ORIGINAL PAPER

APPLICATION OF ABOODH DIFFERENTIAL TRANSFORM METHOD ON SOME HIGHER ORDER PROBLEMS

JAMSHAD AHMAD[1], JAWARIA TARIQ[1]

Manuscript received: 09.10.2017; Accepted paper: 12.12.2017;
Published online: 30.03.2018.

Abstract.*This paper is the witness of coupling of recently developed Aboodh transform and differential transform method. This coupling is known as Aboodh differential transform method. The proposed method is tested on some linear and nonlinear problems for suitable analytical solution and the obtained results demonstrate reliability and efficiency of the proposed method. It is observed that the presented coupling is an alternative approach to overcome the demerit of complex calculation. This new method is more efficient and easy to handle such differential equations in comparison to other methods. The results reveal the complete reliability that this method is very efficient, simple and applications hence can be extended to other problems of diverse nature.*

Keywords: *Analytical approximate solution, Aboodh Transform, Differential Transform Method, Nonlinear Differential Equations*

1. INTRODUCTION

Nonlinear problems are widely used to describe complex physical phenomena in various fields of sciences, and engineering. They also cover the cases of the following types: surface waves in compressible fluids, hydro magnetic waves in cold plasma, acoustic waves in an harmonic crystal, etc. The wide applicability of these equations is the main reason why they have attracted so much attention from many mathematicians. However, such problems are usually very difficult to solve, either numerically or theoretically. Recently, both mathematicians and physicists have devoted considerable effort to the study of exact and approximate solutions of the nonlinear ordinary differential equations corresponding to the nonlinear problems.

Recently, lot of attention is paid to finding appropriate solutions of NLPDEs. In the similar context, various techniques including Adomian's decomposition method (ADM) [1-2], Variational Iteration (VIM) [3-4], Homotopy Perturbation (HPM) [5-6], Homotopy Analysis (HAM) [7], F-Expansion [8], Exp-function [9], sine-cosine [10], differential transform method (DTM) [11-14], and reduced differential transform [10, 15-18] have been applied on wide range of linear and nonlinear problems of diversified physical nature. The differential transform method has been developed for solving the differential and integral equations. For example in [11] this method is used for solving a system of differential equations and in [9] for differential-algebraic equations. In [19-23] this method is applied to partial differential equations and in [24-26] to one-dimensional Volterra integral and integrodifferential equations. New integral transform "Aboodh transform" [25-34] is particularly useful for finding solutions for these problems. Aboodh transform is a useful

[1]University of Gujrat, Faculty of Science, Department of Mathematics, Pakistan.
E-mail: jamshadahmadm@gmail.com.

technique for solving linear Differential equations but this transform is totally incapable of handling nonlinear equations because of the difficulties that are caused by the nonlinear terms. This paper is using differential transforms method to decompose the nonlinear term, so that the solution can be obtained by iteration procedure.

2. ABOODH TRANSFORM

A new transform called the Aboodh transform defined for function of exponential order we consider functions in the set A, defined by:

$$A = \{ f(t) : \exists M, k_1, k_2 > 0, | f(t) | < Me^{-vt} \} \tag{1}$$

For a given function in the set M must be finite number, k_1, k_2 may be finite or infinite.Aboodh transform which is defined by the integral equation

$$A[f(t)] = k(v) = \frac{1}{v}\int_0^{\infty} f(t)e^{-vt}dt, t \geq 0, k_1 \leq v \leq k_2 \tag{2}$$

Let $k(v)$ be Aboodh transform of $f(t)$, $A[f(t)] = K(v)$ then the some results of Aboodh transform are

i. $A[f^{'}(t)] = vK(v) - \dfrac{f(0)}{v}$

ii. $A[f^{''}(t)] = v^2 K(v) - \dfrac{f^{'}(0)}{v} - f(0)$

iii. $A[f^{(n)}(t)] = v^{(n)} K(v) - \sum_{k=0}^{n-1} \dfrac{f^{(k)}(0)}{v^{2-n+k}}$

3. DIFFERENTIAL TRANSFORMMETHOD

Differential transform of the function $Y(x)$ for the k-derivative is defined as follows

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=x_0}, \tag{3}$$

where $Y(x)$ is original function and $y(k)$ is the transformed function. And the inverse differential transform of $y(k)$ is defined as

$$Y(x) = \sum_{k=0}^{\infty} y(k)x^k. \tag{4}$$

The main theorems of the one – dimensional differential transform are:

Theorem 1: If $w(x) = y(x) \pm z(x),$ then $W(k) = Y(k) \pm Z(k).$

Theorem 2: If $w(x) = cy(x),$ then $W(k) = cY(k).$

Theorem 3: If $w(x) = \frac{dy(x)}{dx},$ then $W(k) = (k+1)Y(k+1).$

Theorem 4: If $w(x) = \frac{d^n y(x)}{dx^n},$ then $W(k) = \frac{(k+n)!}{k!}Y(k+n).$

Theorem 5: If $w(x) = y(x)\ z(x),$ then $W(k) = \sum_{r=0}^{k} Y(r)\, Z(k-r).$

Theorem 6: If $w(x) = x^n,$ then $W(k) = \delta(k-n) = \begin{cases} 1, k = n \\ 0, k \neq n \end{cases}.$

Theorem 7: If $w(x) = \exp(\lambda x),$ then $W(k) = \frac{\lambda^k}{k!}.$

Note that c is a constant and n is a nonnegative integer.

4. ANALYSIS OF DIFFERENTIAL TRANSFORM OF NONLINEAR FUNCTION

In this section, we will introduce a reliable and efficient algorithm to calculate the differential transform of nonlinear functions.

Exponential nonlinearity: $f(y) = e^{ay}$

From the definition of transform

$$F(0) = \left[e^{ay(x)}\right]_{x=0} = e^{ay(0)} = e^{aY(0)} \tag{5}$$

Taking a differential of $f(y) = e^{ay}$ with respect to x, we get

$$\frac{df(y)}{dx} = ae^{ay}\frac{dy(x)}{dx} = af(y)\frac{dy(x)}{dx} \tag{6}$$

Application of the differential transform to Eq. (6) gives

$$(k+1)y(k+1) = a\sum_{m=0}^{k}(m+1)Y(m+1)\,F(k-m) \tag{7}$$

Replacing $(k+1)$ by k gives

$$F(k)=a\sum_{m=0}^{k-1}\left(\frac{(m+1)}{k}\right)Y(m+1)\,F(k-1-m),\; k\geq 1 \tag{8}$$

Then from Eq. (5) and (8), we obtain the recursive relation

$$F(k)=\begin{cases} e^{aY(0)} & ,k=0 \\ a\sum_{m=0}^{k-1}\left(\frac{m+1}{k}\right)y(m+1)\,F(k-m-1), k\geq 1 \end{cases} \tag{9}$$

5. NUMERICAL APPLICATIONS

In this section we consider some nonlinear and linear problems in order to presents the effectiveness and applicability of proposed Aboodh differential transform transforms.

Example 5.1 Consider the simple linear fifth order differential equation [30]

$$u^5(x)-32u(x)=f(x), \tag{10}$$

subject to the conditions

$$u(0)=1,\; u^{'}(0)=3, u^{''}(0)=4,\; u^{'''}(0)=7, u^{iv}(0)=16. \tag{11}$$

First applying Aboodh transform on both sides of Eq. (10), we get

$$v^5k(v)-v^3f(0)-v^2f^{'}(0)-vf^{''}(0)-f^{'''}(0)-\frac{f^{iv}(0)}{v}=A[32u(x)+f(x)]$$

$$k(v)=\frac{1}{v^2}+\frac{3}{v^3}+\frac{4}{v^4}+\frac{7}{v^5}+\frac{16}{v^6}+\frac{1}{v^5}\;A[32u(x)+f(x)] \tag{12}$$

$k(v)$ is the Aboodh transform of u(x).

The standard Aboodh transformation method defines the solution u(x) by the series

$$u=\sum_{n=0}^{\infty}u(n). \tag{13}$$

Operating with Aboodh inverse on both sides of Eq. (12) gives

$$u(x)=1+3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+A^{-1}[\frac{1}{v^5}\;A[32u+f]] \tag{14}$$

Substituting Eq. (13) into Eq. (14) we find:

$$u(n+1)=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+A^{-1}[\frac{1}{v^5}\ A[32u(n)+f(n)]] \tag{15}$$

Here,

$$u(0)=1, \qquad n=0$$

$$u(1)=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+A^{-1}[\frac{1}{v^5}\ A[32u(0)+f(0)]]$$
$$=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+A^{-1}[\frac{1}{v^5}\ A[32(1)+(1)]]$$
$$=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+\frac{11}{4}x^5$$

Here,

$$u(1)=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+\frac{11}{4}x^5,\ n=1$$

$$u(2)=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+A^{-1}[\frac{1}{v^5}\ A[32u(1)+f(1)]]$$
$$=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+A^{-1}[\frac{1}{v^5}\ A[32(3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+\frac{11}{4}x^5)+(3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+\frac{11}{4}x^5)]]$$
$$=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+\frac{11}{4}x^5+\frac{11}{80}x^6+\frac{11}{420}x^7+\frac{11}{1920}x^8+\frac{11}{7560}x^9+\frac{11}{403200}x^{10}$$

The solution in a series form is given by

$$u(x)=u(0)+u(1)+u(2)+...$$

$$u(x)=1+2(3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+\frac{11}{4}x^5)+\frac{11}{80}x^6+\frac{11}{420}x^7+\frac{11}{1920}x^8+\frac{11}{7560}x^9+\frac{11}{403200}x^{10}+... \tag{16}$$

Example 5.2 Consider the simple second order differential equation

$$u''+u=0, \tag{17}$$

subject to the conditions

$$u(0)=2,\ u'(0)=3. \tag{18}$$

First applying Aboodh transform on both sides to find

$$v^2k(v)-\frac{f^{'}(0)}{v}-f(0)+A[u]=0$$

$$k(v)=\frac{3}{v^3}+\frac{2}{v^2}-\frac{1}{v^2}\ A[u] \tag{19}$$

$k(v)$ is the Aboodh transform of u(x),

The standard Aboodh transformation method defines the solution u(x) by the series

$$u=\sum_{n=0}^{\infty}u(n) \tag{20}$$

The inverse of Aboodh transform implies that

$$u(x)=2+3x-A^{-1}[\frac{1}{v^2}\ A[u]] \tag{21}$$

Substituting Eq. (20) into Eq. (21) we find:

$$u(n+1)=3x-A^{-1}[\frac{1}{v^2}\ A[u(n)]] \tag{22}$$

Here, $u(0)=2,\ n=0$

$$u(1)=3x-A^{-1}[\frac{1}{v^2}\ A[u(0)]]\quad =3x-A^{-1}[\frac{1}{v^2}\ A[2]]=3x-x^2$$

Here,

$u(1)=3x-x^2,\ n=1$

$$u(2)=3x-A^{-1}[\frac{1}{v^2}\ A[u(1)]]=3x-A^{-1}[\frac{1}{v^2}\ A[3x-x^2]]=3x-\frac{x^3}{2}-\frac{x^4}{12}$$

$u(2)=3x-\frac{x^3}{2}-\frac{x^4}{12},\ n=2$

$$u(3)=3x-A^{-1}[\frac{1}{v^2}\ A[u(2)]]=3x-A^{-1}[\frac{1}{v^2}\ A[3x-\frac{x^3}{2}-\frac{x^4}{12}]]=3x-\frac{x^3}{2}-\frac{x^5}{240}-\frac{x^6}{8640}$$

The solution in a series form is given by

$$u(x)=u(0)+u(1)+u(2)+u(3)+...$$

$$u(x)=2+9x-x^2-x^3-\frac{x^4}{12}-\frac{x^5}{240}-\frac{x^6}{8640}-... \tag{23}$$

Example 5.3 Consider the nonlinear BVP of order five [31]

$$u^{5}(x)=e^{-x}(u(x))^{3}, \tag{24}$$

subject to the conditions

$$u(0)=1,\ u^{'}(0)=\frac{1}{2}, u^{''}(0)=\frac{1}{4}, u(1)=e^{-1/3},\ y^{'}(1)=\left(\frac{-1}{3}\right)e^{-1/3} \tag{25}$$

$$t=\frac{u^{'''}(0)}{3!} \text{ And } w=\frac{u^{iv}(0)}{4!}$$

The constants *t* and *w* can be easily evaluated by using boundary conditions at *x=1*

$$t=0.0304127 \, and \, w=-0.0157735$$

By the value of *t* and *w* remaining initial conditions can be finding

$$u^{'''}(0)=0.1824, u^{iv}(0)=-0.378564.$$

First applying Aboodh transform on both sides to find

$$v^{5}k(v)-v^{3}f(0)-v^{2}f^{'}(0)-vf^{''}(0)-f^{'''}(0)-\frac{f^{iv}(0)}{v}=A[e^{-x}(u(x))^{3}]$$

$$k(v)=\frac{1}{v^{2}}+\frac{1}{2v^{3}}+\frac{1}{4v^{4}}+\frac{0.1824}{v^{5}}-\frac{0.378564}{v^{6}}+\frac{1}{v^{5}}\ A[e^{-x}(u(x))^{3}] \tag{26}$$

$k(v)$ is the Aboodh transform of u(x) ,

Operating with Aboodh inverse on both sides of Eq. (26) gives

$$u(t)=1+\frac{x}{2}+\frac{x^{2}}{8}+(0.0304)x^{3}-(0.01577)x^{4}+A^{-1}[\frac{1}{v^{5}}A[e^{-x}(u)^{3}]] \tag{27}$$

The recursive relation is given by:

$$u(n+1)=\frac{x}{2}+\frac{x^{2}}{8}+(0.0304)x^{3}-(0.01577)x^{4}+A^{-1}[\frac{1}{v^{5}}A[F(n).(u(n))^{3}]] \tag{28}$$

Here,

$$u(0)=1, n=0;\ F(n)=\begin{cases} u(0) & ,n=0 \\ \sum_{m=0}^{n-1}\frac{m+1}{n}u(m+1)F(n-m-1) & \end{cases}$$

$$u(1)=\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+A^{-1}[\frac{1}{v^5}A[F(0).(u(0))^3]]$$

$$u(1)=\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+A^{-1}[\frac{1}{v^5}A[1]]$$
$$=\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+(0.008334)x^5$$

Here,

$$u(1)=3x+2x^2+\frac{7}{6}x^3+\frac{2}{3}x^4+\frac{11}{4}x^5,\ n=1$$

$$F(1)=\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+(0.008334)x^5$$

$$u(2)=\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+A^{-1}[\frac{1}{v^5}A[F(1).(u(1))^3]]$$

$$\begin{aligned}u(2)=&\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+\\ &A^{-1}[\frac{1}{v^5}A[(\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+(0.008334)x^5)\\ &(\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+(0.008334)x^5)^3]]\end{aligned}$$

$$\begin{aligned}u(2)=&\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4+(4.1335978\times10^{-6})x^9+(2.066798\times10^{-6})x^{10}\\ &+(6.9692460\times10^{-7})x^{11}+(7.8085543\times10^{-8})x^{12}+(1.770027\times10^{-8})x^{13}\\ &+(4.741966367\times10^{-9})x^{14}+(3.371435\times10^{-9})x^{15}+(1.41321895\times10^{-10})x^{16}\\ &+(6.52172215\times10^{-11})x^{17}-(7.0205220\times10^{-12})x^{18}+(1.4321895\times10^{-11})x^{19}\\ &-(1.1987336\times10^{-12})x^{20}+(2.93944419\times10^{-13})x^{21}-(7.622876636\times10^{-14})x^{22}\\ &+(4.309835\times10^{-14})x^{23}-(7.158816425\times10^{-14})x^{24}+(7.5664564\times10^{-16})x^{25}\end{aligned}$$

The solution in a series form is given by

$$u(x)=u(0)+u(1)+u(2)+...$$

$$\begin{aligned}u(x)=&1+2(\frac{x}{2}+\frac{x^2}{8}+(0.0304)x^3-(0.01577)x^4)+(4.1335978\times10^{-6})x^9+(2.066798\times10^{-6})x^{10}\\ &+(6.9692460\times10^{-7})x^{11}+(7.8085543\times10^{-8})x^{12}+(1.770027\times10^{-8})x^{13}\\ &+(4.741966367\times10^{-9})x^{14}+(3.371435\times10^{-9})x^{15}+(1.41321895\times10^{-10})x^{16}\\ &+(6.52172215\times10^{-11})x^{17}-(7.0205220\times10^{-12})x^{18}+(1.4321895\times10^{-11})x^{19}\\ &-(1.1987336\times10^{-12})x^{20}+(2.93944419\times10^{-13})x^{21}-(7.622876636\times10^{-14})x^{22}\\ &+(4.309835\times10^{-14})x^{23}-(7.158816425\times10^{-14})x^{24}+(7.5664564\times10^{-16})x^{25}+...\end{aligned}\tag{29}$$

Example 5.4 Consider the nonlinear problem of order five [31]

$$u^{5}(x) = e^{-x}(u(x))^{3}, \quad 0 < x < 1\,, \tag{30}$$

subject to the conditions

$$u(0) = 1,\ u^{'}(0) = \frac{1}{2}, u^{''}(0) = \frac{1}{4}, u^{'''}(0) = 0.376764, u^{iv}(0) = -2.154638929. \tag{31}$$

First applying Aboodh transform on both sides to find:

$$v^{5}k(v) - v^{3} f(0) - v^{2} f^{'}(0) - vf^{''}(0) - f^{'''}(0) - \frac{f^{iv}(0)}{v} = A[e^{-x}(u(x))^{3}]$$

$$k(v) = \frac{1}{v^{2}} + \frac{1}{2v^{3}} + \frac{1}{4v^{4}} + \frac{0.376764}{v^{5}} - \frac{2.154638929}{v^{6}} + \frac{1}{v^{5}}\ A[e^{-x}(u(x))^{3}] \tag{32}$$

$k(v)$ is the Aboodh transform of u(x).

Operating with Aboodh inverse on both sides of Eq. (32) gives

$$u(x) = 1 + \frac{x}{2} + \frac{x^{2}}{8} + (0.062794)x^{3} - (0.104776622)x^{4} + A^{-1}[\frac{1}{v^{5}} A[e^{-x}(u)^{3}]]$$

The recursive relation is given by

$$u(n+1) = \frac{x}{2} + \frac{x^{2}}{8} + (0.062794)x^{3} - (0.104776622)x^{4} + A^{-1}[\frac{1}{v^{5}} A[F(n).(u(n))^{3}]] \tag{33}$$

Here,

$$u(0) = 1, n = 0;\ F(n) = \begin{cases} u(0) & , n = 0 \\ \sum_{m=0}^{n-1} \frac{m+1}{n} u(m+1) F(n-m-1) \end{cases}$$

$$u(1) = \frac{x}{2} + \frac{x^{2}}{8} + (0.062794)x^{3} - (0.104776622)x^{4} + A^{-1}[\frac{1}{v^{5}} A[F(0).(u(0))^{3}]]$$

$$u(1) = \frac{x}{2} + \frac{x^{2}}{8} + (0.062794)x^{3} - (0.104776622)x^{4} + A^{-1}[\frac{1}{v^{5}} A[1]]$$
$$= \frac{x}{2} + \frac{x^{2}}{8} + (0.062794)x^{3} - (0.104776622)x^{4} + \frac{x^{5}}{120}$$

Here,

$$u(1) = \frac{x}{2} + \frac{x^{2}}{8} + (0.062794)x^{3} - (0.104776622)x^{4} + \frac{x^{5}}{120}, \qquad n = 1$$

$$F(1)=\frac{x}{2}+\frac{x^2}{8}+(0.062794)x^3-(0.104776622)x^4+\frac{x^5}{120}$$

$$u(2)=\frac{x}{2}+\frac{x^2}{8}+(0.062794)x^3-(0.104776622)x^4+A^{-1}[\frac{1}{v^5}A[F(1).(u(1))^3]]$$

$$\begin{aligned}u(2)=&\frac{x}{2}+\frac{x^2}{8}+(0.062794)x^3-(0.104776622)x^4\\&+A^{-1}[\frac{1}{v^5}A[(\frac{x}{2}+\frac{x^2}{8}+(0.062794)x^3-(0.104776622)x^4+(0.008334)x^5)\\&(\frac{x}{2}+\frac{x^2}{8}+(0.062794)x^3-(0.104776622)x^4+(0.008334)x^5)^3]]\end{aligned}$$

$$\begin{aligned}u(2)=&\frac{x}{2}+\frac{x^2}{8}+(0.062794)x^3-(0.104776622)x^4+(4.1335978\times10^{-6})x^9+(2.06679894\times10^{-6})x^{10}\\&+(9.8907829\times10^{-7})x^{11}-(2.6235585\times10^{-7})x^{12}-(1.494360269\times10^{-7})x^{13}\\&-(9.568722944\times10^{-8})x^{14}+(2.496281496\times10^{-8})x^{15}+(5.485977564\times10^{-9})x^{16}\\&+(4.916639733\times10^{-9})x^{17}-(2.0332535\times10^{-9})x^{18}+(7.879830295\times10^{-11})x^{19}\\&-(1.23687973\times10^{-10})x^{20}+(7.426327256\times10^{-11})x^{21}-(1.377661325\times10^{-11})x^{22}\\&+(1.168826216\times10^{-12})x^{23}-(4.7551995\times10^{-14})x^{24}+(7.564042387\times10^{-16})x^{25}\end{aligned}$$

The solution in a series form is given by

$$u(x)=u(0)+u(1)+u(2)+...$$

$$\begin{aligned}u(x)=&1+2(\frac{x}{2}+\frac{x^2}{8}+(0.062794)x^3-(0.104776622)x^4)+\frac{x^5}{120}+(4.1335978\times10^{-6})x^9+(2.06679894\times10^{-6})x^{10}\\&+(9.8907829\times10^{-7})x^{11}-(2.6235585\times10^{-7})x^{12}-(1.494360269\times10^{-7})x^{13}\\&-(9.568722944\times10^{-8})x^{14}+(2.496281496\times10^{-8})x^{15}+(5.485977564\times10^{-9})x^{16}\\&+(4.916639733\times10^{-9})x^{17}-(2.0332535\times10^{-9})x^{18}+(7.879830295\times10^{-11})x^{19}\\&-(1.23687973\times10^{-10})x^{20}+(7.426327256\times10^{-11})x^{21}-(1.377661325\times10^{-11})x^{22}\\&+(1.168826216\times10^{-12})x^{23}-(4.7551995\times10^{-14})x^{24}+(7.564042387\times10^{-16})x^{25}+...\end{aligned}\tag{34}$$

Example 5.5 Consider the nonlinear problem of order Six [31]

$$u^6(x)=e^x(u(x))^3,\quad 0<x<1,\tag{35}$$

subject to the conditions

$$\begin{aligned}&u(0)=1,\ u^{'}(0)=-\frac{1}{2},u^{''}(0)=\frac{1}{4},u^{'''}(0)=-14.69918030,u^{iv}(0)=102.1542319\\&u^{v}(0)=-219.2146156.\end{aligned}\tag{36}$$

First applying Aboodh transform on both sides to find

$$v^6k(v)-v^4 f(0)-v^3 f^{'}(0)-v^2 f^{''}(0)-vf^{'''}(0)-f^{iv}(0)-\frac{f^{v}(0)}{v}=A[e^x(u(x))^3]$$

$$k(v)=\frac{1}{v^2}-\frac{1}{2v^3}+\frac{1}{4v^4}-\frac{14.69918030}{v^5}+\frac{102.1542319}{v^6}-\frac{219.2146156}{v^7}+\frac{1}{v^6}A[e^x(u(x))^3] \quad (37)$$

$k(v)$ is the Aboodh transform of u(x).

Operating with Aboodh inverse on both sides of Eq. (37) gives

$$u(t)=1-\frac{x}{2}+\frac{x^2}{8}-(2.449863383)x^3+(4.256426329)x^4-1.826788463x^5+A^{-1}[\frac{1}{v^6}A[e^x(u)^3]]$$

The recursive relation is given by

$$u(n+1)=-\frac{x}{2}+\frac{x^2}{8}-(2.449863383)x^3+(4.256426329)x^4-1.826788463x^5+A^{-1}[\frac{1}{v^6}A[F(n).(u(n))^3]] \quad (38)$$

Here,

$$u(0)=1,\ n=0;\ F(n)=\begin{cases}u(0) & ,n=0\\ \sum_{m=0}^{n-1}\frac{m+1}{n}u(m+1)F(n-m-1)\end{cases}$$

$$u(1)=-\frac{x}{2}+\frac{x^2}{8}-(2.449863383)x^3+(4.256426329)x^4-1.826788463x^5+A^{-1}[\frac{1}{v^6}A[F(0).(u(0))^3]]$$

$$u(1)=-\frac{x}{2}+\frac{x^2}{8}-(2.449863383)x^3+(4.256426329)x^4-1.826788463x^5+A^{-1}[\frac{1}{v^6}A[1]]$$
$$=-\frac{x}{2}+\frac{x^2}{8}-(2.449863383)x^3+(4.256426329)x^4-1.826788463x^5+\frac{x^6}{720}$$

Here,

$$u(1)=-\frac{x}{2}+\frac{x^2}{8}-(2.449863383)x^3+(4.256426329)x^4-1.826788463x^5+\frac{x^6}{720}, \qquad n=1$$

$$F(1)=-\frac{x}{2}+\frac{x^2}{8}-(2.449863383)x^3+(4.256426329)x^4-1.826788463x^5+\frac{x^6}{720}$$
$$u(2)=-\frac{x}{2}+\frac{x^2}{8}-(2.449863383)x^3+(4.256426329)x^4-1.826788463x^5+A^{-1}[\frac{1}{v^6}A[F(1).(u(1))^3]]$$

$$u(2) = -\frac{x}{2} + \frac{x^2}{8} - (2.449863383)x^3 + (4.256426329)x^4 - 1.826788463x^5$$
$$+ A^{-1}[\frac{1}{v^5} A[(-\frac{x}{2} + \frac{x^2}{8} - (2.449863383)x^3 + (4.256426329)x^4 - 1.826788463x^5 + \frac{x^6}{720})$$
$$(-\frac{x}{2} + \frac{x^2}{8} - (2.449863383)x^3 + (4.256426329)x^4 - 1.826788463x^5 + \frac{x^6}{720})^3]]$$

$$u(2) = -\frac{x}{2} + \frac{x^2}{8} - (2.449863383)x^3 + (4.256426329)x^4 - 1.826788463x^5$$
$$+ (4.1335978 \times 10^{-7})x^{10} - (1.8789081 \times 10^{-7})x^{11} + (1.87645803 \times 10^{-6})x^{12}$$
$$-(2.469259907 \times 10^{-6})x^{13} + (5.430772931 \times 10^{-6})x^{14} - (1.023651349 \times 10^{-5})x^{15}$$
$$+(1.498820624 \times 10^{-5})x^{16} - (2.31448188 \times 10^{-5})x^{17} + (3.184650997 \times 10^{-5})x^{18}$$
$$-(3.386754673 \times 10^{-5})x^{19} + (3.95489336 \times 10^{-5})x^{20} - (3.745014088 \times 10^{-5})x^{21}$$
$$+(2.709332005 \times 10^{-5})x^{22} - (1.3553605 \times 10^{-5})x^{23} + (4.370785 \times 10^{-6})x^{24}$$
$$-(8.19381705 \times 10^{-7})x^{25} + (6.8613150 \times 10^{-8})x^{26} - (1.597553011 \times 10^{-10})x^{27}$$
$$+(1.42561109 \times 10^{-13})x^{28} - (5.72408647 \times 10^{-9})x^{29} + (8.7039344 \times 10^{-21})x^{30}$$

The solution in a series form is given by

$$u(x) = u(0) + u(1) + u(2) + ...$$

$$u(x) = 1 + 2(-\frac{x}{2} + \frac{x^2}{8} - (2.449863383)x^3 + (4.256426329)x^4 - 1.826788463x^5) + \frac{x^6}{720}$$
$$+ (4.1335978 \times 10^{-7})x^{10} - (1.8789081 \times 10^{-7})x^{11} + (1.87645803 \times 10^{-6})x^{12}$$
$$-(2.469259907 \times 10^{-6})x^{13} + (5.430772931 \times 10^{-6})x^{14} - (1.023651349 \times 10^{-5})x^{15}$$
$$+(1.498820624 \times 10^{-5})x^{16} - (2.31448188 \times 10^{-5})x^{17} + (3.184650997 \times 10^{-5})x^{18}$$
$$-(3.386754673 \times 10^{-5})x^{19} + (3.95489336 \times 10^{-5})x^{20} - (3.745014088 \times 10^{-5})x^{21}$$
$$+(2.709332005 \times 10^{-5})x^{22} - (1.3553605 \times 10^{-5})x^{23} + (4.370785 \times 10^{-6})x^{24}$$
$$-(8.19381705 \times 10^{-7})x^{25} + (6.8613150 \times 10^{-8})x^{26} - (1.597553011 \times 10^{-10})x^{27}$$
$$+(1.42561109 \times 10^{-13})x^{28} - (5.72408647 \times 10^{-9})x^{29} + (8.7039344 \times 10^{-21})x^{30} + ... \quad (39$$

6. CONCLUSION

In this paper, the analytical solutions of nonlinear higher order initial value problems subject to the appropriate initial conditions which arise frequently in mathematical physics are obtained by using the powerful tool new Aboodh differential transform method. The coupling works fine and yields remarkable solutions for the considered linear and nonlinear initial value problems.

REFERENCES

[1] Adomian, G., *Journal of Mathematical Analysis and Applications*, **102**(2), 420, 1984.
[2] Ahmad, J., Mushtaq, M., Sajjad, N., *Journal of Science and Arts*, **1**(30), 5, 2015
[3] Abbasbandy, S., *International Journal for Numerical Methods in Engineering*, **70**(7), 876, 2007.
[4] Ahmad, J., Aniqa, *Journal of Science and Arts*, **3**(40), 443, 2017.
[5] Noor, M.A., Mohyud-Din, S.T., *International Journal of Nonlinear Science*, **8**(1), 27, 2009.
[6] Ahmad, J., Nosher, H., *Journal of Science and Arts*, **1**(38), 5, 2017.
[7] Liao, S.J., *Beyond Perturbation: Introduction to the Homotopy Analysis Method*, Chapman & Hall/CRC, Boca Raton, USA, 2003.
[8] Zhang, J.L., Wang, M.L., Wang, Y.M., Fang, Z.D., *Physics Letters, Section A: General, Atomic and Solid State Physics*, **350**(1-2), 103, 2006.
[9] Mohyud-Din, S.T., Noor, M.A., Noor, K., Hosseini, M.M., *International Journal of Nonlinear Sciences and Numerical Simulation*, **11**(2), 87, 2010.
[10] Wazwaz, A.M., *Mathematical and Computer Modelling*, **40**(5-6), 499, 2004.
[11] Zhou, J.K., *Differential Transformation and Its Application for Electrical Circuits*, Huazhong University Press, Wuhan, China, 1986.
[12] Ahmad, J., Mohyud-Din, S.T., *PLoS ONE*, **9**(12), Article ID e109127, 2014.
[13] Arikoglu, A., Ozkol, I., *Chaos, Solitons& Fractals*, **34**(5), 1473, 2007.
[14] Kurnaz, A., Oturanç, G., *International Journal of Computer Mathematics*, **82**(6), 709, 2005.
[15] Saravanan, A., Magesh, N., *Journal of the Egyptian Mathematical Society*, **21**(3), 259, 2013.
[16] Abazari, R., Abazari, M., *Computational & Applied Mathematics*, **32**(1), 1, 2013.
[17] Neog, B.C., *IOSR Journal of Mathematics*, **11**(5), 37, 2015.
[18] Abazari, R., Soltanalizadeh, B., *Thai Journal of Mathematics*, **11**(1), 199, 2013.
[19] Ayaz, F., *Appl. Math. Comput.*, **147**, 547, 2004.
[20] Ayaz, F., *Appl. Math. Comput.*, **152**, 649, 2004.
[21] Chen, C.K., *Appl. Math. Comput.*, 106, 171, 1999.
[22] Guoqiang, H., Jiong, W., *J. Comput. Appl. Math.*, **134**,259, 2001.
[23] Guoqiang, H., Ruifang, W., *J. Comput. Appl. Math.*, **139**, 49, 2002.
[24] Arikoglu, A., Ozkol, I., *Appl. Math. Comput.*, **168**, 1145, 2005.
[25] Alamri, S., *Journal of Progressive Research in Mathematics*, **5**(3), 535, 2015.
[26] Odibat, Z.M., *Math. Comput. Model.*, **48**(7-8), 1144, 2008.
[27] Aboodh, K.S., *Global Journal of pure and Applied Mathematics*, **9**(1), 35, 2013.
[28] Aboodh, K.S., *Global Journal of pure and Applied Mathematics*, **10**(2), 249, 2014.

[29] Mohand, M., Mahgob, A., Hassan Sedeeg, A.K., *American Journal of Applied Mathematics*, **4**(5), 217, 2016.
[30] Hassan Sedeeg, A.K., Abdelrahim Mahgoub, M.M., *Mathematical Theory and Modeling*, **6**(8), 108, 2016.
[31] Hassan Sedeeg, A.K., Abdelrahim Mahgoub, M.M., *International Journal of Development Research*, **6**(8), 9085, 2016.
[32] Alshikh, A.A., Abdelrahim Mahgoub, M.M., *Pure and Applied Mathematics Journal*, **5**(5), 145, 2016.
[33] Bhrawy, A.H., Alghamdi, M.A., *Abstract and Applied Analysis*, **2012**, Article ID 364360, 2012.
[34] Hussin, C.H.C., Kilicman, A., *Mathematical Problems in Engineering*, **2011**, Article ID 724927, 2011.