ORIGINAL PAPER

ANALYTICAL STUDY OF FRACTIONAL NEWELL–WHITEHEAD–SEGEL EQUATION USING AN EFFICIENT METHOD

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Abstract. *In this work, four variants of time-fractional nonlinear Caputo-type Newell–Whitehead–Segel (NWS) problem are solved by fractional variational iteration method. This equation results as mathematical model of many important physical reaction-diffusion processes and pattern formation theory in diverse fields of chemical engineering, mechanical engineering, bio-engineering, biology and fluid mechanics, and the numerical treatments of these models are of great scientific significance and practical values. The received outcomes and plotted graphs express that the carried out technique is unpretentious, competent, and easily applicable for nonlinear fractional-order Partial differential equations (PDEs).*

Keywords: *Fractional Newell-Whitehead equation; Fractional VIM; Fractional calculus; Caputo' derivative*

1. INTRODUCTION

Over the previous few years due to remarkable importance and precise representation of widespread natural physical phenomena, fractional differential equations (FDE) have quantity of endless applications within the discipline of science, engineering and economics [1]. Research on handling non-linear fractional PDEs has attracted many researchers in these days. In the last two centuries different definitions of fractional derivative has been introduced but few are as Riemann–Liouville derivative, Jumarie's derivative, Caputo derivative, Caputo-Fabrizio derivative are in common. Two of them are based on RL fractional derivative and integral definition. In order to good insight for understanding of these fractional PDEs, their solutions are very important. Many useful methods have been established in the literature. Unfortunately, most of PDEs do not have precise solutions .The nonlinear NWS equation was originated through Newell and Whitehead [2] and it has been practiced to model innumerable kinds of problems in research regions and has applications in biological system, chemical, dynamical system, mechanical engineering and many other fields. The variational iteration technique (VIM) has received large interest due the rapidly proliferating use and latest traits of fractional calculus in these fields [3]. The VIM is basically settled by the act of obliged adaptations and correction functional (CF) which has watched a comprehensive applications for the clarification of linear just as nonlinear differential equations [4-7]. The VIM is constructed by the Lagrange multiplier progressed by methods for Inokuti et al. [8]. This strategy is, a difference in the general LM approach into an emphasis procedure, which is known as CF. This approach resolve efficaciously, effortlessly,

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and efficiently a big elegance of nonlinear issues, commonly one or more iterations bring about immoderate precise solutions.

To resolve equations of various classes like integer fractional, linear or nonlinear, ordinary differential or PDEs, and many others, a number of techniques were used, such as Adomian decomposition method (ADM) [9-10], reduced differential transform method (RDTM) [11-14], homotopy perturbation technique (HPM) [15-16], He's variational iteration method (HVIM) [16-18], homotopy analysis technique (HAM) [19], Galerkin approach [20], and different strategies

The NSW equation is a famous equation to direct development of almost one-dimensional (1-D) nonlinear styles created through a finite-wavelength uncertainty in isotropic 2-D media, a classical instance being the Rayleigh-B´enard convection (RBC). NWS additionally designates the dynamical action close to the bifurcation point of RBC of binary fluid combinations. RBC is rising up in a smooth fluid heated from bottom, wherein the fluid improves a consistent arrangement of cells called Bénard cells. RBC is one of the most regularly taken into deliberation phenomena because of its systematical and tentative approachability. These forms are the supreme prudently inspected instance of self-organizing nonlinear systems. Most recently, in [21] abundantly of real and complex valued explicit solutions of NWS equation are carried out.

The essential goal of present article is to find the inexact solution of different cases of fractional NWS equation through the use of the VIM in sense of Caputo's derivative approach. The NWS equation with fractional derivatives is written in operator shape as [22-23]:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = k\frac{\partial^\gamma u}{\partial x^\gamma} + cu - du^p, 0 < \alpha \le 1, 1 < \gamma \le 2. \quad (1)$$

where $\alpha > 0$ and $\gamma > 1$ are parameter describing the order of the derivatives, $c, d, k > 0$ and p is a positive integer.

2. BASIC CONCEPT FOR FRACTIONAL CALCULUS

Some primary definitions of fractional calculus are recalled that are practiced in the evaluation. It is famous that there are specific definitions of fractional Integral and fractional derivatives, such as, Grünwald-Letnikov, Riesz, Riemann-Liouville (RL), Caputo, Hadamard and Erdélyi-Kober and many others [24-26]. However, this paper offers interest to two of them which can be Reimann-louville integral operator and Caputo fractional derivative.

Definition 2.1 The RL fractional integral operator of order $\alpha \ge 0$of a function $f \in C\mu, \mu \ge -1$ is defined as [24]

$$J^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-\mu)^{\alpha-1} f(\mu) d\mu, \;\; \alpha > 0, \;\; x > 0, \quad (2)$$

$$J^0 f(x) = f(x). \quad (3)$$

When we verbalize the version of real world problems with fractional calculus, the RL operator have positive negative aspects. Caputo proposed in his research a modified fractional differential operator $D{*}\alpha$.

Definition 2.2 The Caputo's fractional derivative in [27] known as

$$D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^n(\xi)}{(t-\xi)^{\alpha-n+1}} d\xi, & n-1 < \alpha < n, \mathbb{N}, \\ \frac{d^n}{dt^n} f(t), & n = \alpha, \end{cases} \tag{4}$$

where $\alpha > 0$ and n is the smallest integer greater than α. In addition, we have the following for Caputo's derivative

$$D^{\alpha} t^{\zeta} = \begin{cases} \frac{\Gamma(\zeta+1)}{\Gamma(\zeta+1-\alpha)} t^{\zeta-\alpha}, & \zeta \in \mathbb{N}_0, \zeta \geq \lceil \alpha \rceil \\ 0, & \zeta \in \mathbb{N}_0, \zeta \geq \lceil \alpha \rceil, \end{cases} \tag{5}$$

$$D^{\alpha} c = 0, \qquad c \in \mathbb{R}. \tag{6}$$

Some properties of fractional derivative in Caputo's sense are

$$D^{\alpha}\big(\lambda f(t) + \mu g(t)\big) = \lambda D^{\alpha} f(t) + \mu D^{\alpha} g(t), \tag{7}$$

$$I^{\alpha} D_t^{\alpha} f(t) = f(t) - \sum_{k=0}^{m-1} \frac{f^{(k)}(0^+)}{k!} t^k, \ t > 0. \tag{8}$$

3. METHOD ANALYSIS

In direction to interpret the solution technique of the fractional VIM, taking into consideration the time-fractional PDE [21]

$$\frac{\partial^{\alpha}}{\partial t^{\alpha}} u(x,t) = C[x]u(x,t) + q(x,t), \qquad t > 0, x \in \mathbb{R}, \tag{9}$$

with condition

$$u(x,0) = f(x), \tag{10}$$

The CF for Eq. (9) is formed as

$$u_{n+1}(x,t) = u_n(x,t) + I^{\alpha}\left[\lambda\left(\frac{\partial^{\alpha}}{\partial t^{\alpha}} u(x,t) - C[x]u(x,t) - q(x,t)\right)\right], \tag{11}$$

$$u_{n+1}(x,t) = u_n(x,t) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\xi)^{\alpha-1} \lambda(\xi) \left(\frac{\partial^{\alpha}}{\partial t^{\alpha}} u(x,\xi) - C[x]u(x,\xi) - q(x,\xi)\right) d\xi, \tag{12}$$

$$u_{n+1}(x,t) = u_n(x,t) + \frac{1}{\Gamma(\alpha+1)} \int_0^t \lambda(\xi) \left(\frac{\partial^{\alpha}}{\partial t^{\alpha}} u(x,\xi) - C[x]u(x,\xi) - q(x,\xi)\right) d\xi, \tag{13}$$

From Eq. (13), we can obtain successive approximations $u_n(x,t).,n \geq 0.$ As a result

$$u(x,t) = \lim_{n\to\infty} u_n(x,t). \tag{14}$$

The convergence and error estimation of the fractional VIM is well addressed in Ref. [28-29].

Case 1: In Eq. (1) if $c = 1, d = 1, k = 1 \; and \; p = 3$ the time fractional NWS equation is written as:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + u - u^3, \quad 0 < \alpha \leq 1, \tag{15}$$

with the constraints

$$\mathrm{u(x,0)} = \frac{-1+e^{\frac{x}{\sqrt{2}}}}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{16}$$

The CF for Eq. (15) is

$$u_{n+1}(x,t) = u_n(x,t) + \frac{1}{\Gamma(\alpha)}\int_0^{\mathrm{t}} (t-\xi)^{\alpha-1}\lambda(\xi)\left(\frac{\partial^\alpha u_{\mathrm{n}}(\mathrm{x},\xi)}{\partial \xi^\alpha} - \frac{\partial^2 \mathrm{u_n}(\mathrm{x},\xi)}{\partial^2 \mathrm{x}} + \mathrm{u_n}^3(\mathrm{x},\xi) - \mathrm{u_n}(\mathrm{x},\xi)\right)\mathrm{d}\xi, \tag{17}$$

The LM can be determined $\lambda(\xi) = -1$ therefore the CF (17) becomes as

$$u_{n+1}(x,t) = u_n(x,t) - \frac{1}{\Gamma(\alpha)}\int_0^{\mathrm{t}} (t-\xi)^{\alpha-1}\left(\frac{\partial^\alpha u_{\mathrm{n}}(\mathrm{x},\xi)}{\partial \xi^\alpha} - \frac{\partial^2 \mathrm{u_n}(\mathrm{x},\xi)}{\partial^2 \mathrm{x}} + \mathrm{u_n}^3(\mathrm{x},\xi) - \mathrm{u_n}(\mathrm{x},\xi)\right)\mathrm{d}\xi. \tag{18}$$

Consequently, we have

$$u_0(x,t) = \frac{-1+e^{\frac{x}{\sqrt{2}}}}{1+e^{\frac{x}{\sqrt{2}}}},$$

$$u_1(x,t) = \frac{-1+e^{\frac{x}{\sqrt{2}}}}{1+e^{\frac{x}{\sqrt{2}}}} + \frac{1}{\Gamma(\alpha+1)}\left(\frac{3(-e^{-\frac{x}{2\sqrt{2}}}+e^{\frac{x}{2\sqrt{2}}})^4 t^\alpha}{(-e^{-\frac{x}{\sqrt{2}}}+e^{\frac{x}{\sqrt{2}}})^3}\right),$$

$$u_2(x,t) = \frac{-1+e^{\frac{x}{\sqrt{2}}}}{1+e^{\frac{x}{\sqrt{2}}}} + \frac{1}{\Gamma(\alpha+1)}\left(\frac{3(-e^{-\frac{x}{2\sqrt{2}}}+e^{\frac{x}{2\sqrt{2}}})^4 t^\alpha}{(-e^{-\frac{x}{\sqrt{2}}}+e^{\frac{x}{\sqrt{2}}})^3}\right) + \frac{1}{2(1+e^{\frac{x}{\sqrt{2}}})^9}\left(3e^{\frac{x}{\sqrt{2}}}(-1+e^{\frac{x}{\sqrt{2}}})t^\alpha(-\frac{3(-1+e^{\frac{x}{\sqrt{2}}})^2(1+e^{\frac{x}{\sqrt{2}}})^4 t^\alpha}{\Gamma[1+2\alpha]} + \frac{1}{\Gamma(\alpha+1)}\left(2((1+e^{\frac{x}{\sqrt{2}}})^6 - \frac{94^\alpha e^{\frac{x}{\sqrt{2}}}(-1+e^{\sqrt{2}x})^2 t^{2\alpha}\Gamma[\frac{1}{2}+\alpha]}{\sqrt{\pi}\Gamma[1+3\alpha]}\right) - \frac{18e^{\sqrt{2}x}(-1+e^{\frac{x}{\sqrt{2}}})^2 t^{3\alpha}\Gamma[1+3\alpha]}{\Gamma[1+\alpha]^3\Gamma[1+4\alpha]}\right),$$

$$u_n(x,t)=\frac{3\left(-1+e^{\frac{x}{\sqrt{2}}}\right)}{1+e^{\frac{x}{\sqrt{2}}}}+\frac{1}{\Gamma(\alpha+1)}\left(\frac{6(-e^{-\frac{x}{2\sqrt{2}}}+e^{\frac{x}{2\sqrt{2}}})^4t^{\alpha}}{(-e^{-\frac{x}{\sqrt{2}}}+e^{\frac{x}{\sqrt{2}}})^3}\right)+\frac{1}{2\left(1+e^{\frac{x}{\sqrt{2}}}\right)^9}\left(3e^{\frac{x}{\sqrt{2}}}(-1+e^{\frac{x}{\sqrt{2}}})t^{\alpha}(-\frac{3\left(-1+e^{\frac{x}{\sqrt{2}}}\right)^2\left(1+e^{\frac{x}{\sqrt{2}}}\right)^4t^{\alpha}}{\Gamma[1+2\alpha]}+\frac{1}{\Gamma(\alpha+1)}\left(2(\left(1+e^{\frac{x}{\sqrt{2}}}\right)^6-\frac{94^{\alpha}e^{\frac{x}{\sqrt{2}}}\left(-1+e^{\sqrt{2}x}\right)^2t^{2\alpha}\Gamma[\frac{1}{2}+\alpha]}{\sqrt{\pi}\Gamma[1+3\alpha]}\right)-\frac{18e^{\sqrt{2}x}\left(-1+e^{\frac{x}{\sqrt{2}}}\right)^2t^{3\alpha}\Gamma[1+3\alpha]}{\Gamma[1+\alpha]^3\Gamma[1+4\alpha]}\right)+\cdots, \tag{19}$$

The exact solution of Eq. (15) and (16) is

$$u(x,t)=\frac{e^{\frac{x}{\sqrt{2}}}-e^{\frac{-x}{\sqrt{2}}}}{e^{\frac{x}{\sqrt{2}}}+e^{\frac{-x}{\sqrt{2}}}+2e^{\frac{-3t}{2}}}. \tag{20}$$

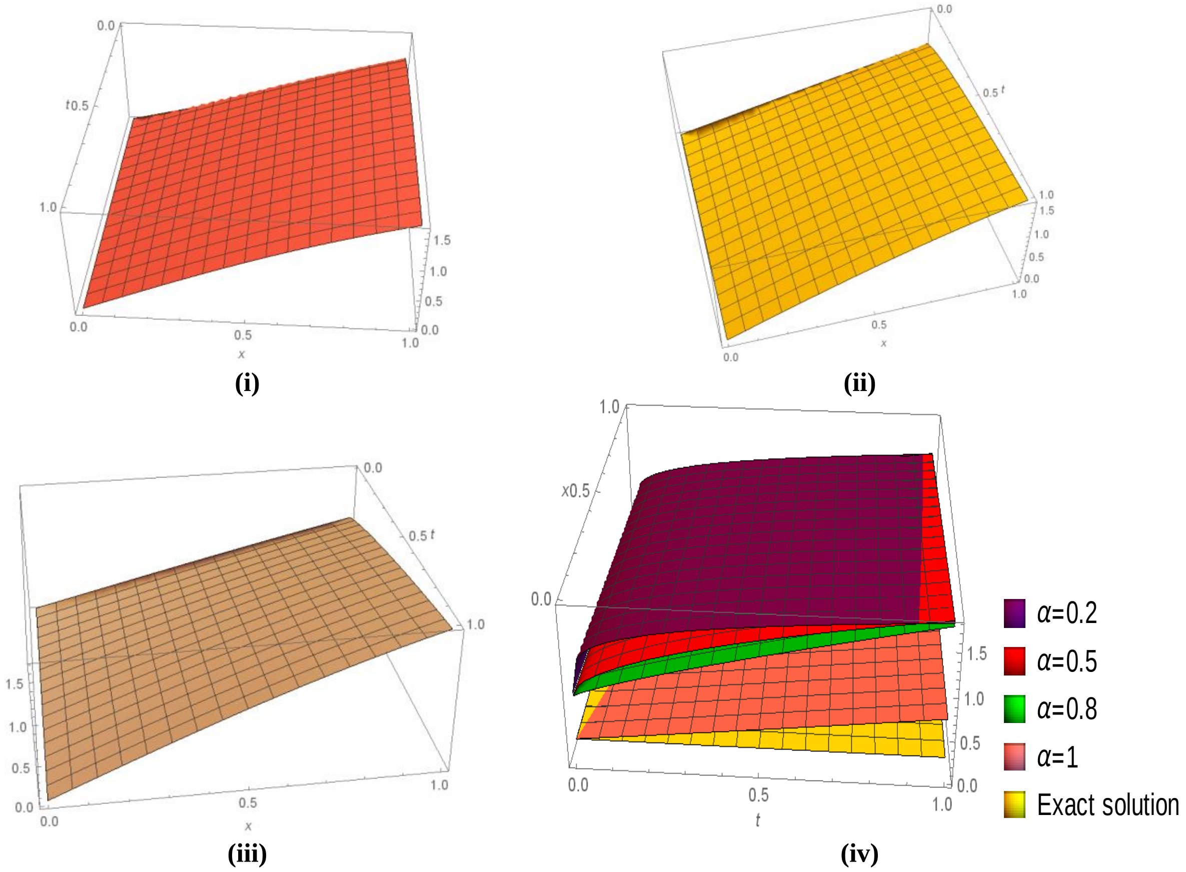

Figure 1. (i)-(iv) represents the approximate solutions given in (28) for α = 0.2, α = 0.5, α = 0.8 and α = 1 in (29). Figure (iv) shows the evolution solution of the cases of $0 \leq \alpha \leq 1$. Graph shows that when α decreases, the evolution solution u(x, t) splits for small values of x. Also comparison of approximate as well as exact solutions is presented in the graph.

Case 2: In Eq. (1) if $c = 1, d = 1, k = 1$ and $p = 2$ the time fractional NWS equation is written as:

$$\frac{\partial^{\alpha}u}{\partial t^{\alpha}}=\frac{\partial^2 u}{\partial x^2}+u-u^2, \quad 0<\alpha\leq 1, \tag{21}$$

subjected to constraints

$$\mathrm{u}(\mathrm{x},0)=\frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^{2}}. \tag{22}$$

According to above defined steps, the CF for Eq. (35) is

$$u_{n+1}(x,t)=u_n(x,t)+\frac{1}{\Gamma(\alpha)}\int_0^{\mathrm{t}}(t-\xi)^{\alpha-1}\lambda(\xi)\left(\frac{\partial^{\alpha}u_{\mathrm{n}}(\mathrm{x},\xi)}{\partial\xi^{\alpha}}-\frac{\partial^{2}\mathrm{u}_{\mathrm{n}}(\mathrm{x},\xi)}{\partial^{2}\mathrm{x}}+\mathrm{u_n}^{2}(\mathrm{x},\xi)-\mathrm{u_n}(\mathrm{x},\xi)\right)\mathrm{d}\xi, \tag{23}$$

with $\lambda(\xi)=-1$the iteration formula (23) takes the following form

$$u_{n+1}(x,t)=u_n(x,t)-\frac{1}{\Gamma(\alpha)}\int_0^{\mathrm{t}}(t-\xi)^{\alpha-1}\left(\frac{\partial^{\alpha}u_{\mathrm{n}}(\mathrm{x},\xi)}{\partial\xi^{\alpha}}-\frac{\partial^{2}\mathrm{u}_{\mathrm{n}}(\mathrm{x},\xi)}{\partial^{2}\mathrm{x}}+\mathrm{u_n}^{2}(\mathrm{x},\xi)-\mathrm{u_n}(\mathrm{x},\xi)\right)\mathrm{d}\xi. \tag{24}$$

As a result, from Eq. (2) we have

$$u_0(x,t)=\frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^{2}},$$

$$u_1(x,t)=\frac{1}{(1+e^{\frac{x}{\sqrt{6}}})^{2}}+\frac{5e^{\frac{x}{\sqrt{6}}}t^{\alpha}}{3(1+e^{\frac{x}{\sqrt{6}}})^{3}\Gamma[\alpha+1]},$$

$$u_2(x,t)=\frac{1}{(1+e^{\frac{x}{\sqrt{6}}})^{2}}+\frac{5e^{\frac{x}{\sqrt{6}}}t^{\alpha}}{3(1+e^{\frac{x}{\sqrt{6}}})^{3}\Gamma[\alpha+1]}+\frac{1}{18(1+e^{\frac{x}{\sqrt{6}}})^{6}}\left(5e^{\frac{x}{\sqrt{6}}}t^{\alpha}\left(\frac{5(1+e^{\frac{x}{\sqrt{6}}})^{2}(-1+2e^{\frac{x}{\sqrt{6}}})t^{\alpha}}{\Gamma[1+2\alpha]}+\frac{1}{\Gamma[1+\alpha]}\left(2\left(3(1+e^{\frac{x}{\sqrt{6}}})^{3}-\frac{5(4^{\alpha})e^{\frac{x}{\sqrt{6}}}t^{2\alpha}\Gamma[\frac{1}{2}+\alpha]}{\sqrt{\pi}\Gamma[1+3\alpha]}\right)\right)\right)\right),$$

$$u_n(x,t)=\frac{1}{(1+e^{\frac{x}{\sqrt{6}}})^{2}}+\frac{1}{18(1+e^{\frac{x}{\sqrt{6}}})^{6}}\left(5e^{\frac{x}{\sqrt{6}}}t^{\alpha}\left(\frac{5(1+e^{\frac{x}{\sqrt{6}}})^{2}(-1+2e^{\frac{x}{\sqrt{6}}})t^{\alpha}}{\Gamma[1+2\alpha]}+\frac{1}{\Gamma[1+\alpha]}\left(2\left(6(1+e^{\frac{x}{\sqrt{6}}})^{3}-\frac{5(4^{\alpha})e^{\frac{x}{\sqrt{6}}}t^{2\alpha}\Gamma[\frac{1}{2}+\alpha]}{\sqrt{\pi}\Gamma[1+3\alpha]}\right)\right)\right)\right)+\cdots, \tag{25}$$

The exact solution of Eq. (21) and (22) is

$$u(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}\frac{5}{6}t}\right)^2}. \tag{26}$$

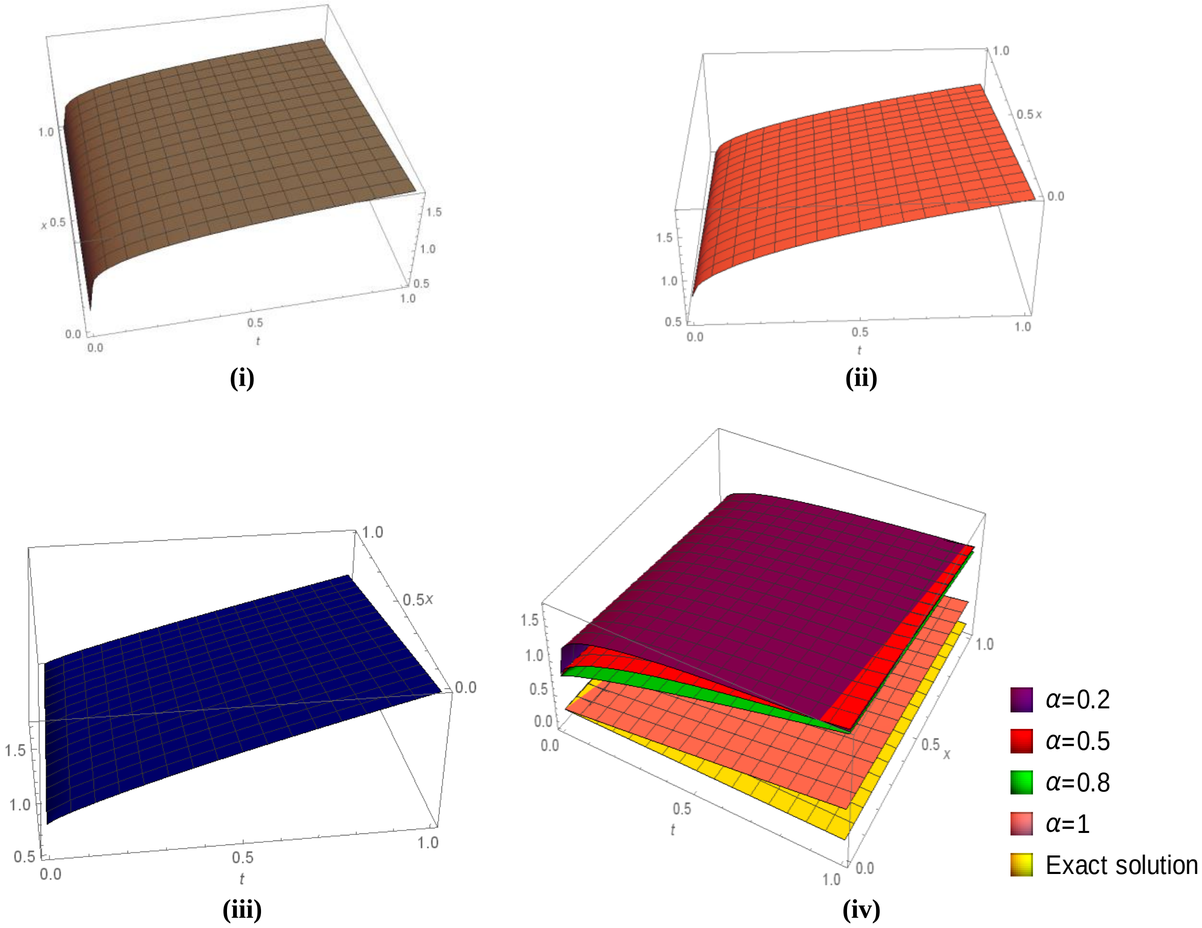

Figure 2. (i)-(iv) presents the approximate solutions given in (25) for α = 0.2, α = 0.5, α = 0.8 and α = 1. Figure (iv) shows the evolution solution for different cases of $0 \leq \alpha \leq 1$. Graph shows that when α decreases, the evolution solution $u(x, t)$ splits for small values of x. Also comparison of approximate as well as exact solutions is presented in the graph (d).

Case 3: In Eq. (1) if $c = 1, d = 1, k = 1$ and $p = 4$ the time fractional NWS equation is written as:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + u - u^4, \quad 0 < \alpha \leq 1, \tag{27}$$

Subjected to the condition

$$u_0(x,t) = x^2. \tag{28}$$

The CF formula for Eq. (27) is

$$u_{n+1}(x,t) = u_n(x,t) - \frac{1}{\Gamma(\alpha)}\int_0^{t}(t-\xi)^{\alpha-1}\lambda(\xi)\left(\frac{\partial^{\alpha} u_{n}(x,\xi)}{\partial \xi^{\alpha}} - \frac{\partial^2 u_{n}(x,\xi)}{\partial^2 x} + u_{n}^{4}(x,\xi) - u_{n}(x,\xi)\right)d\xi, \tag{29}$$

Consequently, we have

$$u_0(x,t) = x^2,$$

$$u_1(x,t) = x^2 - \frac{t^{\alpha}(-2-x^2+x^8)}{\Gamma[\alpha+1]},$$

$$u_2(x,t) = x^2 - \frac{t^{\alpha}(-2-x^2+x^8)}{\Gamma[\alpha+1]} - \frac{1}{\Gamma[1+\alpha]^5}\left(t^{2\alpha}\left(\frac{4^{-\alpha}\sqrt{\pi}(-2+56x^6)\Gamma[1+\alpha]^3}{\Gamma\left[\frac{1}{2}+\alpha\right]} + \frac{4^{-\alpha}\sqrt{\pi}(-2-x^2+x^8)\Gamma[1+\alpha]^3}{\Gamma\left[\frac{1}{2}+\alpha\right]} + \frac{t^{3\alpha}(2+x^2-x^8)^4\Gamma[1+4\alpha]}{\Gamma[1+5\alpha]}\right)\right),$$

$$u_n(x,t) = 3x^2 - \frac{2t^{\alpha}(-2-x^2+x^8)}{\Gamma[\alpha+1]} - \frac{1}{\Gamma[1+\alpha]^5}\left(t^{2\alpha}\left(\frac{4^{-\alpha}\sqrt{\pi}(-2+56x^6)\Gamma[1+\alpha]^3}{\Gamma\left[\frac{1}{2}+\alpha\right]} + \frac{4^{-\alpha}\sqrt{\pi}(-2-x^2+x^8)\Gamma[1+\alpha]^3}{\Gamma\left[\frac{1}{2}+\alpha\right]} + \frac{t^{3\alpha}(2+x^2-x^8)^4\Gamma[1+4\alpha]}{\Gamma[1+5\alpha]}\right)\right) + \cdots, \tag{30}$$

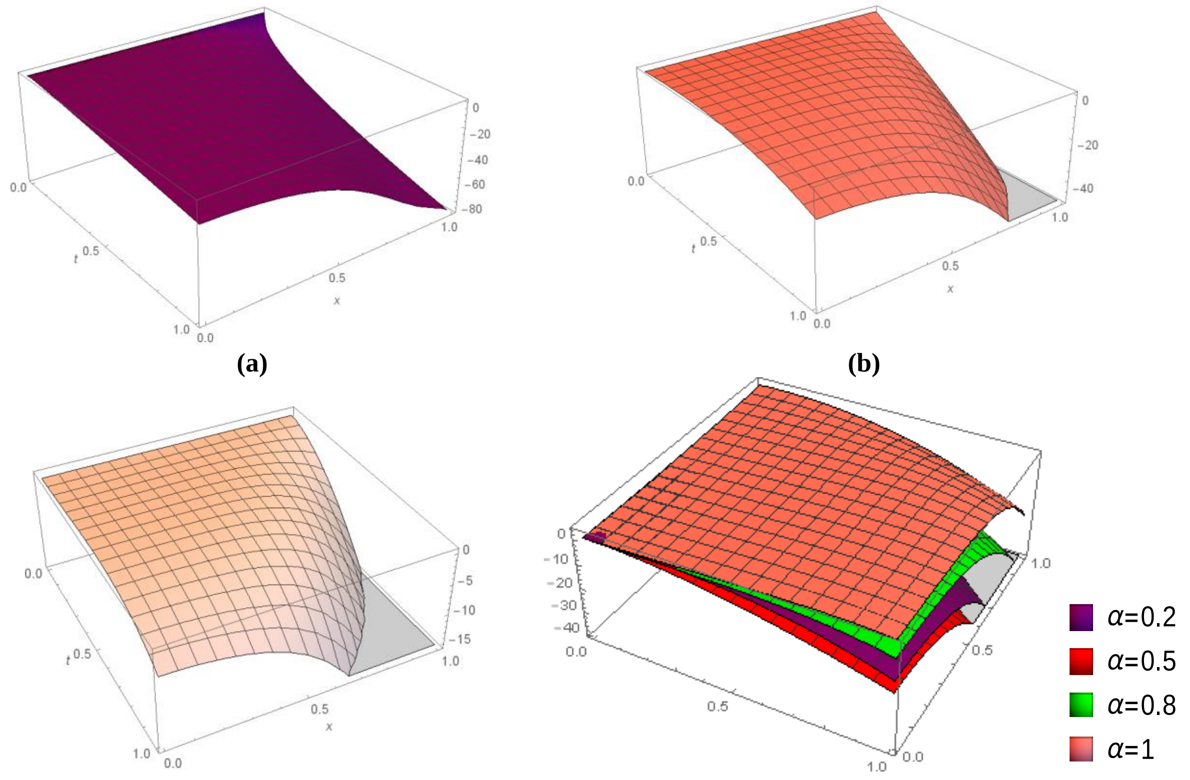

Figure 3. (a)-(d) presents the approximate solutions given in (30) for α = 0.2, α = 0.5, α = 0.8 and α = 1. Figure (d) shows the evolution solution for different cases of 0 ≤ α ≤ 1. Graph shows that when α decreases, the evolution solution u(x, t) splits for small values of x.

Case 4: In Eq. (1) if $c = 1, d = 1, k = 1$ *and* $p = 4$ the time fractional NWS equation is written as:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + u - u^4, \quad 0 < \alpha \le 1, \tag{31}$$

subjected to constraint

$$u(x,0) = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}}. \tag{32}$$

The CF formula for Eq. (31) is

$$u_{n+1}(x,t) = u_n(x,t) - \frac{1}{\Gamma(\alpha)}\int_0^t (t-\xi)^{\alpha-1}\lambda(\xi)\left(\frac{\partial^\alpha u_n(x,\xi)}{\partial\xi^\alpha} - \frac{\partial^2 u_n(x,\xi)}{\partial^2 x} + u_n{}^4(x,\xi) - u_n(x,\xi)\right)d\xi, \tag{33}$$

Consequently, we have

$$u_0(x,t) = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}},$$

$$u_1(x,t) = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} + \frac{7e^{\frac{3x}{\sqrt{10}}}t^\alpha}{\Gamma[\alpha+1]\left(5\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}\right)},$$

$$u_2(x,t) = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} + \frac{7e^{\frac{3x}{\sqrt{10}}}t^\alpha}{\Gamma[\alpha+1]\left(5\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}\right)} - \frac{7e^{\frac{3x}{\sqrt{10}}}\left(-19+66e^{3\sqrt{\frac{2}{5}}x}+7e^{\frac{3x}{\sqrt{10}}}\right)t^{2\alpha}}{\Gamma[\alpha+1]^2\left(50\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{11}{3}}\right)},$$

$$u_n(x,t) = \frac{3}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} + \frac{14e^{\frac{3x}{\sqrt{10}}}t^\alpha}{\Gamma[\alpha+1]\left(5\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}\right)} - \frac{7e^{\frac{3x}{\sqrt{10}}}\left(-19+66e^{3\sqrt{\frac{2}{5}}x}+7e^{\frac{3x}{\sqrt{10}}}\right)t^{2\alpha}}{\Gamma[\alpha+1]^2\left(50\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{11}{3}}\right)} + \cdots, \tag{34}$$

The exact solution of Eq. (31) and (32) is

$$u(x,t) = \left(\frac{1}{2}tanh\left(-\frac{3}{2\sqrt{10}}\left(x - \frac{7}{\sqrt{10}}t\right)\right) + \frac{1}{2}\right)^{\frac{2}{3}}. \tag{35}$$

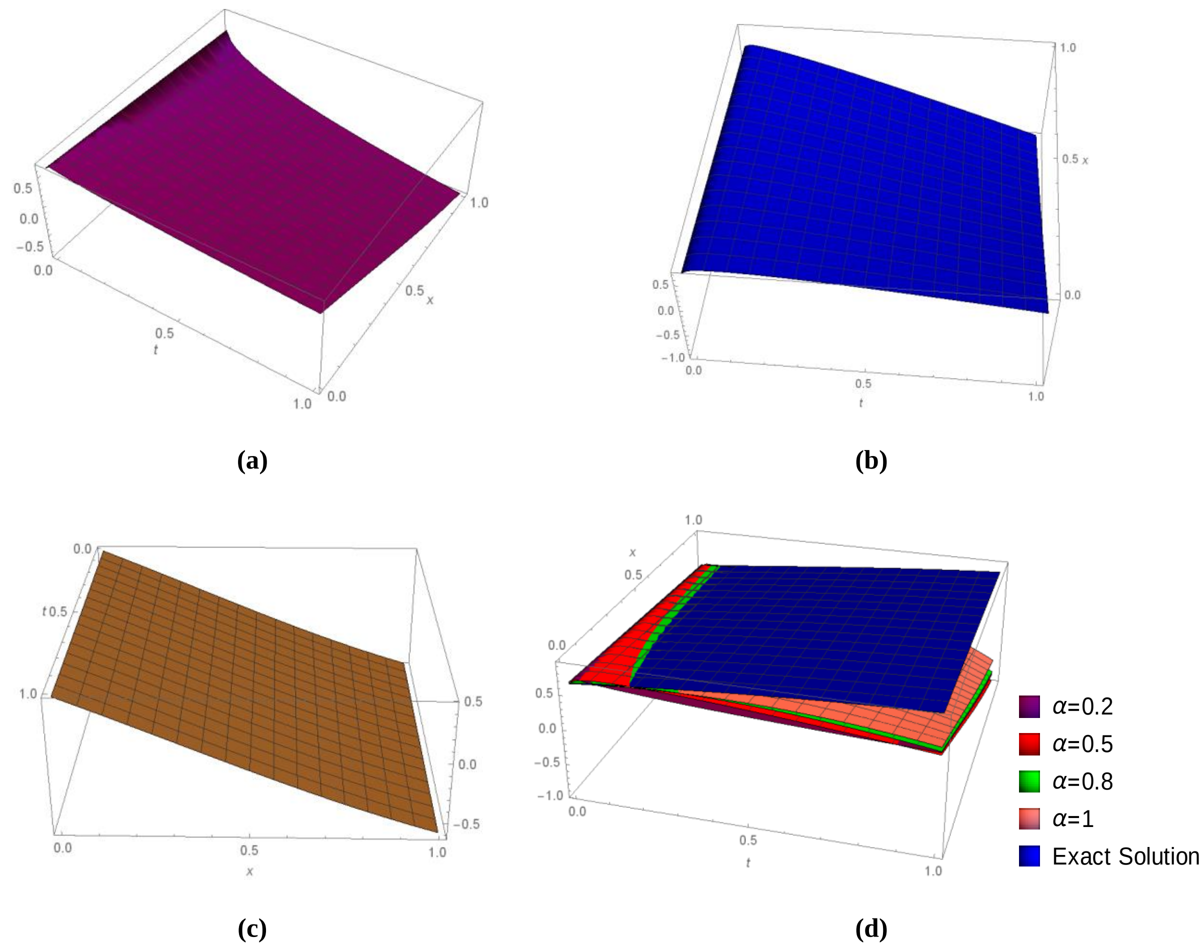

Figure 4. (a)-(b) presents the approximate solutions given in (30) for α = 0.2, α = 0.5, α = 0.8 and α = 1. Figure (d) shows the evolution solution for different cases of $0 \leq \alpha \leq 1$. Graph shows that when α decreases, the evolution solution u(x, t) splits for small values of x.

4. CONCLUSION

In this work, fractional variational iteration approach is effectively applied to resolve time-fractional Newell-Whitehead-Segel equation. We have discussed the three variants of fractional NWS equations in this paper with preliminary conditions. The proposed approximated solutions are acquired without using any sort of discretization, perturbation, or any preventive conditions and are compared graphically with the exact solutions. It is ostensibly seen from demonstrative examples along with graphical depiction that the method is easy to employ, dominant and efficient to find the approximate solutions.

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