ORIGINAL PAPER

STUDY OF NUMERICAL ACCURACY OF ONE DIMENSIONAL HEAT EQUATION BY BENDER-SCHMIDT METHOD, CRANK-NICHOLSON DIFFERENCE METHOD AND DU FORT AND FRANKEL METHOD

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Abstract. *In this research paper we introduce the numerical accuracy of solution one dimensional heat equation by different numerical scheme.The error analysis of different method also discussed. This study can be used to suggest the better accuracy for solving a partial differential equation. This comparative study provides better result for numerical solution of parabolic type equation.*

Keywords: *Partial differential equation, one dimensional heat equation, numerical accuracy.*

1. INTRODUCTION

There are many problems in the field of science, engineering and technology which can be solved by partial differential equations formulation. This research will compare the accuracy of various method Bender-Schmidt Method, Crank-Nicholson Difference Method and Du Fort and Frankel Method, in completing numerical solutions of partial differential equations, which is limited to certain boundary condition [1, 3, 7-9].

The general linear partial differential equation of the second order in two independent variables is of the form:

$$A(x,y)\frac{\partial^2 u}{\partial x^2} + B(x,y)\frac{\partial^2 u}{\partial x \partial y} + C(x,y)\frac{\partial^2 u}{\partial y^2} + F\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0$$

The one dimensional heat equation, namely,

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

where $\alpha^2 = \dfrac{k}{pc}$ is an example of parabolic equation. If $\alpha^2 = \dfrac{1}{a}$, the equation becomes

$$\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}$$

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The solution of this equation is a temperature function $u(x,t)$ which is defined for values of x from 0 to ℓ and for values of time t from 0 to ∞. The solution is not defined in a closed domain but advances in an open-ended region from initial values, satisfying the prescribed boundary satisfying the prescribed boundary conditions in general, the study of pressure waves in a fluid propagation of heat and unsteady state problems lead to parabolic type of equations. In this paper we are studying the finite difference algorithm for solving parabolic partial differential equation[2].

2. METHODS

Example 1 (solution of one dimensional heat equation):

Solve the boundary value problem $u_t = u_{xx}$ under the conditions u(0, t) = u(1, t) = 0 and u(x, 0) = $\sin \pi x$, $0 \le x \le 1$ taking h = 0.2 and α =1/2.

Solution:

We know that the suitable solution of one dimensional heat equation $u_t = c^2 u_{xx}$ is given by

$$u = (c_1 \cos px + c_2 \sin px).e^{-c^2 p^2 t} \quad , \quad \text{where } c^2 = 1 \tag{1}$$

Therefore,

$$u = (c_1 \cos px + c_2 \sin px).e^{-p^2 t} \tag{2}$$

Conditions are:

(i) u = 0, x = 0
(ii) u = 0, x = 1
(iii) u = $\sin \pi x$ at t = 0, $0 \le x \le 1$

Using condition (i) in equation (2), we get

$$0 = c_1 e^{-p2t}$$

$$c_1 = 0$$

From equation (2)

$$u = (c_2 \sin px).e^{-p^2 t} \tag{3}$$

Now using condition (ii) in equation (3)

$$0 = (c_2 \sin p).e^{-p^2 t}$$

$$\Rightarrow \sin p = 0 = \sin n\pi, \qquad n \in I$$

$$\Rightarrow p = n\pi$$

From (3)

$$u = (c_2 \sin p).e^{-n^2\pi^2 t} \tag{4}$$

Most general solution of given equation is

$$u = \sum_{n=1}^{\infty} a_n \sin n\pi x.e^{-n^2\pi^2 t} \tag{5}$$

Now applying condition (iii) in equation (5), we get

$$\sin \pi x = \sum_{n=1}^{\infty} a_n \sin n\pi x$$

$$\Rightarrow \sin \pi x = a_1 \sin \pi x + a_2 \sin 2\pi x + a_3 \sin 3\pi x + ...$$

$$\Rightarrow a_1 = 1, \qquad \mathrm{a}_2 = a_3 = ... = 0$$

Now putting these value in equation (5)

$$u = \sin \pi x.e^{-\pi^2 t}$$

	$x\rightarrow$	0	0.2	0.4	0.6	0.8	1
$t\downarrow$	$j \setminus i$	0	1	2	3	4	5
0	0	0	0.587785252	0.951056516	0.951056516	0.587785252	0
0.02	1	0	0.482494526165092	0.780692542720894	0.780692542720894	0.482494526165092	0
0.04	2	0	0.396064662853158	0.640846086239177	0.640846086239177	0.396064662853158	0
0.06	3	0	0.325117091809890	0.526050504871922	0.526050504871922	0.325117091809890	0
0.08	4	0	0.266878450163855	0.431818403230012	0.431818403230012	0.266878450163855	0
0.10	5	0	0.219072171091850	0.354466218815846	0.354466218815846	0.219072171091850	0

2.1. BENDER-SCHMIDT METHOD

Consider one dimensional heat equation, namely,

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \tag{a}$$

where $\alpha^2 = \dfrac{k}{pc}$ is an example of parabolic equation. $\alpha^2 = \dfrac{1}{a}$, the equation becomes,

$$\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}$$

With boundary conditions, $u(0, t) = T_0$, $u(\ell, t) = T_\ell$ and with initial condition $u(x, 0) = f(x)$, $0<x<\ell$.

Consider a rectangular mesh in the x-t plane with spacing h along x direction and k along time t direction. Denoting a mesh point $(x, t) = (ih, jk)$ as simply i, j, we have

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{k}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

Substituting these in (a), we obtain

$$u_{i,j+1} - u_{i,j} = \frac{k}{ah^2}\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right]$$

or $u_{i,j+1} - u_{i,j} = \lambda\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right]$ where $\lambda = \frac{k}{ah^2}$

i.e., $u_{i,j+1} = \lambda u_{i+1,j} + (1-2\lambda)u_{i,j} + \lambda u_{i-1,j}$ (b)

Equation (b) is called Explict Formula. It is valid if $0<\lambda \le \frac{1}{2}$

If we take, λ =1/2, the coefficient of $u_{i,j}$ vanishes.

Hence equation (b) becomes $u_{i,j} = \frac{1}{2}\left[u_{i-1,j} + u_{i+1,j}\right]$ [2, 6].

	$x\rightarrow$	0	0.2	0.4	0.6	0.8	1
$t\downarrow$	$j \setminus i$	0	1	2	3	4	5
0	0	0	0.587785252	0.951056516	0.951056516	0.587785252	0
0.02	1	0	0.475528258	0.769420884	0.769420884	0.475528258	0
0.04	2	0	0.384710442	0.622474571	0.622474571	0.384710442	0
0.06	3	0	0.311237285	0.503592506	0.503592506	0.311237285	0
0.08	4	0	0.251796253	0.407414895	0.407414895	0.251796253	0
0.10	5	0	0.203707447	0.329605574	0.329605574	0.203707447	0

2.2. CRANK-NICHOLSON DIFFERENCE METHOD

To solve the parabolic equation $u_{xx} = au_t$ with the boundary conditions u(0, t) = T_0, u(ℓ, t) = T_1 and the initial condition u(x, 0) = f(x) [5, 10]. At $u_{i,j}$

$$u_{xx} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

and at $u_{i,j+1}$

$$u_{xx} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2}$$

Taking average of these two values

$$u_{xx} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1} + u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{2h^2}$$

Using $u_t = \dfrac{u_{i,j+1} - u_{i,j}}{k^2}$

$$a\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1} + u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{2h^2}$$

Setting $\dfrac{k}{ah^2} = \lambda$, the above equation reduces to

$$\frac{1}{2}\lambda u_{i+1,j+1} + \frac{1}{2}\lambda u_{i-1,j+1} - (\lambda + 1)u_{i,j+1} = -\frac{1}{2}\lambda u_{i+1,j} - \frac{1}{2}\lambda u_{i-1,j} + (\lambda - 1)u_{i,j}$$

This can be written as,

$$\lambda(u_{i+1,j+1} + u_{i-1,j+1}) - 2(\lambda + 1)u_{i,j+1} = 2(\lambda - 1)u_{i,j} - \lambda(u_{i+1,j} + u_{i-1,j}) \qquad \text{(c)}$$

Equation (c) is called Crank-Nicholson difference scheme or method.
Setting λ =1 i.e. $k = ah^2$ The Crank-Nicholson formula reduces to

$$u_{i,j+1} = \frac{1}{4}\left[u_{i-1,j+1} + u_{i+1,j+1} + u_{i-1,j} + u_{i+1,j}\right] \qquad \text{(d)}$$

	$x\rightarrow$	0	0.2	0.4	0.6	0.8	1
$t\downarrow$	$j \setminus i$	0	1	2	3	4	5
0	0	0	0.587785252	0.951056516	0.951056516	0.587785252	0
0.02	1	0	0.485313444827586	0.785253648965517	0.785253648965517	0.485313444827586	0
0.04	2	0	0.400706106448276	0.648356099689655	0.648356099689655	0.400706106448276	0
0.06	3	0	0.330848826586207	0.535324646517241	0.535324646517241	0.330848826586207	0
0.08	4	0	0.273170147103448	0.441998582620690	0.441998582620690	0.273170147103448	0
0.10	5	0	0.225546906034483	0.364942560206897	0.364942560206897	0.225546906034483	0

2.3. DU FORT AND FRANKEL METHOD

If we replace the derivative in (a) by the central difference approximations,

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k}$$

and

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

We obtain $u_{i,j+1} - u_{i,j-1} = \frac{2kc^2}{h^2}\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right]$

i.e., $u_{i,j} = u_{i,j-1} + 2\alpha\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right]$ (e)

where $\alpha = kc^2/h^2$. This difference equation is called the Richardson scheme which is a 3-level method.

If we replace $u_{i,j}$ by the mean of the values $u_{i,j-1}$ and $u_{i,j+1}$ i. e. $u_{i,j} = \frac{1}{2}\left(u_{i,j-1} + u_{i,j+1}\right)$ in (e), we get $u_{i,j+1} = u_{i,j-1} + 2\alpha\left[u_{i-1,j} - 2(u_{i,j-1} + u_{i,j+1}) + u_{i+1,j}\right]$.

On simplification, it can be written as

$$u_{i,j+1} = \frac{1-2\alpha}{1+2\alpha}u_{i,j-1} + \frac{2\alpha}{1+2\alpha}\left[u_{i-1,j} + u_{i+1,j}\right]$$

This difference scheme is called Du Fort-Frankel method [4, 7].

	x→	0	0.2	0.4	0.6	0.8	1
t↓	j \ i	0	1	2	3	4	5
0	0	0	0.587785252	0.951056516	0.951056516	0.587785252	0
0.02	1	0	0.475528258	0.769420884	0.769420884	0.475528258	0
0.04	2	0	0.384710442	0.622474571	0.622474571	0.384710442	0
0.06	3	0	0.311237285	0.503592506	0.503592506	0.311237285	0
0.08	4	0	0.251796253	0.407414895	0.407414895	0.251796253	0
0.10	5	0	0.203707447	0.329605574	0.329605574	0.203707447	0

Example 2 (solution of one dimensional heat equation):

Find the temperature in a bar of length 2 whose ends are kept at zero and lateral surface insulated if the initial temperature is $\sin\frac{\pi x}{2}+3\sin\frac{5\pi x}{2}$. Given that *u(0, t) = 0 = u(2, t)* for any *t*.

Solution:

The initial condition is $u(x,0)=\sin\frac{\pi x}{2}+3\sin\frac{5\pi x}{2}$.

One dimensional heat flow equation is $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, where $c^2 = 1$.

Its solution is

$$u(x,t)=(c_1\cos px+c_2\sin px).e^{-p^2t} \tag{6}$$

Using condition $u(x,t)=c_1e^{-p^2t}$

$$\Rightarrow c_1 = 0$$

From (6)

$$u(x,t)=c_2\sin px.e^{-p^2t} \tag{7}$$

$$u(2,t)=0=c_2\sin 2p.e^{-p^2t}$$

$$\sin 2p=0=\sin n\pi$$

$$p=\frac{n\pi}{2},\qquad n\in I$$

Hence from (7)

$$u(x,t)=b_n\sin\frac{n\pi x}{2}.e^{-\frac{n^2\pi^2t}{4}}$$

The most general solution is

$$u(x,t)=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}.e^{-\frac{n^2\pi^2 t}{4}} \tag{8}$$

$$u(x,0)=\sin\frac{\pi x}{2}+3\sin\frac{5\pi x}{2}=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$$

$$u(x,0)=b_1\sin\frac{\pi x}{2}+b_2\sin\frac{2\pi x}{2}+.....+b_5\sin\frac{5\pi x}{2}+.......$$

Comparing , we get $b_1 = 1$ and $b_5 = 3$.
Hence from (8),

$$u(x,t)=\sin\frac{\pi x}{2}.e^{-\frac{\pi^2 t}{4}}+3\sin\frac{5\pi x}{2}.e^{-\frac{25\pi^2 t}{4}} \tag{9}$$

	x→	0	0.2	0.4	0.6	0.8	1
t↓	j \ i	0	1	2	3	4	5
0	0	0	0.587785252292473	0.951056516295153	0.951056516295153	0.587785252292473	0
0.02	1	0	0.532544051503428	0.861674375839121	0.861674375839121	0.532544051503428	0
0.04	2	0	0.482494526165092	0.780692542720894	0.780692542720894	0.482494526165092	0
0.06	3	0	0.437148752524894	0.707321539724895	0.707321539724895	0.437148752524894	0
0.08	4	0	0.396064662853158	0.640846086239177	0.640846086239177	0.396064662853158	0
0.10	5	0	0.358841735804913	0.580618125114360	0.580618125114360	0.358841735804913	0

2.4. BENDER-SCHMIDT METHOD

The initial condition is $u(x,0)=\sin\frac{\pi x}{2}+3\sin\frac{5\pi x}{2}$.

One dimensional heat flow equation is $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, where $c^2 = 1$

h = 2/5 , α = ¼, k = 1/25, u(0, t) = 0 = u(2, t) for any t

	x→	0	0.2	0.4	0.6	0.8	1
t↓	j \ i	0	1	2	3	4	5
0	0	0	0.587785252292473	0.951056516295153	0.951056516295153	0.587785252292473	0
0.02	1	0	0.531656755220025	0.860238700294483	0.860238700294483	0.531656755220025	0
0.04	2	0	0.480888052683633	0.778093214025869	0.778093214025869	0.480888052683633	0
0.06	3	0	0.434967329848284	0.703791923690310	0.703791923690310	0.434967329848284	0
0.08	4	0	0.393431645846720	0.636585775229804	0.636585775229804	0.393431645846720	0
0.10	5	0	0.355862266730811	0.575797242884033	0.575797242884033	0.355862266730811	0

2.5. CRANK-NICHOLSON DIFFERENCE METHOD

The initial condition is $u(x,0) = \sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$. One dimensional heat flow equation is $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $c^2 = 1$, h = 2/5 , α = ¼, k = 1/25, u(0, t) = 0 = u(2, t) for any *t*.

	x→	0	0.2	0.4	0.6	0.8	1
t↓	*j* \ *i*	0	1	2	3	4	5
0	0	0	0.587785252292473	0.951056516295153	0.951056516295153	0.587785252292473	0
0.02	1	0	0.534254429627061	0.861377266220623	0.861377266220623	0.534254429627061	0
0.04	2	0	0.481526240326048	0.781597959277497	0.781597959277497	0.481526240326048	0
0.06	3	0	0.440275399622164	0.710998594987854	0.710998594987854	0.440275399622164	0
0.08	4	0	0.395017578640727	0.641924793686430	0.641924793686430	0.395017578640727	0
0.10	5	0	0.360505208137336	0.581781815842564	0.581781815842564	0.360505208137336	0

2.6. DU FORT AND FRANKEL METHOD

The initial condition is $u(x,0) = \sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$. One dimensional heat flow equation is $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $c^2 = 1$, h = 2/5 , α = ¼, k = 1/25, u(0, t) = 0 = u(2, t) for any *t*.

	x→	0	0.2	0.4	0.6	0.8	1
t↓	*j* \ *i*	0	1	2	3	4	5
0	0	0	0.587785252292473	0.951056516295153	0.951056516295153	0.587785252292473	0
0.02	1	0	0.531656755220025	0.860238700294483	0.860238700294483	0.531656755220025	0
0.04	2	0	0.482674650862319	0.780983990603220	0.780983990603220	0.482674650862319	0
0.06	3	0	0.437546915274415	0.707965780586674	0.707965780586674	0.437546915274415	0
0.08	4	0	0.396880143816331	0.642165562154770	0.642165562154770	0.396880143816331	0
0.10	5	0	0.359904159143062	0.582337162185925	0.582337162185925	0.359904159143062	0

2.7. ERROR ANALYSIS

	x→	0	0.2	0.4	0.6	0.8	1
t↓	*j* \ *i*	0	1	2	3	4	5
0	0	0	u_1	u_2	u_3	u_4	0
0.02	1	0	u_5	u_6	u_7	u_8	0
0.04	2	0	u_9	u_{10}	u_{11}	u_{12}	0
0.06	3	0	u_{13}	u_{14}	u_{15}	u_{16}	0
0.08	4	0	u_{17}	u_{18}	u_{19}	u_{20}	0
0.10	5	0	u_{21}	u_{22}	u_{23}	u_{24}	0

3. RESULTS AND DISCUSSION

Example 1: Solve the boundary value problem $u_t = u_{xx}$ under the conditions u(0, t) = u(1, t) = 0 and u(x, 0) = $\sin \pi$ x, $0 \le x \le 1$ taking h = 0.2 and α =1/2.

	Relative Percentage Error Table		
	Bender-Schmidt Value	**Crank-Nicholson Value**	**Du Fort and Frankel**
u_1	0.000000000000000000	0.000000000000000000	0.000000000000000000
u_2	0.000000000000000000	0.000000000000000000	0.000000000000000000
u_3	0.000000000000000000	0.000000000000000000	0.000000000000000000
u_4	0.000000000000000000	0.000000000000000000	0.000000000000000000
u_5	0.000069662681650920	0.000028189186624940	0.000069662681650920
u_6	0.000112716587208940	0.000045611062446230	0.000112716587208940
u_7	0.000112716587208940	0.000045611062446230	0.000112716587208940
u_8	0.000069662681650920	0.000028189186624940	0.000069662681650920
u_9	0.000113542208531580	0.000046414435951180	0.000113542208531580
u_{10}	0.000183715152391770	0.000075100134504780	0.000183715152391770
u_{11}	0.000183715152391770	0.000075100134504780	0.000183715152391770
u_{12}	0.000113542208531580	0.000046414435951180	0.000113542208531580
u_{13}	0.000138798068098900	0.000057317347763170	0.000138798068098900
u_{14}	0.000224579988719219	0.000092741416453190	0.000224579988719219
u_{15}	0.000224579988719219	0.000092741416453190	0.000224579988719219
u_{16}	0.000138798068098900	0.000057317347763170	0.000138798068098900
u_{17}	0.000150821971638549	0.000062916969395930	0.000150821971638549
u_{18}	0.000244035082300120	0.000101801793906779	0.000244035082300120
u_{19}	0.000244035082300120	0.000101801793906779	0.000244035082300120
u_{20}	0.000150821971638549	0.000062916969395930	0.000150821971638549
u_{21}	0.000153647240918500	0.000064747349426330	0.000153647240918500
u_{22}	0.000248606448158460	0.000104763413910510	0.000248606448158460
u_{23}	0.000248606448158460	0.000104763413910510	0.000248606448158460
u_{24}	0.000153647240918500	0.000064747349426330	0.000153647240918500

Example 2: Find the temperature in a bar of length 2 whose ends are kept at zero and lateral surface insulated if the initial temperature is $\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$. Given that *u(0, t) = 0 = u(2, t)* for any *t*.

	Relative Percentage Error Table		
	Bender-Schmidt Value	Crank-Nicholson Value	Du Fort and Frankel
u_1	0.0000000000000000	0.0000000000000000	0.0000000000000000
u_2	0.0000000000000000	0.0000000000000000	0.0000000000000000
u_3	0.0000000000000000	0.0000000000000000	0.0000000000000000
u_4	0.0000000000000000	0.0000000000000000	0.0000000000000000
u_5	0.0000088729628340	0.0000171037812363	0.0000088729628340
u_6	0.0000143567554464	0.0000029710961850	0.0000143567554464
u_7	0.0000143567554464	0.0000029710961850	0.0000143567554464
u_8	0.0000088729628340	0.0000171037812363	0.0000088729628340
u_9	0.0000160647348146	0.0000096828583904	0.0000018012469723
u_{10}	0.0000259932869502	0.0000090541655660	0.0000029144788233
u_{11}	0.0000259932869502	0.0000090541655660	0.0000029144788233
u_{12}	0.0000160647348146	0.0000096828583904	0.0000018012469723
u_{13}	0.0000218142267661	0.0000312664709727	0.0000039816274952
u_{14}	0.0000352961603459	0.0000367705526296	0.0000064424086178
u_{15}	0.0000352961603459	0.0000367705526296	0.0000064424086178
u_{16}	0.0000218142267661	0.0000312664709727	0.0000039816274952
u_{17}	0.0000263301700644	0.0000103708421243	0.0000081548096317
u_{18}	0.0000426031100937	0.0000107870744725	0.0000131947591559
u_{19}	0.0000426031100937	0.0000107870744725	0.0000131947591559
u_{20}	0.0000263301700644	0.0000104708421243	0.0000081548096317
u_{21}	0.0000297946907410	0.0000166347233242	0.0000106242333815
u_{22}	0.0000482088223033	0.0000116369072820	0.0000171903707157
u_{23}	0.0000482088223033	0.0000116369072820	0.0000171903707157
u_{24}	0.0000297946907410	0.0000166347233242	0.0000106242333815

4. CONCLUSION

On the basis of the above discussion we get the result obtained by analytical methods is always providing accurate solution but numerical solution always providing approximate result. But among these numerical methods Crank-Nicolsan method was providing fast convergence in comparison to Bender-Schdimit method and Du Fort and Frankel method. Since it is not possible to solve every partial differential equation analytically so numerical methods providing a good agreement in those cases where solutions not exist or we are unable to solve partial differential equation analytically. This comparative study provides better result for numerical solution of one dimentional heat equation.

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REFERENCES

[1] Agyeman, E., Folson, D., *International Journal of Computer Applications*, **79**(5), 11, 2013.
[2] Wani, S.S., Thakar, S.H., *International Journal of Applied Physics and Mathematics*, **3**(5), 324, 2013.
[3] Liu, T., *International Journal of Hybrid Information Techonology*, **8**(8), 91, 2015.
[4] Smith, G.D., *Numerical Solutions of Partial Differential Equations*, Oxford University Press, Oxford, 1965.
[5] Pandey, K., Verma, L., *Applied Mathematics*, **2**(7), 883, 2011.
[6] Sastry, S.S, *Introduction Methods of Numerical Analysis*, 5th Ed., PHI Learning Private Ltd., New Delhi, 2012.
[7] Grewal, B.S., *Numerical methods in engineering and science: with programs in C and C++*, Khanna Publisher, Nai Sarak - Delhi, 2013.
[8] Rao, K.S., *Numerical methods for Scientists and Engineers*, 3rd Ed., PHI Learning Private Ltd., New Delhi, 2006.
[9] Robinson, J.C., *Infinite-dimensional dynamical systems: An introduction to dissipative parabolic PDEs and the theory of global attractors*, Cambridge University Press, Cambridge, 2001.
[10] Kandasamy, P., Thilagavathy, K., Gunavathi, K., *Numerical methods*, S. Chand & Co. Ltd., New Delhi, 2006.