ORIGINAL PAPER

MATHEMATICAL CALCULUS FOR ANALOG SIGNAL PROCESSING ALGORITHM OF SENSORS USED IN HOUSEHOLD POWER MONITORING SYSTEM

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Abstract. *This paperwork represents a mathematical calculus method for deriving the RMS value of an analog signal which has a DC offset. The analog signal is generated either by a current transformer based sensor or a voltage transformer based one, which is used for an off-grid household power consumption monitoring system. The received analog signal values are converted by the microcontroller's ADC into digital values. Based on a mathematical modelling of the signal, an algorithm is thus developed to be uploaded into a microcontroller based embedded device, which further converts the digital values into comprehensive numerical values about the measured power consumption.*
Keywords: *analog sensor, rms, analog signal, ADC, power monitoring, algorithm.*

1. INTRODUCTION

Modern electrical sensors play an important role in measuring different physical quantities, processes or characteristics from our real world, which are then translated into electrical signals that can be processed and converted into final usable numerical data [1]. Although nowadays both analog sensors and digital sensors are widely used, there is no ultimate delimitation on which type of sensor is better or has the most advantages as each measuring process require a certain type of sensor with a certain characteristics set.

As a general analogy between sensors with different type of signal output, we would mention that the digital sensors are more accurate, they can provide only a finite or discrete number of approximate read values (conditioned by the internal/external ADC range set), they can be very expensive and also can be damaged easily in electrical overloads or short-circuits; the analog sensors are not always accurate, sometimes they need to be calibrated by hardware/software means, they can virtually provide an infinite set of read values, they are simple, cheap and robust, but sometimes require additional circuitry for signal filtering or conditioning and ADC in a more complex systems [2].

The objective of this article is to experiment and develop a viable aplication algorithm used for electrical power consumption monitoring systems in off-grid housholds.

In order to conduct the experiments we have chosen two, low cost, aftermarket current and voltage sensors based on small inductive transformers that output a proportional readable voltage to the sensed current or voltage on the primary phase circuit. With these two elements we could calculate the power consumption on that particular power grid. The system is based on an Arduino data acquisition board which uses an Atmega328 microcontroller for data processing. By the help of an internal 10-bit ADC module, the microcontroller aquires and

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converts the signal data from the analog inputs. The detailed graphic diagram of the involved modules for signal processing is best described in Fig. 1.

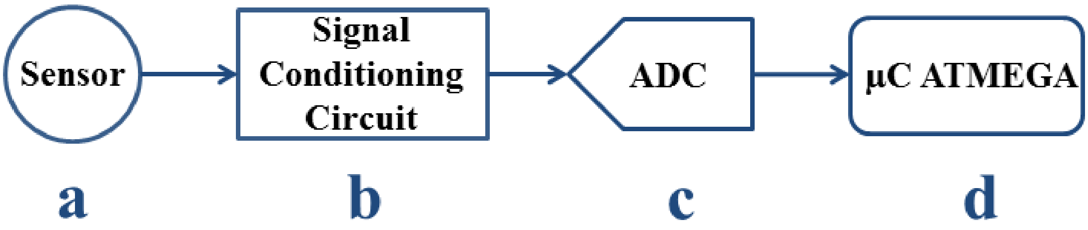

Figure 1. Block diagram representation of the electronic modules used for the analog signal processing from generation stage to numerical data conversion stage; thus we have the following modules: *a*-represents the *analog sensor*, *b*-represents the electronic *signal conditioning circuit*, *c*-represents the 2^{10} resolution, *ADC* module of the microcontroller, *d*-represents the *microcontroller* that converts digital data from ADC to numerical data by customised algorithm means.

The opportunity of using this sort of electronic board is that it can further be upgraded from a monitoring device, to a full automated system, where it can increase the energy efficiency of a household, by scheduling the operation of some large power consuming appliances that don't need owner supervision during peak energy production from a renewable energy source.

2. EXPERIMENT DEVELOPMENT, RESULTS AND CALCULATIONS

In order to develop a mathematical expression of the analog signal and in consequence, a calculation algorithm for the data output, we will first have to understand how the Arduino board is working in converting the input analog signal into output numerical data.

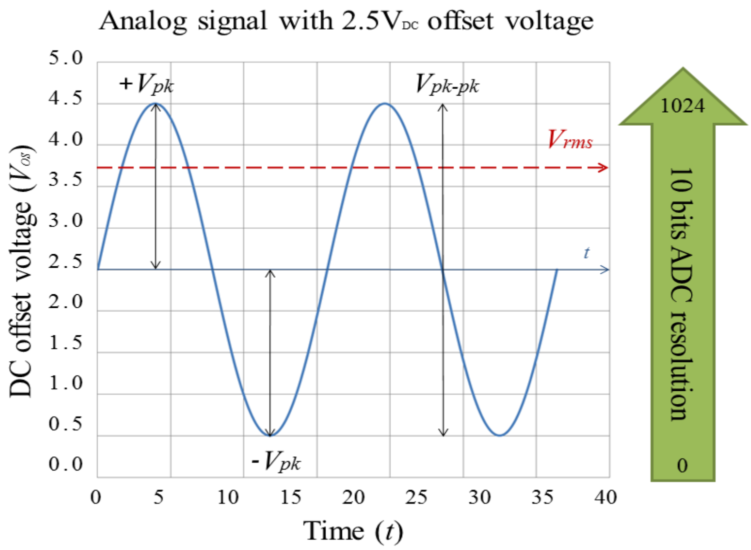

Figure 2. AC analog signal representation with 2,5V$_{DC}$ offset adaptation for Arduino.

According to the producer's datasheets, the Arduino board and the Atmega328 microcontroller is working only with positive voltages (the highest voltage reference is 5V$_{DC}$

while the lowest voltage reference is $0V_{DC}$), thus it is intuitive to conclude that the 10-bit range ADC values, from 0 to 1024, are directly proportional to the reference voltage of $0V_{DC}$ to $5V_{DC}$. Since the analog signal is composed of both positive and negative alternating values, generated at the 50Hz frequency in correspondence to the power line frequency to which the sensor is attached to, a voltage divider is implemented in our experiment in order to change the signal offset of $0V_{DC}$ to $2,5V_{DC}$. By that, the AC signal will have a DC component represented by the offset value, and thus, the ADC will read alternating positive values above and below $2,5V_{DC}$ or above and below 512 numerical values (Fig. 2).

In order to prove our previously described signal theory, we developed a virtual electronic simulation circuit using Multisim and Proteus software (Fig. 3).

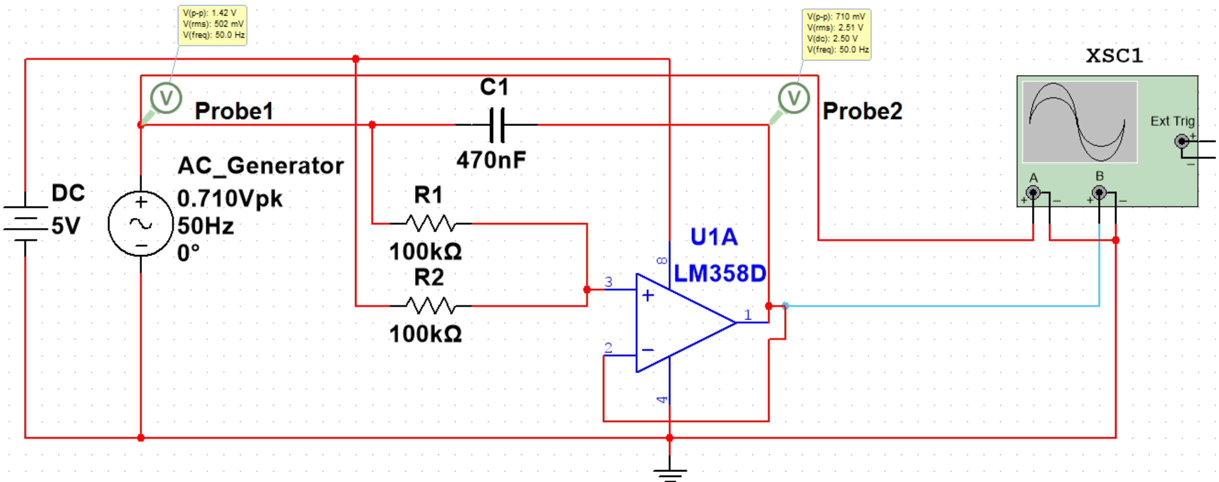

Figure 3. Electronic circuit simulation schematic for the analog signal emulation having – 50Hz AC generator/sensor, LM358 op-amp configured as *non inverting-summing attenuator* and gain reduction of the output signal, R1 and R2 voltage divider for DC offset, C1 noise filter, P1 and P2 input-output reading voltage probes and XSC1 virtual oscilloscope.

The circuit also contains an operational amplifier, but not to increase the gain of the signal but rather to reduce it. Although this op-amp configuration is not very common (non inverting-summing attenuator) [3,4], it helps our experiment functionality in three ways:

- the first being the $2,5V_{DC}$ offset component added from the voltage divider for the ADC reading;
- the second being the filtered, low gain signal output, in order to eliminate electrical noise and spikes, that are generated by the phase line and picked up by the ADC;
- the third, being the Arduino analog inputs protection either from a large voltage spike or from large voltage sinewaves (over $2,5V_{pk}$).

The disadvantage of this gain reduction technique is that the output signal resolution is smaller but without serious impacting on the reading.

In theory, the *rms* voltage value of a constant DC signal is the respective DC voltage value, but the *rms* voltage value of the output signal registered by the simulation was of 2,51V as it contained an AC signal with a DC component.

In Figure 4 it is represented an oscilloscope measurement capture of the input analog signal for a 20ms period. The peak to peak voltage of the signal is of 1,42V with 0V offset which is not in compliance with the Arduino ADC because of the negaive part of the waveform.

In Fig. 5 it is represented an oscilloscope measurement capture of the output analog signal for the same 20ms period but with attenuation and offset. The peak to peak voltage of the signal is of 710mV and with 2,5V offset. The waveform is suited for the Arduino ADC reading.

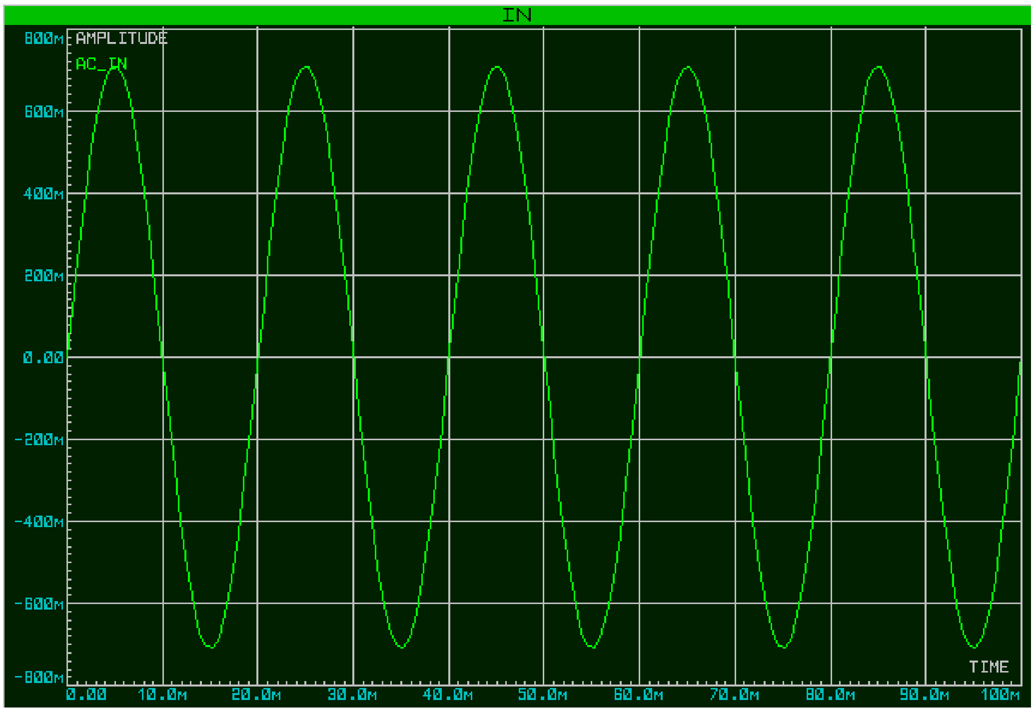

Figure 4. Oscilloscope signal capture - 20ms period, input analog signal generated by the AC sensor with $0{,}710V_{pk}$ and 0V offset.

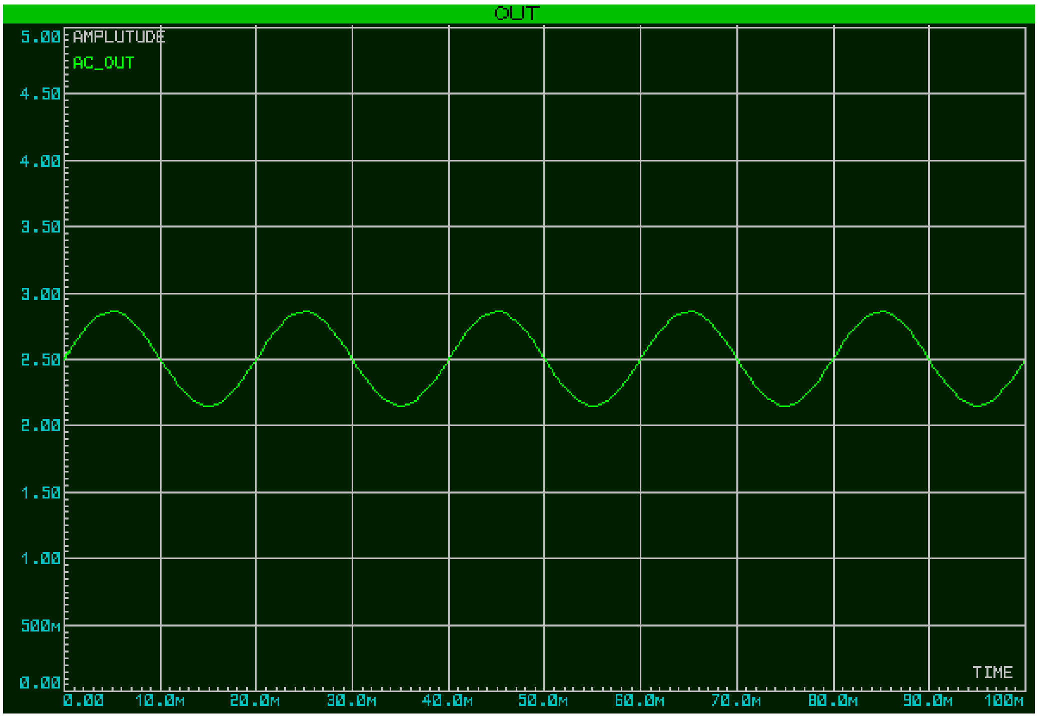

Figure 5. Oscilloscope signal capture - 20ms period, output analog signal processed by the conditioning circuit with $0{,}355V_{pk}$ and $2{,}5V_{DC}$ offset.

To verify our theory and the simulation results, we have developed a calculation method by deriving the *rms* value of the studied analog signal which has a DC offset.

The signal time dependency function is described in relation (1):

$$v(t) = V_{DC} + V_m \sin(\omega t) \tag{1}$$

where V_{DC} is the DC offset value of 2,5V while V_m is the peak voltage of the signal.
By integrating the *rms* definition we have the squared *rms* according to (2):

$$v_{RMS}^2 = \frac{1}{T}\int_0^T v(t)^2 \cdot dt \tag{2}$$

Thus the integral form will become (3):

$$v_{RMS}^2 = \frac{1}{T}\int_0^T \left[V_{DC} + V_m \sin(\omega t)\right]^2 \cdot dt \tag{3}$$

$$v_{RMS}^2 = \frac{1}{T}\int_0^T \left[V_{DC}^2 + 2V_{DC}V_m \sin(\omega t) + \left(V_m \sin(\omega t)\right)^2\right] \cdot dt \tag{4}$$

$$v_{RMS}^2 = \frac{1}{T}\left[V_{DC}^2 t\Big|_0^T + \frac{2}{\omega}V_{DC}V_m \cos(\omega t)\Big|_0^T + V_m^2 \int_0^T \left[\frac{1}{2} - \frac{\cos(2\omega t)}{2}\right] \cdot dt\right] \tag{5}$$

$$v_{RMS}^2 = \frac{1}{T}\left(V_{DC}^2 T + \frac{2}{\omega}V_{DC}V_m \left(\cos(2\pi) - \cos(0)\right)\right) + \frac{1}{T}\left(V_m^2 \frac{1}{2} t\Big|_0^T - V_m^2 \frac{1}{4\omega}\sin(2\omega t)\Big|_0^T\right) \tag{6}$$

$$v_{RMS}^2 = V_{DC}^2 + \frac{1}{T}\left(V_m^2 \frac{T}{2} - V_m^2 \frac{1}{4\omega}\left(\sin(4\pi) - \sin(0)\right)\right) \tag{7}$$

$$v_{RMS}^2 = V_{DC}^2 + \frac{V_m^2}{2} \tag{8}$$

The final result of the rms value of the output signal with a DC component is represented by the (9) expression:

$$v_{RMS} = \sqrt{V_{DC}^2 + \frac{V_m^2}{2}} \tag{9}$$

Therefore the theoretical *rms* value of the signal with offset is 2,51V as calculated in relation (10):

$$v_{RMS} = 2,51V \tag{10}$$

which confirms the theoretical simulation results.
Judging by the experimental results and the calculations, we have generated an algorithm (Figure 6) to be uploaded into the microntroller in order to display the power consumption according to (11):

$$P = V \times I \cos\varphi \tag{11}$$

where P is the real power, V is the voltage value registered by the voltage sensor, I is the current value registered by the current sensor and $cos\varphi$ is the power factor $pf=0,99$ for a resistive load.

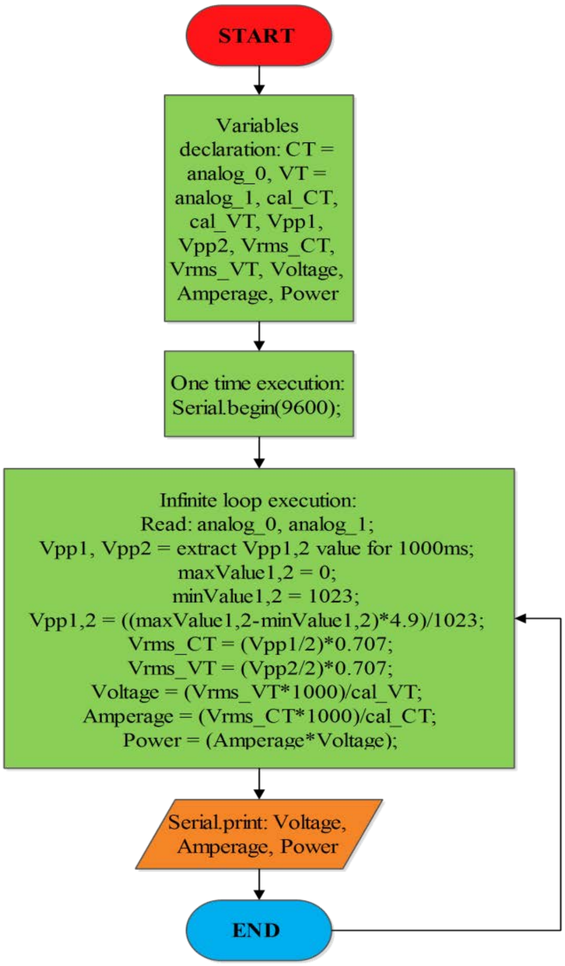

Figure 6. Algorithm diagram description with calculation blocks for converting the two analog signal values into digital numeric values for displaying the voltage, amperage and power consumption.

The calculation method of the analog signal values is the same both for the voltage sensor and for the current sensor as each output a proportional smaller voltage to the sensed primary voltage or current.

The algorithm calculation and conversion method of the analog signal values from the sensors is represented by (12):

$$a_{pk-pk} = \frac{(max_{val} - min_{val}) \cdot V_{ref}}{ADC_{val}} \tag{12}$$

where $a_{pk\text{-}pk}$ is the peak to peak analog signal value, max_{val} is the maximum threshold value of the ADC of 1024, min_{val} is the minimum threshold value of the ADC of 0, V_{ref} is the Arduino reference voltage of $5V_{DC}$, ADC_{val} is the instantaneous registered value of the respective sensor.

In order to convert the processed signal values into *rms* values, relation (13) is used:

$$V_{rms}, I_{rms} = \left(\frac{a_{pk-pk}}{2}\right) \cdot 0,707 \tag{13}$$

where V_{rms} is the *rms* value of voltage, I_{rms} is the *rms* value of current and $a_{pk\text{-}pk}$ is the peak to peak analog signal value.

The power consumption calculation done in the algorithm is the product of V_{rms} and I_{rms} according to (14):

$$P = V_{rms} \times I_{rms} \tag{14}$$

Due to reading errors generated by the native imperfection of the analog sensors, we have also implemented in the algorithm, a calibration factor variable (*cal*), which has a predefined value in order to have an accurate conversion of the signal values into the *rms* values.

Measurements were conducted on a resistive load to test the capability of the system to register the correct values (Table 1.).

Table 1. Arduino measurements.

n	*Volts (V_{rms})*	*Amperes (A_{rms})*	*Power (P)*
1	223.85	8.34	1867.64
2	223.13	8.34	1861.65
3	223.85	8.34	1867.64
4	223.13	8.34	1861.65
5	223.13	8.34	1861.65
6	223.13	8.34	1861.65
7	223.13	8.34	1861.65
8	224.57	8.34	1873.62
9	223.13	8.34	1861.65
10	223.13	8.34	1861.65
11	223.13	8.34	1861.65
12	223.13	8.34	1861.65
13	223.85	8.34	1867.64
14	223.13	8.34	1861.65
15	224.57	8.34	1873.62
16	223.13	8.34	1861.65
17	222.41	8.34	1855.66
18	223.13	8.34	1861.65
19	222.41	8.34	1855.66
20	223.13	8.34	1861.65
RMSE	***0.5518***	***0***	***4.5889***

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(p_i - a_i)^2} \tag{15}$$

where n is the number of samples, p is the predicted target and a is the actual target.

According to (15) calculations, both sensors outputted similar results in comparison with the profesional multimeter measurements conducted in parallel.

4. CONCLUSIONS AND OBJECTIVES

The objective of the article was to develop an algorithm suited for a low cost, accessible, electronic aquisition board by using low cost electrical analog sensors in order to read the power consumption of an off-grid household.

Original contributions:

- Identification of what type of sensors are best suited for our practical experiment.
- Designing of an electronic circuit adapted to a specific task.
- Conducting a simulated process for signal aquisition and interpretation.
- Development a calculation method that confirms the theoretical simulation results.
- Building a complex calculation algorithm that translates the input analog values into numerical values for registered power consumption, voltage and amperage.
- Conducting measurements and interpretation of the results.

The developed algorithm and calculation method can easily be used for any other types of analog sensors and aplications such as biometrics, with minor modifications to the source code according to the requirements [5]. Also the applicability of the entire system is represented by the fact that an automation algorithm can also be developed for energy efficiency by using the Arduino output pins in order to control appliances scheduled consumption and operation.

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REFERENCES

[1] Fraden, J., *Handbook of Modern Sensors: Physics, Designs, and Applications*, 4th Edition, Springer-Verlag, New York, 2010.

[2] Pelgrom Marcel, J.M., *Analog-to-Digital Conversion*, 2nd Edition, Springer-Verlag, New York, 2013.

[3] Toochinda, V., *Signal Conditioning Basics using Op-amps*, online publication - www.controlsystemslab.com/doc/electronics/analog/sigcon_opamp.pdf, 2011.

[4] Yasuda, T., Andoh, H., *Analog Integrated Circuits and Signal Processing*, **25**(3), 245, 2000.

[5] Vasile, I, Andrei, H, Ardeleanu, M, Vasile, V, *Proceedings of 7th International Conference on Electronics, Computers and Artificial Intelligence (ECAI)*, 71, 2015.