ORIGINAL PAPER

A GENERAL EXPONENTIAL FAMILY OF ESTIMATORS FOR POPULATION MEAN USING AUXILIARY ATTRIBUTE

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Abstract: *The present article addresses the problem of improved estimation of population mean of the study variable. For elevated estimation, we suggest a new generalized exponential ratio type family of estimators of population mean using auxiliary attribute. The bias and mean squared errors (MSE) of the suggested family are derived up to the approximation of order one. The optimum value of the characterizing scalar is obtained and the minimum MSE of the suggested family is also obtained. Further more generalized class is also proposed and the suggested family is its special case. The minimum MSE of this class is also obtained for the optimum value of the scalar. A theoretical comparison of the suggested class has made with the competing estimators of population mean. An empirical study has also been performed to show that the suggested class is better than the competing estimators of population mean.*

Keywords: *Study variable, Auxiliary Attribute, Exponential ratio estimator, Bias, MSE.*

1. INTRODUCTION

Sampling becomes Indispensable while dealing with the large population to save time, money and manpower. Thus in such situation estimation of parameters are well-advised rather than to calculate them. The most pertinent estimator for any parameter is the commensurate statistic. Thus the most germane estimator for estimating population mean is the sample mean. It is also desired from the estimator of the parameter under consideration that it should possess the properties of good estimator. Albeit the sample mean is unbiased for population mean but its sampling variance is reasonably large. So we quest for even biased estimator but having its sampling distribution very close to true parameter under consideration that is the estimator with lesser MSE. This objective is achieved through the use of the auxiliary information supplied by the auxiliary variable which is highly positively or negatively correlated with study variable. The auxiliary information may be quantitative as well as qualitative. Many times we come across the problems that auxiliary information is not available in quantitative form but is available in the form of an attribute. There are so many examples in real life where auxiliary information is available in the form of an attribute. For more details the latest references can be made of Singh and Kumar [1], Singh and Solanki [2], Sharma et al. [3], Sharma and Singh [4], Zaman [5] and Zaman and Toksoy [6].

Let the finite population under consideration has N units which are distinct and identifiable. Let $(Y_i, \phi_i), i = 1, 2, ..., N$ be the observation values on the i^{th} population unit for

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study variable y and the auxiliary attribute ϕ respectively. Further it is assume that $\phi_i = 1$ and $\phi_i = 0$, $i = 1, 2, ..., N$, if it possesses a particular characteristic or does not possess it. Let $A = \sum_{i=1}^{N} \phi_i$ and $a = \sum_{i=1}^{n} \phi_i$ represents the total number of units in the population and sample respectively which possess the attribute ϕ. Let $P = \frac{A}{N}$ and $p = \frac{a}{n}$ denote the proportion of units in the population and sample respectively possessing attribute ϕ. Let a sample of size n is drawn from the above population using simple random sampling without replacement (SRSWOR) technique with sample values $(y_i, \phi_i), i = 1, 2, ..., n.$

2. MATERIALS AND METHODS

2.1. REVIEW OF EXISTING ESTIMATORS

The vastly justified estimator for the population mean is the commensurate sample mean of study variable given by:

$$t_0 = \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \tag{1}$$

The estimator t_0 is unbiased for population mean and its variance up to the approximation of order one is:

$$V(t_0) = \lambda \bar{Y}^2 C_y^2 \tag{2}$$

where, $\lambda = \left(\frac{1}{n} - \frac{1}{N} \right)$, $\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} y_i$, $S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \bar{Y})^2$, $C_y = \frac{S_y}{\bar{Y}}$

Naik and Gupta [7] suggested the ratio type estimator of population mean $\bar{Y}$ of the main variable using the known information on auxiliary attribute as:

$$t_1 = \bar{y} \left(\frac{P}{p} \right) \tag{3}$$

The above estimator t_1 is biased estimator. The bias and mean squared error of t_1 respectively are:

$$\begin{aligned} B(t_1) &= \lambda \bar{Y} [C_p^2 - C_{yp}] \\ MSE(t_1) &= \lambda \bar{Y}^2 [C_y^2 + C_p^2 - 2C_{yp}] \end{aligned} \tag{4}$$

where, $C_p = \frac{S_p}{\bar{Y}}$, $S_p^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\phi_i - P)^2$, $C_{yp} = \rho_{pb} C_y C_p$.

Singh et al. [8] suggested exponential ratio type estimator of population mean using auxiliary attribute as:

$$t_2 = \bar{y}\exp\left(\frac{P-p}{P+p}\right) \tag{5}$$

The bias and the MSE of the estimator t_2 respectively are:

$$\begin{aligned} B(t_2) &= \lambda\bar{Y}\left[\frac{C_p^2}{4} - \frac{C_{yp}}{2}\right] \\ MSE(t_2) &= \lambda\bar{Y}^2\left[C_y^2 + \frac{C_p^2}{4} - C_{yp}\right] \end{aligned} \tag{6}$$

Zaman and Kadilar [9] suggested the following exponential ratio type family of estimator using known parameters of auxiliary attribute as:

$$t_{3(i)} = \bar{y}\exp\left[\frac{(a_iP+b_i)-(a_ip+b_i)}{(a_iP+b_i)+(a_ip+b_i)}\right] \ , \ i = 1, 2, ..., 9 \tag{7}$$

where, a_i and b_i are either constants or the known parameters of auxiliary attribute along with $a \neq 0$.

Zaman and Kadilar [9] have given some members of their family which are presented in Table 1, given below.

Table 1. Members of Zaman and Kadilar [9] family of estimators

Estimator	**Values of** a_i	b_i	**Estimator**	**Values of** a_i	b_i
$t_{3(1)} = \bar{y}\exp\left[\frac{P-p}{P+p+2\beta_2}\right]$	1	β_2	$t_{3(5)} = \bar{y}\exp\left[\frac{C_p(P-p)}{C_p(P+p)+2\beta_2}\right]$	C_p	β_2
$t_{3(2)} = \bar{y}\exp\left[\frac{P-p}{P+p+2C_p}\right]$	1	C_p	$t_{3(6)} = \bar{y}\exp\left[\frac{C_p(P-p)}{C_p(P+p)+2\rho_{pb}}\right]$	C_p	ρ_{pb}
$t_{3(3)} = \bar{y}\exp\left[\frac{P-p}{P+p+2\rho_{pb}}\right]$	1	ρ_{pb}	$t_{3(7)} = \bar{y}\exp\left[\frac{\rho_{pb}(P-p)}{\rho_{pb}(P+p)+2C_p}\right]$	ρ_{pb}	C_p
$t_{3(4)} = \bar{y}\exp\left[\frac{\beta_2(P-p)}{\beta_2(P+p)+2C_p}\right]$	β_2	C_p	$t_{3(8)} = \bar{y}\exp\left[\frac{\beta_2(P-p)}{\beta_2(P+p)+2\rho_{pb}}\right]$	β_2	ρ_{pb}
			$t_{3(9)} = \bar{y}\exp\left[\frac{\rho_{pb}(P-p)}{\rho_{pb}(P+p)+2\beta_2}\right]$	ρ_{pb}	β_2

Note: *More members can be found for different values of* a_i *and* b_i.

The bias and the MSE of Zaman and Kadilar [9] estimators respectively are given by:

$$B(t_{3(i)})=\lambda\bar{Y}[\theta_i^2C_p^2-\theta_iC_{yp}],\ i=1,2,\ldots,9 \tag{8}$$

$$MSE(t_{3(i)})=\lambda\bar{Y}^2[C_y^2+\theta_i^2C_p^2-2\theta_iC_{yp}],\ i=1,2,\ldots,9 \tag{9}$$

where,

$$\theta_1=\frac{P}{2(P+\beta_2)},\ \theta_2=\frac{P}{2(P+C_p)},\ \theta_3=\frac{P}{2(P+\rho_{pb})},\ \theta_4=\frac{\beta_2P}{2(\beta_2P+C_p)},\ \theta_5=\frac{C_pP}{2(C_pP+\beta_2)},$$

$$\theta_6=\frac{C_pP}{2(C_pP+\rho_{pb})},\ \theta_7=\frac{\rho_{pb}P}{2(\rho_{pb}P+C_p)},\ \theta_8=\frac{\beta_2P}{2(\beta_2P+\rho_{pb})},\ \theta_9=\frac{\rho_{pb}P}{2(\rho_{pb}P+\beta_2)}.$$

2.2. PROPOSED FAMILY OF ESTIMATORS

Motivated by Searls [10] and Zaman and Kadilar [9], we suggest the following new family of estimator of population mean of study variable using known population parameters of auxiliary attribute as:

$$\eta_i=\kappa_i\bar{y}\exp\left[\frac{(a_iP+b_i)-(a_ip+b_i)}{(a_iP+b_i)+(a_ip+b_i)}\right]\ i=1,2,\ldots,9 \tag{10}$$

where, κ_i are the characterizing scalars to be determined such that the MSE of the suggested estimator η_i are minimum.

Table 2 represents the members of the suggested family corresponding to the members of the Zaman and Kadilar [9] family of estimators.

Table 2. Members of the proposed family of estimators

Estimator	Values of a_i	Values of b_i	Estimator	Values of a_i	Values of b_i
$\eta_1=\kappa_1\bar{y}\exp\left[\frac{P-p}{P+p+2\beta_2}\right]$	1	β_2	$\eta_5=\kappa_5\bar{y}\exp\left[\frac{C_p(P-p)}{C_p(P+p)+2\beta_2}\right]$	C_p	β_2
$\eta_2=\kappa_2\bar{y}\exp\left[\frac{P-p}{P+p+2C_p}\right]$	1	C_p	$\eta_6=\kappa_6\bar{y}\exp\left[\frac{C_p(P-p)}{C_p(P+p)+2\rho_{pb}}\right]$	C_p	ρ_{pb}
$\eta_3=\kappa_3\bar{y}\exp\left[\frac{P-p}{P+p+2\rho_{pb}}\right]$	1	ρ_{pb}	$\eta_7=\kappa_7\bar{y}\exp\left[\frac{\rho_{pb}(P-p)}{\rho_{pb}(P+p)+2C_p}\right]$	ρ_{pb}	C_p
$\eta_4=\kappa_4\bar{y}\exp\left[\frac{\beta_2(P-p)}{\beta_2(P+p)+2C_p}\right]$	β_2	C_p	$\eta_8=\kappa_8\bar{y}\exp\left[\frac{\beta_2(P-p)}{\beta_2(P+p)+2\rho_{pb}}\right]$	β_2	ρ_{pb}
			$\eta_9=\kappa_9\bar{y}\exp\left[\frac{\rho_{pb}(P-p)}{\rho_{pb}(P+p)+2\beta_2}\right]$	ρ_{pb}	β_2

Note: *More members can be found for different values of* a_i *and* b_i.

To derive the bias and the MSE of the suggested family of estimators, we use the following standard approximations given as:

$\bar{y} = \bar{Y}(1+e_0)$, $p = P(1+e_1)$, such that $E(e_0) = E(e_1) = 0$, and $E(e_0^2) = \lambda C_y^2$, $E(e_1^2) = \lambda C_p^2$, $E(e_0 e_1) = \lambda C_{yp}$.

Using the above approximations, simplifying and taking expectations, we get the bias and MSE of the suggested class respectively as,

$$B(\eta_i) = (\kappa_i - 1)\bar{Y} + \kappa_i \bar{Y}\left[\frac{3}{2}\theta_i^2 \lambda C_p^2 - \theta_i \lambda C_{yp}\right]$$

$$MSE(\eta_i) = \bar{Y}^2\left[1 + \kappa_i^2(1 + \lambda C_y^2 + 4\theta_i^2 \lambda C_p^2 - 4\theta_i \lambda C_{yp}) - 2\kappa_i(1 + \frac{3}{2}\theta_i^2 \lambda C_p^2 - \theta_i \lambda C_{yp})\right] \quad (11)$$

The optimum value of κ_i, which minimizes $MSE(\eta_i)$ is:

$$\kappa_i = \frac{(1 + \frac{3}{2}\theta_i^2 \lambda C_p^2 - \theta_i \lambda C_{yp})}{(1 + \lambda C_y^2 + 4\theta_i^2 \lambda C_p^2 - 4\theta_i \lambda C_{yp})} = \frac{A_i}{B_i} \quad (12)$$

where, $A_i = (1 + \frac{3}{2}\theta_i^2 \lambda C_p^2 - \theta_i \lambda C_{yp})$ and $B_i = (1 + \lambda C_y^2 + 4\theta_i^2 \lambda C_p^2 - 4\theta_i \lambda C_{yp})$

The bias of the suggested family of estimators η_i and the minimum value of $MSE(\eta_i)$ for the optimum value of κ_i respectively are:

$$B(\eta_i) = -\bar{Y}\left[1 - \frac{A_i^2}{B_i}\right]$$

$$MSE_{\min}(\eta_i) = \bar{Y}^2\left[1 - \frac{A_i^2}{B_i}\right] \quad (13)$$

2.3. MORE GENERAL CLASS OF SUGGESTED ESTIMATORS

We further suggest a more general class of estimators of population mean of primary variable using known parameters of auxiliary attribute as:

$$\eta_i^* = \kappa_1 \bar{y} + \kappa_2 \bar{y} \exp\left[\frac{(a_i P + b_i) - (a_i p + b_i)}{(a_i P + b_i) + (a_i p + b_i)}\right] \quad (14)$$

where, κ_1 and κ_2 are scalar constants with $\kappa_1 + \kappa_2 = 1$ and to be determined such that $MSE(\eta_i^*)$ is minimum.

Some special known members of the above general class are presented in Table3.

Table 3. Some known special members of the more general class

Estimator	Value of	
	κ_1	κ_2
$t_0 = \bar{y}$, Mean per unit estimator	1	0
$\kappa\bar{y}$, Seals (1964) estimator	κ	0
$t_{3(i)} = \bar{y}\exp\left[\frac{(a_i P + b_i) - (a_i p + b_i)}{(a_i P + b_i) + (a_i p + b_i)}\right]$ Zaman and Kadilar (2019) family of estimators	0	1
$\eta_i = \kappa_i \bar{y}\exp\left[\frac{(a_i P + b_i) - (a_i p + b_i)}{(a_i P + b_i) + (a_i p + b_i)}\right]$ Proposed family of estimators	0	κ_i

Note: *More members can be found for different values of* κ_1 *and* κ_2.

Expressing the suggested class in (14) in terms of e_i's and simplifying, we have:

$$\eta_i^* = \bar{Y}[1 + e_0 - (1+\kappa_1)(\theta_i e_1 + \theta_i e_0 e_1 - \frac{3}{2}\theta_i^2 e_1^2)]$$

Subtracting $\bar{Y}$ both sides of above equation, we have:

$$\eta_i^* - \bar{Y} = \bar{Y}[e_0 - (1+\kappa_1)(\theta_i e_1 + \theta_i e_0 e_1 - \frac{3}{2}\theta_i^2 e_1^2)] \tag{15}$$

Taking expectations on both sides and putting the values of various expectations, we get the bias of η_i^* as:

$$B(\eta_i^*) = -\kappa_3 \theta_i \lambda \bar{Y}(C_{yp} - \frac{3}{2}\theta_i C_p^2), \text{ where, } \kappa_3 = (1+\kappa_1) \tag{16}$$

From (15), up to the first order of approximation, we have the MSE of η_i^* as:

$$MSE(\eta_i^*) \cong \bar{Y}^2 E[e_0 - \kappa_3 \theta_i e_1]^2$$

Expanding, simplifying and putting values of various expectations, we have:

$$MSE(\eta_i^*) = \lambda \bar{Y}^2 [C_y^2 - \kappa_3^2 \theta_i^2 C_p^2 - 2\kappa_3 \theta_i C_{yp}] \tag{17}$$

The optimum value of κ_3, which minimizes the $MSE(\eta_i^*)$ is:

$$\kappa_3 = \frac{C_{yp}}{\theta_i C_p^2} = \kappa_{3(opt)} \text{ (say)}$$

The bias and minimum MSE of η_i^* for the above optimum value of κ_3, respectively are:

$$B(\eta_i^*) = \lambda \bar{Y} C_{yp}^2 \left[\frac{3}{2} - \frac{1}{C_p^2}\right]$$

$$MSE_{\min}(\eta_i^*) = \lambda \bar{Y}^2 \left[C_y^2 - \frac{C_{yp}^2}{C_p^2}\right] \quad (18)$$

3. RESULTS AND DISCUSSION

3.1. BIAS AND MSE EFFICIENCY COMPARISON

In the following Table4 the bias and MSE comparison of the suggested estimator with the competing estimators have been presented and the conditions for the proposed estimator to perform better than the competing estimators are given.

Table 4. Bias and MSE comparison.

Estimator	**Bias comparison** $B(t_i) - B(\eta_i^*) > 0$	**MSE comparison** $MSE(t_i) - MSE_{\min}(\eta_i^*) > 0$
$t_0 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ Mean per unit estimator	-	$\lambda \bar{Y}^2 \frac{C_{yp}^2}{C_p^2} > 0$
$t_1 = \bar{y}\left(\frac{P}{p}\right)$ Naik and Gupta [7]	$[C_p^2 - C_{yp}] - C_{yp}^2\left[\frac{3}{2} - \frac{1}{C_p^2}\right] > 0$	$[C_p^2 - 2C_{yp}] + \frac{C_{yp}^2}{C_p^2} > 0$
$t_2 = \bar{y}\exp\left(\frac{P-p}{P+p}\right)$ Singh *et al.* [8]	$\left[\frac{C_p^2}{4} - \frac{C_{yp}}{2}\right] - C_{yp}^2\left[\frac{3}{2} - \frac{1}{C_p^2}\right] > 0$	$\left[\frac{C_p^2}{4} - C_{yp}\right] + \frac{C_{yp}^2}{C_p^2} > 0$
$t_{3(i)} = \bar{y}\exp\left[\frac{(a_i P + b_i) - (a_i p + b_i)}{(a_i P + b_i) + (a_i p + b_i)}\right]$ Zaman and Kadilar [9]	$[\theta_i^2 C_p^2 - \theta_i C_{yp}] - C_{yp}^2\left[\frac{3}{2} - \frac{1}{C_p^2}\right] > 0$	$[\theta_i^2 C_p^2 - 2\theta_i C_{yp}] + \frac{C_{yp}^2}{C_p^2} > 0$
$\eta_i = \kappa_i \bar{y}\exp\left[\frac{(a_i P + b_i) - (a_i p + b_i)}{(a_i P + b_i) + (a_i p + b_i)}\right]$ Proposed Estimator	$\left[\frac{A_i^2}{B_i} - 1\right] - \lambda C_{yp}^2\left[\frac{3}{2} - \frac{1}{C_p^2}\right] > 0$	$\left[1 - \frac{A_i^2}{B_i}\right] - \lambda\left[C_y^2 - \frac{C_{yp}^2}{C_p^2}\right] > 0$

3.2. NUMERICAL ILLUSTRATION

For the theoretical justification, we have considered the two natural populations in Zaman and Kadilar [9]. The parameters of the populations are given in Table-5.

Table 5. Parameters of two natural populations

Population I				
$N = 89$	$\bar{Y} = 3.3596$	$\lambda_1 = 0.0171$	$\lambda_5 = 0.0433$	$\lambda_9 = 0.0132$
$n = 20$	$P = 0.1236$	$\lambda_2 = 0.0221$	$\lambda_6 = 0.1508$	
$\beta_2 = 3.4920$	$C_y = 0.6008$	$\lambda_3 = 0.0695$	$\lambda_7 = 0.0171$	
$\rho_{pb} = 0.7660$	$C_p = 2.6779$	$\lambda_4 = 0.0694$	$\lambda_8 = 0.1802$	
Population II				
$N = 111$	$\bar{Y} = 29.2790$	$\lambda_1 = 0.0146$	$\lambda_5 = 0.0382$	$\lambda_9 = 0.0117$
$n = 30$	$P = 0.1170$	$\lambda_2 = 0.0203$	$\lambda_6 = 0.1441$	
$\beta_2 = 3.8980$	$C_y = 0.8720$	$\lambda_3 = 0.0640$	$\lambda_7 = 0.0164$	
$\rho_{pb} = 0.7970$	$C_p = 2.7580$	$\lambda_4 = 0.0709$	$\lambda_8 = 0.1819$	

The MSE and the PRE of the suggested estimator with respect to the competing estimators are presented in Table6 and Table7 for the PopulationI and PopulationII respectively.

Table 6.MSE of various estimators and PRE of suggested estimator for PopulationI.

Estimator	MSE	PRE	Estimator	MSE	PRE
t_0	0.1579	253.4510	$t_{3(9)}$	0.1442	231.4607
t_1	2.2168	3558.266	η_1	0.1389	222.9535
t_2	0.4030	646.8700	η_2	0.1342	215.4093
$t_{3(1)}$	0.1404	225.3612	η_3	0.0975	156.5008
$t_{3(2)}$	0.1357	217.8170	η_4	0.0976	156.6613
$t_{3(.3)}$	0.0981	157.4639	η_5	0.1162	186.5169
$t_{3(4)}$	0.0982	157.6244	η_6	0.0659	105.7785
$t_{3(5)}$	0.1171	187.9615	η_7	0.1389	222.9535
$t_{3(6)}$	0.0667	107.0626	η_8	0.0642	103.0498
$t_{3(7)}$	0.1404	225.3612	η_9	0.1426	228.8925
$t_{3(8)}$	0.0654	104.9759	$(\eta_i^*)_{\min}$	0.0623	100.0000

Table 7. MSE of various estimators and PRE of suggested estimator for PopulationII.

Estimator	MSE	PRE	Estimator	MSE	PRE
t_0	15.8557	274.1304	$t_{3(9)}$	14.9436	258.3610
t_1	94.532	1634.3710	η_1	14.4910	250.5360
t_2	15.5403	268.6774	η_2	14.0860	243.5339
$t_{3(1)}$	14.7247	254.5764	η_3	11.2780	194.9862
$t_{3(2)}$	14.2948	247.1438	η_4	10.8840	188.1743
$t_{3(.3)}$	11.3891	196.9070	η_5	12.8720	222.5450
$t_{3(4)}$	10.9827	189.8807	η_6	7.5765	130.9907
$t_{3(5)}$	13.0318	225.3077	η_7	14.3620	248.3057
$t_{3(6)}$	7.6304	131.9225	η_8	6.5039	112.4464
$t_{3(7)}$	14.5909	252.2631	η_9	14.7010	254.1667
$t_{3(8)}$	6.5614	113.4405	$(\eta_i^*)_{\min}$	5.7840	100.0000

Figs. 1 and 2 represents the PRE of the suggested estimators over the competing estimators for the PopulationI and PopulationII respectively.

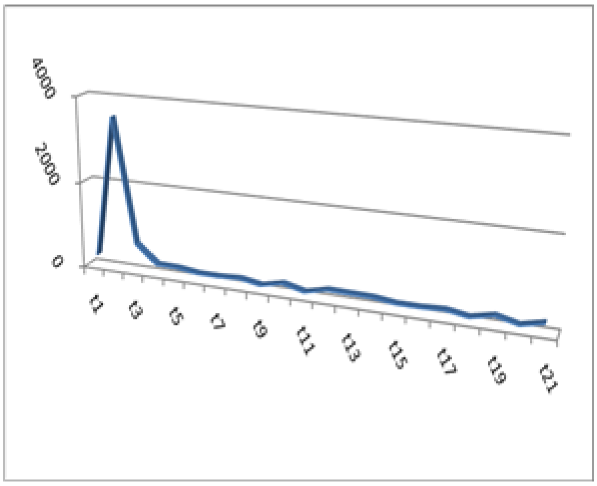

Figure 1. PRE of suggested estimator for Population I.

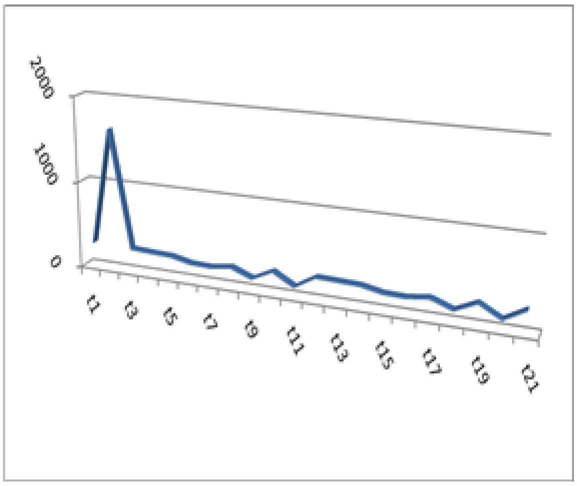

Figure 2. PRE of suggested estimator for Population II.

4. CONCLUSIONS

In the present paper, it was suggested a family of estimators of population mean utilizing the known parameters of auxiliary attribute. It was studied the large sampling properties of the suggested family through the bias and mean squared error. Further the suggested family has been more generalized and some known estimators along with the suggested family have been shown as the members of the more generalized proposed family. Its bias and MSE has been obtained up to the approximation of order one. The least value of the MSE of suggested family has been obtained for the optimum value of the characterizing scalar.

From Tables 6 and 7, it can be observed that the suggested estimator has the least MSE. Thus the sampling distribution of the suggested estimator is most close to the true population mean as per the purpose of the article. Thus the suggested family may be used for the enhanced estimation of population mean using known auxiliary attribute. Further the suggested family may be extended to various sampling designs.

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