ORIGINAL PAPER

THE STUDY OF NEW APPROACHES IN CUBIC SPLINE INTERPOLATION FOR AUTO MOBILE DATA

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Abstract. *This paper presents the theory of Interpolation and its applications in numerical analysis. It specially focuses on cubic spline interpolation. Data interpolation is useful for scientific visualization for data interpretation. One of the efficient methods for data interpolation is cubic spline function. Cubic splines are the most popular. They produce an interpolated function that is continuous through to the second derivative; Splines tend to be stabler than fitting a polynomial through the N +1 points, with less possibility of wild oscillations between the tabulated points.*

Keywords: *Data interpolation, Cubic spline, Polynomial spline, Continuity.*

1. INTRODUCTION

When computers were not available, the draftsman used a device to draw a smooth curve through a given set of points such that the slope and curvature are also continuous along the curve, that is, f(x), f'(x) and f''(x) are continuous on the curve. Such a device was called a spline and plotting of the curve was called spline fitting.

The given interval [a, b] is subdivided into n subintervals $[x_0, x_1]$, $[x_1, x_2]$ .... $[x_{n-1}, x_n]$ where $a= x_0< x_1< x_2< \ldots< x_n = b$. The nodes $x_1, x_2, \ldots, x_{n-1}$ are called internal nodes [1].

Spline interpolation is an alternative approach to data interpolation. Compare to polynomial interpolation using on single formula to correlate all the data points, spline interpolation uses several formulae; each formula is a low degree polynomial to pass through all the data points. These resulting functions are called *splines.*

Spline interpolation is preferred over polynomial interpolation because the interpolation error can be made small even when using low degree polynomials for the spline. Spline interpolation avoids the problem of Runge's phenomenon, which occurs when the interpolating uses high degree polynomials [2].

1.1. CUBIC SPLINE INTERPOLATION

The goal of cubic spline interpolation is to get an interpolation formula that is continuous in both the first and second derivatives, both within the intervals and at the interpolating nodes. This will give us a smoother interpolating function. The continuity of first derivative means that the graph y = F(x) will not have sharp corners. The continuity of second derivative means that the radius of curvature is defined at each point.

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The equation of the cubic spline in the i[th] interval $[x_{i-1}, x_i]$, is show in equation (12).

$$f_i(x) = a_i + b_i x + c_i x^2 + d_i x^3 \; ; i = 1, 2, \dots \dots . n, \tag{1}$$

where a_i, b_i, c_i, d_i is the 4n coefficient for i = 1, 2…n,

The cubic spline interpolation in (1) satisfies the following condition:

$$f_i(x_i) = f_i \tag{2}$$

$$f_i(x_{i+1}) = f_{i+1}(x_{i+1}) \tag{3}$$

$$f_i'(x_{i+1}) = f_{i+1}'(x_{i+1}) \tag{4}$$

$$f_i''(x_{i+1}) = f_{i+1}''(x_{i+1}) \tag{5}$$

Equation (2) until (5) gives a total 4 – 2n conditions. For cubic spline interpolation, we need 4n parameters to be determined.[3] In this paper will be used the natural boundary conditions. It is given as follows:

$$f_0''(x_0) = f_{n-1}''(x_n) = 0 \tag{6}$$

1.2. DATA COLLECTION AND EXPERIMENTAL SETUP

Nowadays, drilling become more challenges day by day. One of the technologies that have been introduced to improve drilling operation is casing while drilling technique. This technique is using casing to drill the well instead of the drill pipe that usually use in conventional drilling. Thus it will save the time and cost from the tripping time. Recently, casing drilling can only be used in shallow depth and soft formations. Because high stress caused by the hard formation will lead the damage of the casing. So, design of the casing need to be analyzed carefully to give the best drilling performance [6].

Below (Table 1) is example for cubic spline interpolation.

Table 1. Data of cubic spline interpolation

Time (second)	Velocity (m.p.h)
2	25
3	36
4	52
5	59

Here we apply cubic spline interpolation on [2, 3], [3, 4], and [4, 5]. Thus we have three cubic spline interpolation that satisfies condition (2) until (6) above. The cubic spline interpolation is given as follows:

$$f(x) = \begin{cases} 1.93x^3 - 11.58x^2 + 32.23x - 8.58, [2,3] \\ -4.67x^3 + 47.83x^2 - 146.02x - 169.68, [3,4] \\ 2.73x^3 - 40.96x^2 + 209.11x - 303.80, [4,5] \end{cases}$$

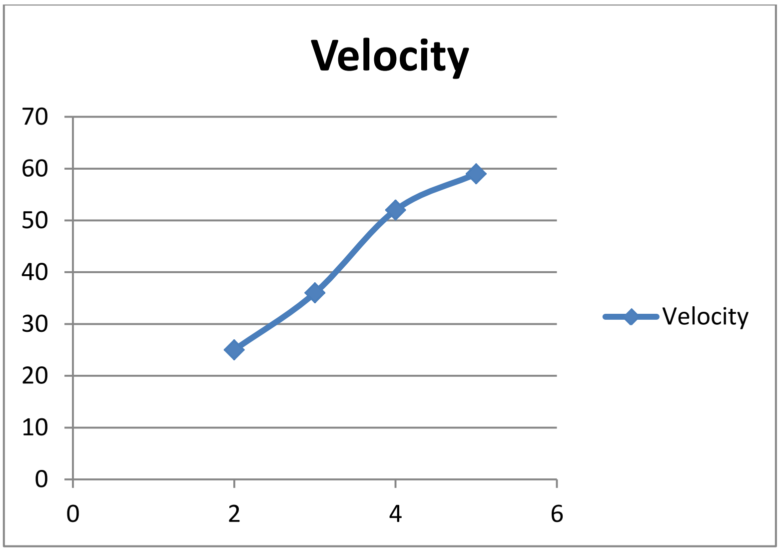

Figure 1. Cubic spline interpolation for data listed in Table 1.

Table 2. Data of cubic spline interpolation according with [4]

Time (second)	Velocity (m.p.h)
0	1
1	2
2	33
3	244

Here we apply cubic spline interpolation on [0,1], [1,2], and [2,3]. Thus we have three cubic spline interpolation that satisfies condition (2) until (6) above. The cubic spline interpolation is given as follows:

$$f(x) = \begin{cases} -4x^3 + 5x + 1, [0,1] \\ 50x^3 - 162x^2 + 167x - 53, [1,2] \\ -46x^3 + 414x^2 - 985x + 715, [2,3] \end{cases}$$

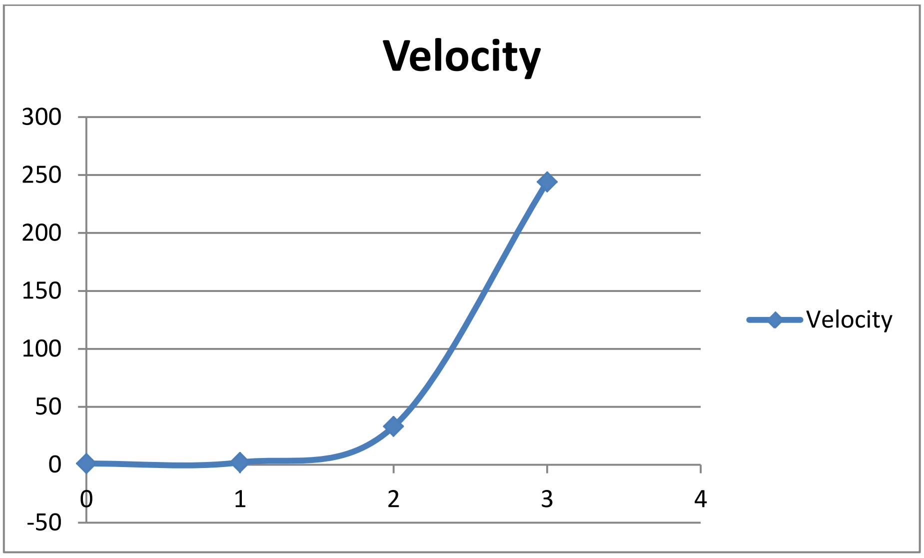

Figure 2. Cubic spline interpolation for data listed in Table 2.

Table 3. Value of x and y according with [5]

x	y
-2	-23
0	1
1	4
3	82
4	193

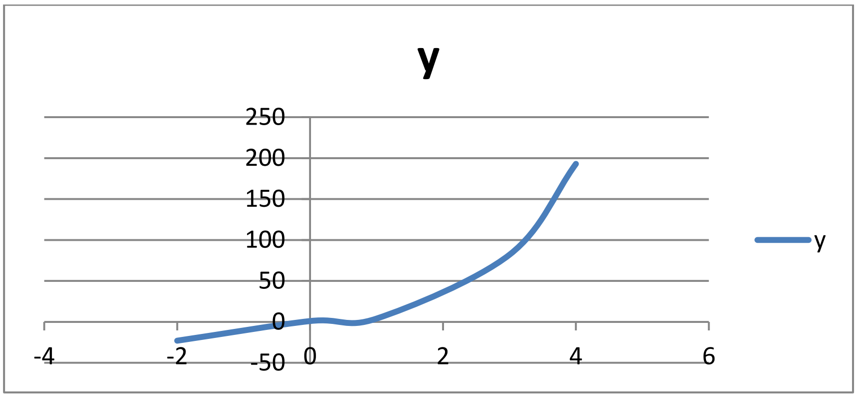

Figure 3. y graphic reprezentation.

2. ERROR ANALYSIS

In this section we will discuss the error analysis for both splines; cubic spline interpolation and PCHIP. We use two sets taken from two function $f(x) = x^2 - 8$ and $f(x) = 2e^x - x^2$. Below table summarized the error analysis for data interpolation by using PCHIP (P(x)) and cubic spline (S(x)) for all two functions.

For the error measurement we use (1) Absolute error and (2) Root Mean Square Error(RMSE) given by the following formula:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \widehat{y_i})^2}{N}}$$

where y_i is a true data and $\widehat{y_i}$ is a observed data and N is a total number of data.

Table 4. Error analysis for data interpolation

x	$f(x)$	$S(x)$	$P(x)$	$S(x) - f(x)$	$(S(x) - f(x))^2$	$P(x) - f(x)$	$(P(x) - f(x))^2$
0.0	-8.00	-8.00	-8.0000	0	0	0.00000	0.000000
0.2	-7.96	-7.96	-7.9440	0	0	0.01600	0.000256
0.4	-7.84	-7.84	-7.7920	0	0	0.04800	0.002304
0.6	-7.64	-7.64	-7.5680	0	0	0.07200	0.005184
0.8	-7.36	-7.36	-7.2960	0	0	0.06400	0.004096
1.0	-7.00	-7.00	-7.0000	0	0	0.00000	0.000000
1.2	-6.56	-6.56	-6.6160	0	0	-0.05600	0.003136
1.4	-6.04	-6.04	-6.0880	0	0	-0.04800	0.002304
1.6	-5.44	-5.44	-5.4520	0	0	-0.01200	0.000144
1.8	-4.76	-4.76	-4.7440	0	0	0.01600	0.000256
2.0	-4.00	-4.00	-4.0000	0	0	0.00000	0.000000

2.2	-3.16	-3.16	-3.1867	0	0	-0.02670	0.000713
2.4	-2.24	-2.24	-2.2600	0	0	-0.02000	0.000400
2.6	-1.24	-1.24	-1.2400	0	0	0.00000	0.000000
2.8	-0.16	-0.16	-0.1467	0	0	0.01330	0.000177
3.0	1.00	1.00	1.0000	0	0	0.00000	0.000000
3.2	2.24	2.24	2.2187	0	0	-0.02130	0.000454
3.4	3.56	3.56	3.5360	0	0	-0.02400	0.000576
3.6	4.96	4.96	4.9440	0	0	-0.01600	0.000256
3.8	6.44	6.44	6.4347	0	0	-0.00530	0.000177
4.0	8.00	8.00	8.0000	0	0	0.00000	0.000000
SUM				0	0	0.0000	0.02029

Error analysis for $f(x) = x^2 - 8$ [7]

where y_i is a true data and $\hat{y_i}$ is a observed data and N is a total number of data.

Table 5. Error analysis for $f(x) = x^2 - 8$

x	$f(x)$	S(x)	P(x)	S$(x) - f(x)$	$(S(x) - f(x))^2$	P$(x) - f(x)$	$(P(x) - f(x))^2$
0.0	2.000	2.000	2.000	0.000000	0.000000	0.000000	0.000000
0.2	2.403	2.453	2.203	0.049895	0.002489	-0.200106	0.040042
0.4	2.824	2.876	2.589	0.052751	0.002783	-0.234249	0.054873
0.6	3.284	3.318	3.118	0.033262	0.001106	-0.165838	0.027502
0.8	3.811	3.822	3.748	0.011018	0.000121	-0.063082	0.003979
1.0	4.437	4.437	4.437	0.000000	0.000000	0.000036	0.000000
1.2	5.200	5.207	5.237	0.007066	0.000050	0.036866	0.001359
1.4	6.150	6.181	6.247	0.030100	0.000906	0.096600	0.009332
1.6	7.346	7.403	7.491	0.056435	0.003185	0.144635	0.020919
1.8	8.859	8.920	8.993	0.060305	0.003637	0.133605	0.017850
2.0	10.778	10.778	10.778	0.000000	0.000000	-0.000012	0.000000
2.2	13.210	13.059	13.155	-0.150827	0.022749	-0.054927	0.003017
2.4	16.286	15.984	16.403	-0.302753	0.091659	0.116547	0.013583
1.6	20.168	19.807	20.502	-0.360876	0.130232	0.334124	0.111639
2.8	25.049	24.784	25.431	-0.265294	0.070381	0.381607	0.145624
3.0	31.171	31.171	31.171	0.000026	0.000000	0.000026	0.000000
3.2	38.825	39.224	38.900	0.398440	0.158754	0.074540	0.005556
3.4	48.368	49.197	49.471	0.828500	0.686412	1.102900	1.216388
3.6	60.237	61.346	62.381	1.109731	1.231503	2.144431	4.598585
3.8	74.962	75.928	77.124	0.965231	0.931671	2.161831	4.673513
4.0	93.196	93.196	93.196	0.000000	0.000000	0.000000	0.000000
SUM				2.523011	3.337638	6.009535	10.944

Error analysis for $f(x) = 2e^x - x^2$

Table 6. Comparision of error between cubic spline and PCHIP

Bil	Function	Root mean square error PCHIP Cubic spline	
1	$f(x) = x^2 - 8$	0.031	0
2	$f(x) = 2e^x - x^2$	0.72	0.40

From the table root mean square error of cubic spline interpolation is less than the error by using PCHIP interpolation for function $f(x) = x^2 - 8$ and $f(x) = 2e^x - x^2$

3. CONCLUSION

This paper discussed the use of cubic spline interpolation and PCHIP. The main difference between both cubic splines is their continuity. Both cubic spline works well for all tested data sets. Error analysis for data interpolation also has been discussed in details.

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