ORIGINAL PAPER

ANALYTICAL PERIODIC SOLUTION OF MHD OSCILLATORY SLIP FLOW REGIME THROUGH POROUS MEDIUM

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Manuscript received: 24.07.2019; Accepted paper: 15.10.2019;
Published online: 30.12.2019.

Abstract: *In the present problem an analytical periodic solution of an electrically conducting viscous, incompressible fluid in a slip flow through porous medium bounded by two parallel infinite porous horizontal plates is studied. The lower plate is motionless and the upper one is oscillatory in time about a costant mean. The fluid motion is in effect of a constant injection/suction, and a magnetic field perpendicular to the plates. The unsteady problem is solved analytically and solution of transient velocity, the amplitude and phase of the skin friction at the stationary plate are found.*
Keywords: *MHD, oscillatory flow, periodic, slip flow, injection, suction.*

1. INTRODUCTION

The study of oscillatory fluid flow in a porous channel has been receiving considerable attention in the last recent years due to its applications in soil mechanics, ground water hydrology, irrigation, drainage, water purification processes, absorption and filltration processes in chemical engineering. Also, the Navier slip flow regime has been receiving attention of many researchers, because of its applications in modern science, technology and industrialization [1]. Understanding to these applications, Reddy et al. [2] presented the effect of slip condition, radiation and chemical reaction on unsteady mhd periodic flow of a viscous fluid through saturated porous medium in a planer channel. Unsteady MHD slip flow with radiative heat and mass transfer over an inclined plate embedded in a porous medium was reported by Venkateswarlu and Makinde [3]. It was assumed that the Navier slip boundary condition effect depends on the shear stress of both lower and upper walls of a channel by Eegunjobi and Makinde [1]. Stuart [4] studied a two dimensional flow past an infinite, porous plate with constant suction when the free stream oscillates in time. Authors Soundalgekar [5-6], Vajravelu and Sastri [7], Soundalgekar and Gupta [8] examined the theory of laminar boundary layer with free stream oscillations. Vajravelu [9] has investigated fluid flow confined between two parallel infinite horizontal plates when one of the plates oscillates intime about a constant mean, he obtain an exact solution of the problem. Analytical solutions for the problem of rotating system also obtained by Mazumder [10], Ganapathy [11] and Singh [12].

The intention of the present study is to analyze the effects of slip flow regime and effect of porosity on the oscillatory flow of an electrically conducting viscous, incompressible fluid, confined between two parallel infinite porous horizontal plates, when the upper plate is oscillatory in time and lower plate is motionless.

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2. MATHEMATICAL ANALYSIS

Consider a two dimensional unsteady slip flow of an incompressible viscous fluid confined between two horizontal plates, the lower plate is at rest and upper plate is oscillatory in time with constant mean. The flow is subjected to a constant injection and suction velocity V_0 . A magnetic field of uniform strength B_0 is also applied perpendicular to the horizontal plate. The governing equations for the flow are written, in a coordinate system fixed with the oscillating plate as:

$$\frac{\partial v*}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u*}{\partial t*} + v*\frac{\partial u*}{\partial y*} = \frac{\partial U*}{\partial t*} + \mu\frac{\partial^2 u*}{\partial y*^2} - \frac{\sigma B_0^2}{\rho}(u* - U*) - \frac{\mu}{K*}(u* - U*) \tag{2}$$

The boundary conditions are

$$\begin{aligned} u* &= -L_1\left(\frac{\partial u*}{\partial y*}\right) \quad at \quad y* = 0 \\ u* &= U*(t*) \quad at \quad y* = d \end{aligned} \tag{3}$$

The unsteady velocity $U*(t*)$, is given by

$$U*(t*) = U_0\left[1 + \frac{1}{2}\varepsilon\left(e^{i\omega* t*} + e^{-i\omega* t*}\right)\right] \tag{4}$$

where U_0 is the constant mean velocity, $\omega*$ is the frequency and ε a positive constant.

Introduce $\eta = \frac{y*}{V_0\, d}$, $u = \frac{u*}{U_0}$, $v = \frac{v*}{V_0}$, $t = \frac{\omega* t*}{V_0^2}$, $\omega = \frac{\omega* d^2}{\mu}$ in equations (1) and (2) gives:

$$\frac{\partial v}{\partial \eta} = 0 \tag{5}$$

$$\frac{\omega}{\mathrm{Re}}\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial \eta} = \frac{\omega}{\mathrm{Re}}\frac{\partial U}{\partial t} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial \eta^2} - \frac{1}{\mathrm{Re}}\left(M^2 + \frac{1}{K}\right)(u - U) \tag{6}$$

here, $\mathrm{Re} = \frac{V_0^2 d}{\mu}$, $M = \frac{B_0\, d}{V_0}\left(\frac{\sigma}{\mu}\right)^{1/2}$, $K = \frac{\mu}{d^2}\frac{K*}{V_0^2}$.

The boundary conditions become

$$\begin{aligned} u &= 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right) - h_1\left(\frac{\partial u}{\partial \eta}\right) \quad \text{at } \eta \to 0 \\ u &= u(t) = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right) \qquad \text{at } \eta = 1 \end{aligned} \tag{7}$$

Integrating equation of continuity (5) gives

$$v = V_0 \tag{8}$$

Using equation (8) in solving equation (6)

$$\frac{\omega}{\mathrm{Re}}\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \eta} = \frac{\omega}{\mathrm{Re}}\frac{\partial U}{\partial t} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial \eta^2} - \frac{1}{\mathrm{Re}}\left(M^2 + \frac{1}{K}\right)(u - U) \tag{9}$$

Taking the solution of equation (9) of the form

$$u(\eta, t) = F_0(\eta) + \frac{\varepsilon}{2}\left[F_1(\eta)e^{it} + \overline{F}_1(\eta)\, e^{-it}\right] \tag{10}$$

where the bar denotes a complex conjugate, substituting for $u(\eta,t)$ from equation (9) and for $U(t)$ from equation (7) into equation (9) and then equating steady and periodic terms separately to zero, we get

$$F_0'' - \mathrm{Re}\, F_0' - \left(M^2 + \frac{1}{K}\right)F_0 = \left(M^2 + \frac{1}{K}\right) \tag{11}$$

$$F_1'' - \mathrm{Re}\, F_1' - \left(M^2 + \frac{1}{K} + i\omega\right)F_1 = -\left(M^2 + \frac{1}{K} + i\omega\right) \tag{12}$$

$$\tilde{F}_1'' - \mathrm{Re}\, \tilde{F}_1' - \left(M^2 + \frac{1}{K} - i\omega\right)\tilde{F}_1 = -\left(M^2 + \frac{1}{K} - i\omega\right) \tag{13}$$

Boundary conditions are

$$F_0(\eta) = -h_1\frac{\partial F_0}{\partial \eta},\quad F_1(\eta) = -h_1\frac{\partial F_1}{\partial \eta},\quad \tilde{F}_1(\eta) = -h_1\frac{\partial \tilde{F}_1}{\partial \eta} \text{ at } \eta = 0$$

$$F_0(\eta) = 1,\quad F_1(\eta) = 1,\quad \tilde{F}_1(\eta) = 1 \text{ at } \eta = 1 \tag{14}$$

The solution of equation (11) and (12) with the boundary conditions (14) can be obtained as

$$F_0(\eta) = C_1\, e^{m_1\eta} + C_2\, e^{-m_2\eta} + 1 \tag{15}$$

$$F_1(\eta) = C_3\, e^{m_3\eta} + C_4\, e^{-m_4\eta} + 1 \tag{16}$$

where,

$$L = M^2 + \frac{1}{K},\quad L_1 = M^2 + \frac{1}{K} + i\omega,\quad m_1 = \frac{\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4L}}{2},\quad m_2 = \frac{\mathrm{Re} - \sqrt{\mathrm{Re}^2 + 4L}}{2},$$

$$m_3 = \frac{\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4L_1}}{2},\quad m_4 = \frac{\mathrm{Re} - \sqrt{\mathrm{Re}^2 + 4L_1}}{2},\quad C_1 = \frac{e^{-m_3 - m_1}}{1 - h_1 m_2 - e^{-m_1 - m_2} - h_1 m_1 e^{-m_1 - m_2}},$$

$$C_2 = \frac{-1}{1-h_1 m_2 - e^{-m_1-m_2} - h_1 m_1 e^{-m_1-m_2}}, \quad C_3 = \frac{e^{-m_3-m_4}}{1-h_1 m_4 - e^{-m_3-m_4} - h_1 m_3 e^{-m_3-m_4}},$$

$$C_4 = \frac{-1}{1-h_1 m_4 - e^{-m_3-m_4} - h_1 m_3 e^{-m_3-m_4}}$$

The transient velocity can be deduced from eqs. (15) and (16) for $t = \frac{\pi}{2}$ as

$$u(\eta) = F_0(\eta) - \varepsilon F_1(\eta)$$

$$F_1(\eta) = F_r(\eta) + i F_i(\eta) \tag{17}$$

Now after knowing the velocity field we can calculate the skin friction at the lower plate as

$$\tau_w = \frac{d\tau_w^*}{\mu U_0} = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} = C_1 m_1 - C_2 m_2 - \varepsilon C_3 m_3 + \varepsilon C_4 m_4 \tag{18}$$

3. RESULTS AND NUMERICAL DISCUSSION

The absolute velocity and skin friction are plotted againt η for different values of K (the permeability parameter), Re (the injection/suction number) and h_1 (rarefaction parameter) in the figs 1 to 6, $V_0 = 1$.

In Figs.1-3, the absolute velocity distribution u is plotted against η for different values of K, Re and h_1. It is observed that u decreases as K or Re increases whereas it increases as h_1 increases.

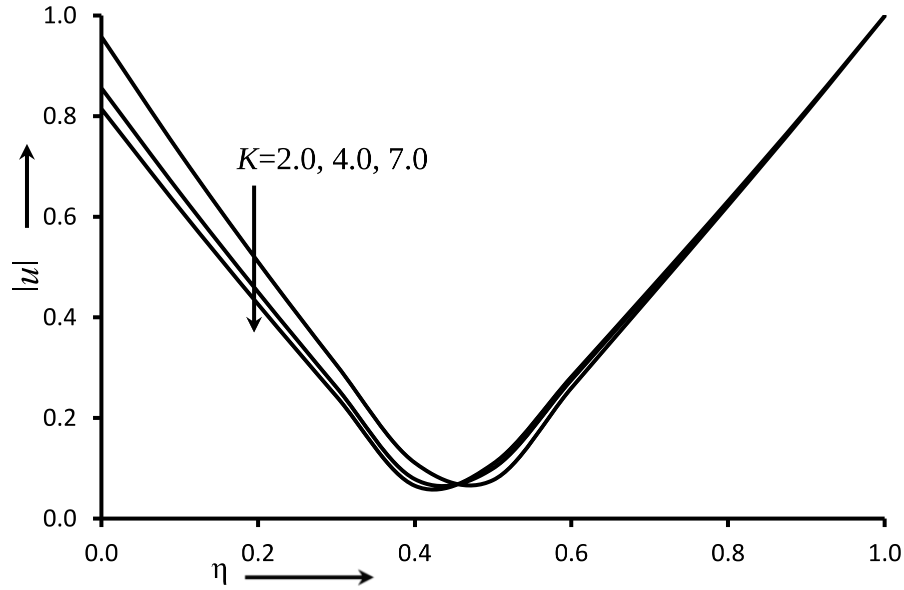

Figure 1. The absolute velocity versus η for different values of K when M=1.0, Re=5.0, h_1=0.4, ε=0.001 and ω=5.0

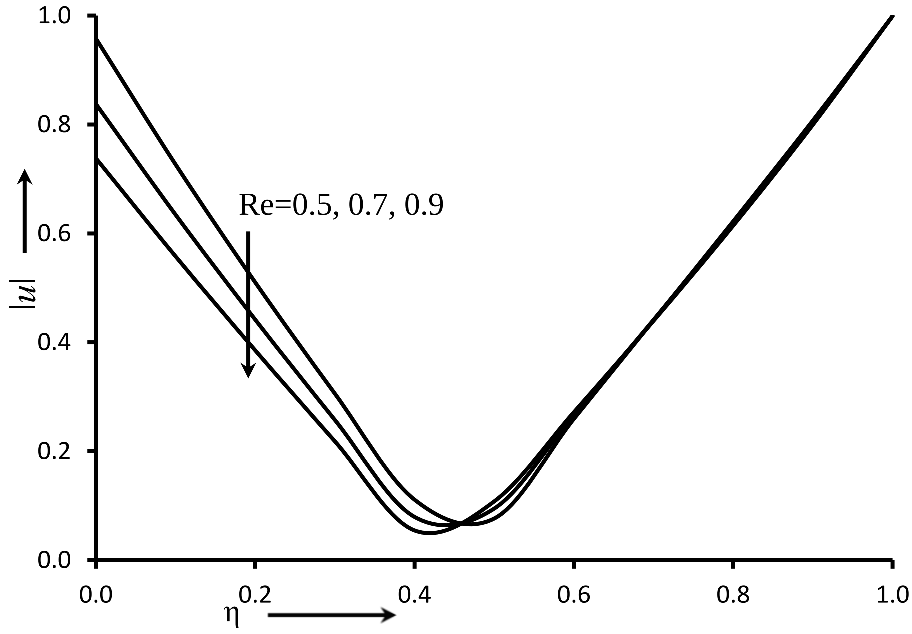

Figure 2. The absolute velocity versus η for different values of Re when M=1.0, K=2.0, h_1=0.4, ε=0.001 and ω=5.0.

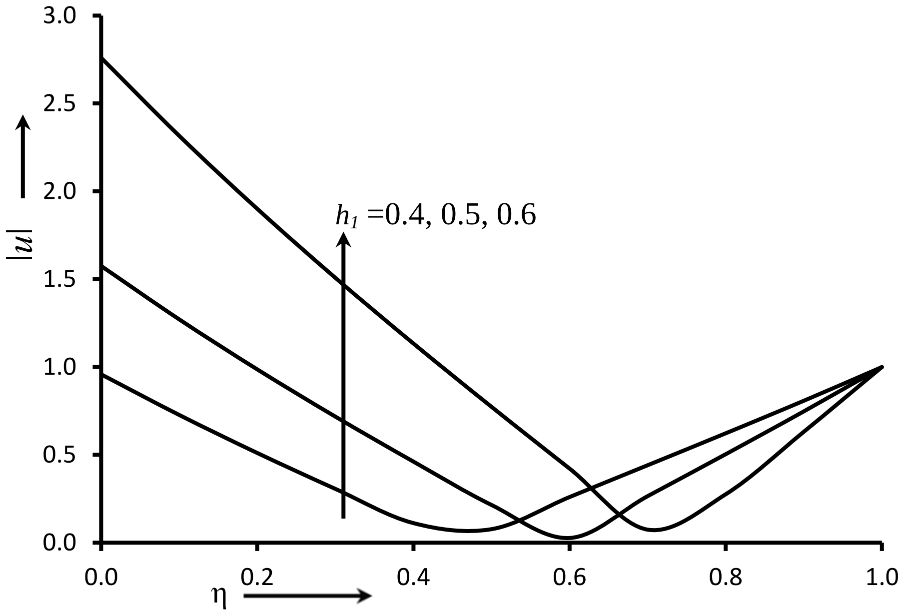

Figure 3. The absolute velocity versus η for different values of h_1 when M=1.0, Re=5.0, K=5.0, ε=0.001 and ω=5.0.

Figs. 4-6 depicts the variations in skin friction coefficient, it has been noted that skin friction decreases as K or Re decreases and it increase with h_1.

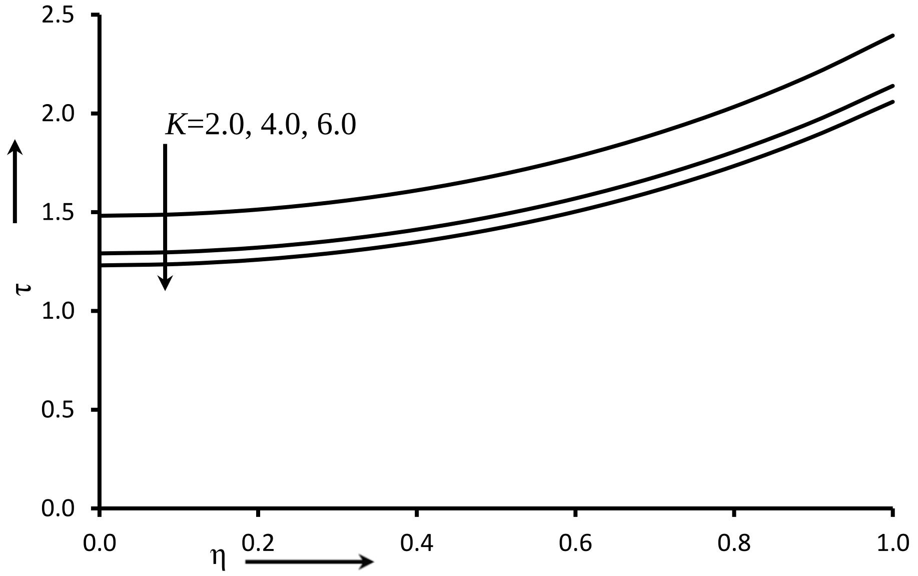

Figure 4. The skin friction coefficient versus η for different values of K when Re=0.05, h_1=0.4, ε=0.001 and ω=5.0.

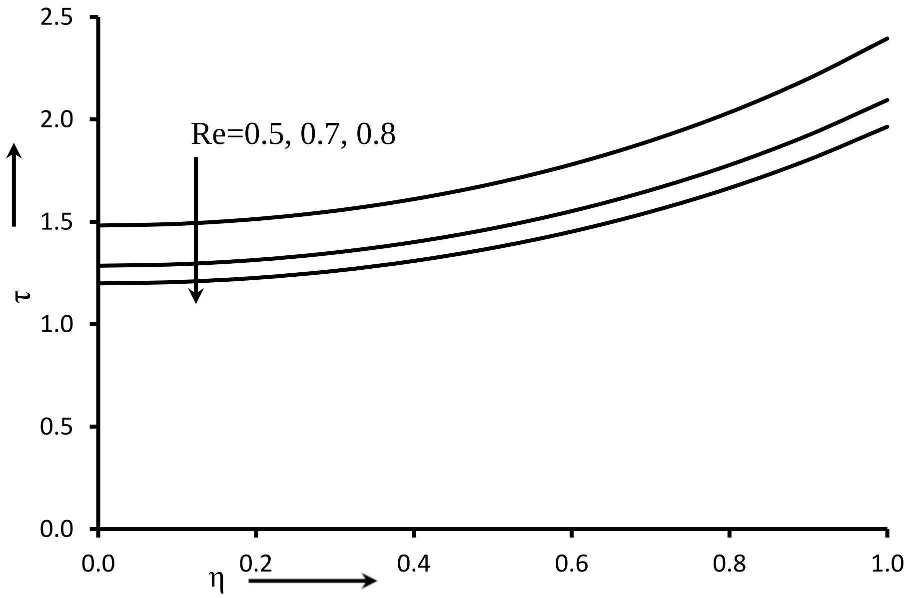

Figure 5. The skin friction coefficient versus η for different values of Re when K=2.0, h_1=0.4, ε=0.001 and ω=5.0.

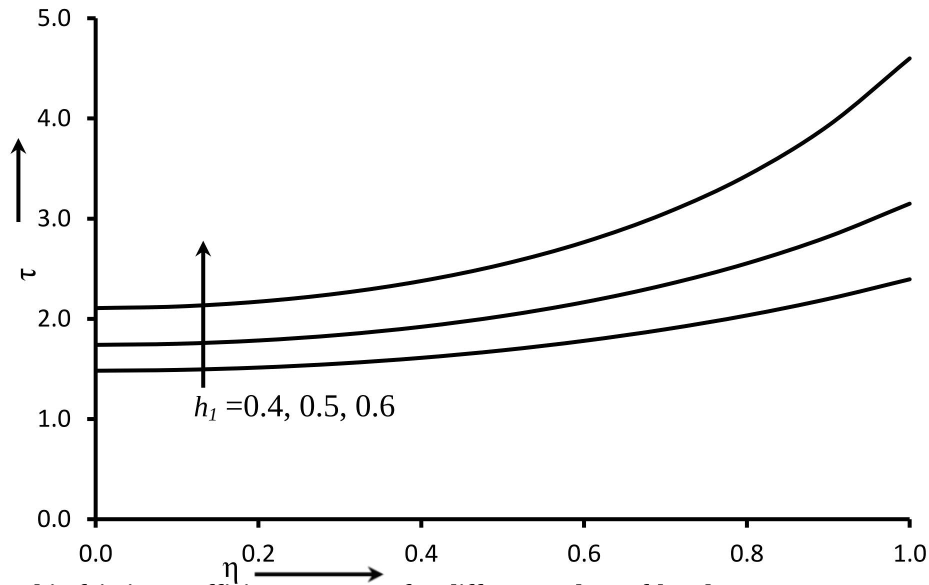

Figure 6. The skin friction coefficient versus η for different values of h_1 when K=2.0, Re=0.5, ε=0.001 and ω=5.0.

4. CONCLUSION

In this manuscript the effects of slip flow regime and effect of porosity on the oscillatory flow of an electrically conducting viscous, incompressible fluid, confined between two parallel infinite porous horizontal plates, when the upper plate is oscillatory in time and lower plate is motionless. The effect of pososity or injection/suction is to reduce the velocity of fluid or the sheer stress, whereas the rarefraction parameter enhance the velocity or the sheer stress.

NOMENCLATURE

y^* horizontal coordinate	m	η dimensionless horizontal coordinate
u^* axial velocity	m/s	u dimensionless axial velocity
v^* transverse velocity	m/s	v dimensionless transverse velocity
t^* time	s	t dimensionless time parameter
U^* velocity of oscillating plate	m/s	ω dimensionless frequency
L_1 mean free path	m	Re dimensionless injection/suction parameter
ρ density	kg/m^3	M dimensionless Hartmann number
μ dynamic coefficient of viscosity	Pa.s	K dimensionless permeability parameter
K^* permeability of porous medium	H/m	ε dimensionless positive constant
σ electrical conductivity	S/m	h_1 dimensionless rarefaction parameter
B_0 magnetic field coefficient	T	F_0 dimensionless axial velocity component
k thermal conductivity	W/m K	F_1 dimensionless axial velocity component
ω^* frequency	s^{-1}	τ_w dimensionless skin friction coefficient
V_0 dimensionless constant transverse velocity		C_1, C_2, C_3, C_4 constants

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