proposition (2.v) we get $f(x)=0$ have some solutions, deduce $\Delta'\geq 0$.

The second technique. Convert a problem to another equivalent problems.

Example 2. Given triangle ABC. Let a,b,c denote the length of edges and let $S=S_{ABC}$. Prove that for all $x>0$ we have the inequality: $(2x-1)a^2+(\frac{2}{x}-1)b^2+c^2\geq 4\sqrt{3}S$.

Proof. The above inequality is equivalent to $2a^2x^2-(a^2+b^2-c^2-4\sqrt{3}S)x+2b^2\geq 0$ for all $x>0$. Consider $\Delta=[a^2+b^2-c^2-4\sqrt{3}S+4ab][a^2+b^2-c^2-4\sqrt{3}S-4ab]$.

Since $1\geq\cos(C-\frac{2\pi}{3})=\frac{c^2-a^2-b^2}{4ab}+\frac{\sqrt{3}S}{ab}$ deduce $a^2+b^2-c^2-4\sqrt{3}S+4ab\geq 0$.

Similary, since $1\geq\cos(C+\frac{\pi}{3})$ deduce $a^2+b^2-c^2-4\sqrt{3}S-4ab\leq 0$. We deduce $\Delta\leq 0$ hence ends the proof.

The third technique. Change the approaching methods to simplify the original problem.

Example 3. Given sequence (a_n) such that:

$$\begin{cases} a_0 = 1 \\ a_{n+1} = 6a_n + \sqrt{35a_n^2 + 2010}, n \geq 0. \end{cases}$$

Prove that

i) $a_{n+1} = 12a_n - a_{n-1}, a_{n+1} = \dfrac{a_n^2 - 2010}{a_{n-1}}$ for all $n \geq 1$.

ii) The sequence (a_n) is not bounded from above.

Proof. i) Since $(a_{n+1} - 6a_n)^2 = 35a_n^2 + 2010$ we deduce the equation: $a_{n+1}^2 - 12a_n a_{n+1} + a_n^2 - 2010 = 0$ for all $n \geq 0$. We change $n+1$ by n deduce $a_{n-1}^2 - 12a_n a_{n-1} + a_n^2 - 2010 = 0$. We deduce a_{n-1} and a_{n+1} are two solutions of this equation $x^2 - 12a_n x + a_n^2 - 2010 = 0$. Applying Viest theorem, we deduce $a_{n+1} = 12a_n - a_{n-1}, a_{n+1} = \dfrac{a_n^2 - 2010}{a_{n-1}}$.

ii) Bcause $a_n > 0$ and $a_{n+1} = 6a_n + \sqrt{35a_n^2 + 2010} > 6a_n$ for all $n \geq 0$ deduce the sequence (a_n) is a monotonically increasing. If the sequence (a_n) is bounded from above then it has a finite limit. Suppose this finite limit is a.

Since $a_{n+1}a_{n-1} = a_n^2 - 2010$ and (i) deduce $a^2 = a^2 - 2010$. Deduce $2010 = 0$: incorrect. We get the sequence (a_n) is not bounded from above.

Example 4. Given the integer sequence (a_n) such that $a_0 = 1, a_1 = 4$ and $a_{n+2} = 4a_{n+1} - a_n$ where $n \geq 0$. Prove that

i) $a_{n+1} = \dfrac{a_n^2 - 1}{a_{n-1}}, n \geq 1$.

ii) The sequence (a_n) is not bounded from above.

Proof. i) We have $a_n^2 + a_{n-1}^2 - 4a_n a_{n-1} - 1 = 0$ for all $n \geq 1$.

Therefore a_{n+1} and a_{n-1} are two solutions of the equation $x^2 - 4a_n x + a_n^2 - 1 = 0$. Using Viest theorem, we have $a_{n+1} = \dfrac{a_n^2 - 1}{a_{n-1}}$.

ii) Because $a_{n+2} = 4a_{n+1} - a_n$ for all $n \geq 0$ and $a_0 = 1, a_1 = 4$, we deduce the sequence (a_n) is a monotonically increasing. If the (a_n) is bounded sequence from above then it has a finite limit. Suppose this finite limit is a. Since $a_{n+1}a_{n-1} = a_n^2 - 1$ and (i) we deduce $a^2 = a^2 - 1$. We get $1 = 0$: incorrect. Therefore, the sequence (a_n) is not bounded from above.

The fourth technique. Use different methods to ultilize given assumptions.

Example 5. Given triangle ABC Let a,b,c denote the length of edges; h_a,h_b,h_c : the length of altitudes; with $a+b+c=2.$ We have

$$2+\frac{3}{\sqrt{2}}\geq T=[\frac{a(a+2h_a)}{2-a}+\frac{b(b+2h_b)}{2-b}+\frac{c(c+2h_c)}{2-c}][\frac{a(2-a)}{a+2h_a}+\frac{b(2-b)}{b+2h_b}+\frac{c(2-c)}{c+2h_c}].$$

Proof. Because $a+2h_a=b(\cos C+\sin C)+c(\cos B+\sin B)$ deduce $b+c<a+2h_a\leq\sqrt{2}(b+c).$ Therefore $1<\dfrac{a+2h_a}{2-a}\leq\sqrt{2}.$

Similar $1<\dfrac{b+2h_b}{2-b}\leq\sqrt{2},\quad 1<\dfrac{c+2h_c}{2-c}\leq\sqrt{2}.$ With $a=1,\quad A=\sqrt{2},\quad t_1=a,\quad t_2=b,$ $t_3=c$ and $a_1=\dfrac{a+2h_a}{2-a},\ a_2=\dfrac{b+2h_b}{2-b},\ a_3=\dfrac{c+2h_c}{2-c}.$

Using proposition 3 we have the inequality $2+\dfrac{3}{\sqrt{2}}\geq T.$

Example 6. Given triangle ABC Let a,b,c denote the length of edges and three real numbers p,q,r so that $p+q+r=0.$ We have $a^2pq+b^2qr+c^2rp\leq 0.$

Proof. Since $p+q+r=0$ we suppose $p,q\leq 0, r\geq 0.$ Because $c^2>(a-b)^2$ deduce

$$a^2pq+b^2qr+c^2rp\leq a^2pq+b^2qr+(a-b)^2rp=a^2p(q+r)+b^2r(p+q)-2abpr.$$

Therefore $a^2pq+b^2qr+c^2rp\leq-(ap+br)^2\leq 0.$

Example 7. Prove that for all real numbers x,y,z and for all triangles ABC we have inequality $x^2+y^2+z^2\geq 2xy\cos C+2yz\cos A+2zx\cos B.$ We deduce

i) $\cos A+\cos B+\cos C\leq\dfrac{3}{2}.$

ii) $\dfrac{1}{3}\cos A+\dfrac{1}{4}\cos B+\dfrac{1}{5}\cos C\leq\dfrac{5}{12}.$

Proof. Bcause the quadratic function $f(x)=x^2-2x(y\cos C+z\cos B)+y^2+z^2-2yz\cos A$ has $\Delta\leq 0$ deduce $f(x)\geq 0$ $\forall x,y,z\in\mathbb{R}$ and for all the triangles ABC.

Choosing $x=y=z=1$ we get the inequality (i).

Choosing $x=\dfrac{6}{\sqrt{6.8.10}}, y=\dfrac{8}{\sqrt{6.8.10}}$ and $z=\dfrac{10}{\sqrt{6.8.10}}$ we get the inequality (ii).

Example 8. Let a,b,c denote the length of edges of a triangle. Prove that if three reals x,y,z and such that $ax+by+cz=0$ then we have $ayz+bzx+cxy\leq 0$ and $yz+zx+xy\leq 0.$

Proof. Since $ax+by+cz=0$ deduce $cz=-ax-by.$ Because $c>0$ therefore $ayz+bzx+cxy\leq 0$ is equivalent to $aycz+bczx+c^2xy\leq 0.$ We prove that $ay(-ax-by)+bx(-ax-by)+c^2xy\leq 0$ or $abx^2+(a^2+b^2-c^2)xy+aby^2\geq 0.$ Consider the function $f(x,y)=abx^2+(a^2+b^2-c^2)xy+aby^2$ where $ab>0.$ Because

$$\Delta = -(a+b+c)(a+b-c)(a-b+c)(-a+b+c)y^2 \le 0$$

therefore $f(x,y) \ge 0$ for all x, y.

Because $z = -\dfrac{ax+by}{c}$ deduce $yz + zx + xy \le 0$ is equivalent to $ax^2 + (a+b-c)xy + by^2 \ge 0.$ One can easily seen that $\Delta \le 0.$ We deduce $yz + zx + xy \le 0.$

The fifth technique. Applying knowledge flexibly. Sometime a problem maybe hard for pupils. It require flexibly, depend on specific problem.

Example 9. Suppose that $f(x) = ax^2 + bx + c$ where $a \ne 0$ such that $|f(x)| \le 1$ with $|x| \le 1.$ Prove that $\max\{4a^2 + 3b^3\} = 16.$

Proof. Because $|f(0) - f(1)| \le |f(0)| + |f(1)| \le 2$ therefore $(a+b)^2 \le 4.$

Because $|f(0) - f(-1)| \le |f(0)| + |f(-1)| \le 2$ therefore $(a-b)^2 \le 4.$

We get $4a^2 + 3b^2 = 2(a+b)^2 + 2(a-b)^2 - b^2 \le 8 + 8 = 16.$ The equality holds if and only if $a = 2, b = 0, c = -1$ or $a = -2, b = 0, c = 1.$

Example 10. Suppose that $f(x) = ax^2 + (c-b)x + (e-d)$ $(a \ne 0)$ has some solutions that are larger than 1. Prove that $g(x) = ax^4 + bx^3 + cx^2 + dx + e$ has some solutions.

Proof. Denoted solutions of $f(x) = 0$ by $x = t^2$ with $t > 0.$ Deduce $at^4 + ct^2 + e = bt^2 + d.$ Because $g(t) = at^4 + ct^2 + e + t(bt^2 + d) = (1+t)(bt^2 + d)$ deduce $g(-t) = at^4 + ct^2 + e - t(bt^2 + d) = (1-t)(bt^2 + d).$ We get $g(t)g(-t) = (1-t^2)(bt^2 + d)^2 < 0$ deduce $g(x) = 0$ has some solutions in $[-t; t].$

Example 11. Suppose $f(x) = x^4 + ax^3 + bx^2 + ax + 1$ has some positive solutions. Prove that $|b| \ge 2(1 - |a|).$

Proof. Suppose that $f(x)$ has solution is $x_0 > 0.$ Because $-|a|x_0^3 \le ax_0^3,$ $-|b|x_0^2 \le bx_0^2,$ $-|a|x_0 \le ax_0$ therefore

$$x_0^4 - |a|x_0^3 - |b|x_0^2 - |a|x_0 + 1 \le x_0^4 + ax_0^3 + bx_0^2 + ax_0 + 1 = 0.$$

Because $2x_0^2 \le x_0^4 + 1$ and $x_0^4 + 1 \ge x_0^3 + x_0$ therefore

$$x_0^4 + 1 \le |a|(x_0^3 + x_0) + |b|x_0^2 \le |a|(x_0^4 + 1) + |b|\frac{x_0^4 + 1}{2}.$$

Therefore, we deduce the inequality $|b| \ge 2(1 - |a|).$

Example 12. Suppose $f(x) = ax^2 + bx + c$ such that $|f(x)| \le 1$ where $|x| \le 1.$ Find the maximun value of $|a| + |b| + |c|.$

Proof. Because $|f(x)| \le 1$ where $|x| \le 1$ deduce

$$\begin{cases} |a+b+c|\le 1 \\ |a-b+c|\le 1 \\ |c|\le 1 \end{cases}$$

We consider $a \ge 0.$ From the above system of inequalities, we deduce

$$\begin{cases} -1-c \le a+b \le 1-c \\ -1-c \le a-b \le 1-c \\ |c|\le 1. \end{cases}$$

If $b \ge 0$ then $|a|+|b| = a+b \le 1-c \le 2.$ We get $|a|+|b|+|c| \le 3.$
If $b \le 0$ then $|a|+|b| = a-b \le 1-c \le 2.$ We get $|a|+|b|+|c| \le 3.$
Therefore, when choosing $a = 2, b = 0, c = -1$ and it such that
$|2x^2 - 1| = |2\cos^2 t - 1| = |\cos 2t| \le 1$ where $|x| \le 1.$
We get the maximun value of $|a|+|b|+|c|$ is 3.

Example 13. Suppose $f(x) = ax^2 + bx + c$ such that $|f(0)|, |f(1)|, |f(-1)| \le 1.$ Prove that $|f(x)| \le \frac{5}{4}$ where $|x| \le 1.$

Proof. Because $|f(0)|, |f(1)|, |f(-1)| \le 1$. Set $A = a+b+c,\quad B = a-b+c$ we have $\begin{cases} |A| = |a+b+c| \le 1 \\ |B| = |a-b+c| \le 1 \\ |c| \le 1 \end{cases}$ and $|f(x)| = |(\frac{A+B}{2} - c)x^2 + \frac{A-B}{2}x + c|.$

We deduce $|f(x)| = |\frac{A}{2}(x^2+x) + \frac{B}{2}(x^2-x) + c(1-x^2)|.$

Therefore, we deduce $|f(x)| \le \frac{1}{2}|x^2+x| + \frac{1}{2}|x^2-x| + (1-x^2) \le \frac{5}{4}$ khi $|x| \le 1.$

Example 14. Given for all real numbers a, b, c such that $|ax^2 + bx + c| \le h$ where $|x| \le 1$ and we always have $|a|+|b|+|c| \le kh.$ Find the minimun value of $k \in R$.

Proof. Set $h = \max\{|ax^2 + bx + c| | x \in [-1;1]\} \ge 0.$ Choosing $x = 0, x = \pm 1,$ we always have inequalities

$$\begin{cases} a+b+c \le h \le 3h \\ -a-b-c \le h \le 3h \\ a-b+c \le h \le 3h \\ -a+b-c \le h \le 3h \\ c \le h - c \le h. \end{cases}$$

From this inequalities, we deduce:

$a + b - c = a + b + c - 2c \le h + 2h = 3h$
$a - b - c = a - b + c - 2c \le h + 2h = 3h$
$-a - b + c = -a - b - c + 2c \le h + 2h = 3h$
$-a + b + c = -a + b - c + 2c \le h + 2h = 3h.$

We always have $|a|+|b|+|c|\leq 3h.$ With $a=2, b=0, c=-1,$ we have $|2x^2-1|\leq 1=h$ where $|x|\leq 1$ and $|a|+|b|+|c|=3=3.1=3.h.$ Therefore $k<3$, k do not satisfy conditions.

Conclusion we get $k_{nn}=3.$

CONCLUSIONS

In mathematics, it is important to teach pupils self-learning and discovering knowledge. This paper proposed some techniques in teaching the lesson "Quadratic equation", mathematics grade 10 in Vietnam. In which, teachers help pupils to applying knowledge flexibly and studying more effectively. Thereby, improving the quality of not only teaching and learning Mathematics but education in Vietnam in general.

REFERENCES

[1] Nhi, D.V., Chin, V.D., Dung, D.N., Phuong, P.M., Tinh, T.T., Tuan, N.A., *Elementary geometry*, Information and Communication Publishing House, Vietnam, 2015.

[2] Nhi, D.V.,Tinh, T.T., Vi, P.T., Hai, P.D., *Inequality, extremum, system equations*. Information and Communication Publishing House, Vietnam, 2013.

[3] Nam, T.T., Nhi, D.V., Thang, L.B., *Some applications of modern algebra in sloving elementary math problems*, University of Education Hochiminh city Publishing House, Vietnam, 2013.

[4] Hai, T.T., Tinh, T.T., *Forming and strengthening teaching competency in Mathematics Pedagogy students*, Journal of Science of Hanoi National University of Education, Vol. 60, No. 1, pp. 30-37, 2015.

[5] Marzano, R.J., *The art and science of teaching: a comprehensive framework for effective instruction,* ASCD, Alexandria-Virginia, USA, 2007.