ORIGINAL PAPER

ON INEXTENSIBLE FLOWS OF Π_1 BISHOP SPHERICAL IMAGES WITH TYPE-2 BISHOP FRAME

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Abstract. *In this paper, we study $\mathbf{\Pi}_1$ Bishop spherical images according to type-2 Bishop frame. Using the type-2 Bishop frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in space.*

Keywords: *Type-2 Bishop frame, Curvatures, Flows.*

1. INTRODUCTION

A large number of aspects of engineering and applied physics utilized gaseous, smooth flows. Gaseous flows are extremely essential with spacecraft, automobiles, aeroplanes. Likewise, many people utilized in the style and design of wind generators and burning machines. Research of smooth flow is extremely important meant for the uses of naviero, just like the style of boats and various tasks in municipal design including the design and style of the roadstead and the safety of seaside [1]. Flows are researched simply by various experts [1-20].

Construction of fluid flows constitutes an active research field with a high industrial impact. Corresponding real-world measurements in concrete scenarios complement numerical results from direct simulations of the Navier-Stokes equation, particularly in the case of turbulent flows, and for the understanding of the complex spatio-temporal evolution of instationary flow phenomena. More and more advanced imaging devices (lasers, highspeed cameras, control logic, etc.) are currently developed that allow to record fully timeresolved image sequences of fluid flows at high resolutions. As a consequence, there is a need for advanced algorithms for the analysis of such data, to provide the basis for a subsequent pattern analysis, and with abundant applications across various areas [21, 22].

In this paper, we study $\mathbf{\Pi}_1$ Bishop spherical images in Euclidean space $\mathbb{E}^3$. Using the type-2 Bishop frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in Euclidean space $\mathbb{E}^3$.

2. MATERIALS AND METHODS

Assume that $\{\mathbf{T},\mathbf{N},\mathbf{B}\}$ be the Frenet frame field along α. Then, the Frenet frame satisfies the following Frenet-Serret equations [23]:

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$$\nabla_{\mathrm{T}}\mathrm{T} = \kappa\mathrm{N},$$

$$\nabla_{\mathrm{T}}\mathrm{N} = -\kappa\mathrm{T} + \tau\mathrm{B},$$

$$\nabla_{\mathrm{T}}\mathrm{B} = -\tau\mathrm{N},$$

where κ is the curvature of α and τ its torsion and

$$g(\mathrm{T},\mathrm{T}) = 1, g(\mathrm{N},\mathrm{N}) = 1, g(\mathrm{B},\mathrm{B}) = 1,$$

$$g(\mathrm{T},\mathrm{N}) = g(\mathrm{T},\mathrm{B}) = g(\mathrm{N},\mathrm{B}) = 0.$$

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative [24]. The Bishop frame is expressed as:

$$\nabla_{\mathrm{T}}\mathrm{T} = k_1\mathrm{M}_1 + k_2\mathrm{M}_2,$$

$$\nabla_{\mathrm{T}}\mathrm{M}_1 = -k_1\mathrm{T},$$

$$\nabla_{\mathrm{T}}\mathrm{M}_2 = -k_2\mathrm{T},$$

where

$$g(\mathrm{T},\mathrm{T}) = 1, g(\mathrm{M}_1,\mathrm{M}_1) = 1, g(\mathrm{M}_2,\mathrm{M}_2) = 1,$$

$$g(\mathrm{T},\mathrm{M}_1) = g(\mathrm{T},\mathrm{M}_2) = g(\mathrm{M}_1,\mathrm{M}_2) = 0.$$

Here, we shall call the set $\{\mathbf{T},\mathbf{M}_1,\mathbf{M}_2\}$ as Bishop trihedra, k_1 and k_2 as Bishop curvatures and $\mathrm{U}(s) = \arctan\frac{k_2}{k_1}$, $\tau(s) = \mathrm{U}'(s)$ and $\kappa(s) = \sqrt{k_1^2 + k_2^2}$.

Bishop curvatures are defined by:

$$k_1 = \kappa(s)\cos\mathrm{U}(s),$$

$$k_2 = \kappa(s)\sin\mathrm{U}(s).$$

Let us express a relatively parallel adapted frame:

$$\nabla_T \Pi_1 = -\varepsilon_1 B,$$

$$\nabla_T \Pi_2 = -\varepsilon_2 B,$$

$$\nabla_T B = \varepsilon_1 \Pi_1 + \varepsilon_2 \Pi_2,$$

where

$$g(B,B) = 1, g(\Pi_1,\Pi_1) = 1, g(\Pi_2,\Pi_2) = 1,$$

$$g(B,\Pi_1) = g(B,\Pi_2) = g(\Pi_1,\Pi_2) = 0.$$

We shall call this frame as Type-2 Bishop Frame. In order to investigate this new frame's relation with Frenet-Serret frame, first we write [25]:

$$\tau = \sqrt{\varepsilon_1^2 + \varepsilon_2^2}.$$

The relation matrix between Frenet--Serret and type-2 Bishop frames can be expressed as:

$$\mathbf{T} = \sin \mathsf{A}(s)\mathbf{\Pi}_1 - \cos \mathsf{A}(s)\mathbf{\Pi}_2,$$

$$\mathbf{N} = \cos \mathsf{A}(s)\mathbf{\Pi}_1 + \sin \mathsf{A}(s)\mathbf{\Pi}_2,$$

$$\mathbf{B} = \mathbf{B}.$$

So by (2.4), we may express as:

$$\varepsilon_1 = -\tau \cos \mathsf{A}(s),$$

$$\varepsilon_2 = -\tau \sin \mathsf{A}(s).$$

By this way, we conclude that:

$$\mathsf{A}(s) = \arctan \frac{\varepsilon_2}{\varepsilon_1}.$$

The frame $\{\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{B}\}$ is properly oriented, and τ and $\mathsf{A}(s) = \int_0^s \kappa(s)ds$ are polar coordinates for the curve α. We shall call the set $\{\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{B}, \boldsymbol{\varepsilon}_1, \varepsilon_2\}$ as type-2 Bishop invariants of the curve α.

Definition 2.1. Let α be a regular curve in E^3. If we translate of the first vector field of type-2 Bishop frame to the center O of the unit sphere S^2, we obtain a spherical image ϕ. This curve is called $\mathbf{\Pi}_1$ Bishop spherical image or indicatrix of the curve α.

3. FLOWS WITH TYPE-2 BISHOP FRAME

Let $\alpha(u,t)$ is a one parameter family of smooth curves in E^3.
The arclength of α is given by:

$$s(u) = \int_0^u \left|\frac{\partial \alpha}{\partial u}\right| du,$$

where

$$\left|\frac{\partial \alpha}{\partial u}\right| = \left|\left\langle \frac{\partial \alpha}{\partial u}, \frac{\partial \alpha}{\partial u}\right\rangle\right|^{\frac{1}{2}}.$$

The operator $\frac{\partial}{\partial s}$ is given in terms of u by:

$$\frac{\partial}{\partial s} = \frac{1}{v}\frac{\partial}{\partial u},$$

where $v = \left|\frac{\partial \alpha}{\partial u}\right|$ and the arclength parameter is $ds = vdu.$
Any flow of α can be represented as $\{\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{B}\}$

$$\frac{\partial \alpha}{\partial t} = \mathsf{b}_1\mathbf{\Pi}_1 + \mathsf{b}_2\mathbf{\Pi}_2 + \mathsf{b}_3\mathbf{B},$$

where $\mathsf{b}_1, \mathsf{b}_2, \mathsf{b}_3 \in C^\infty(\mathsf{E}^3)$.

Definition 3.1. *The flow* $\frac{\partial \alpha}{\partial t}$ *in* E^3 *are said to be inextensible if*

$$\frac{\partial}{\partial t}\left|\frac{\partial \alpha}{\partial u}\right| = 0.$$

Lemma 3.2. *Let* $\frac{\partial \alpha}{\partial t}$ *be a smooth flow of the curve* α *according to new type-2 Bishop frame. The flow is inextensible if and only if*

$$(\frac{\partial b_1}{\partial u}+b_3 v\varepsilon_1)\sin A=(\frac{\partial b_2}{\partial u}+b_3 v\varepsilon_2)\cos A,$$

where $b_1, b_2, b_3 \in C^{\infty}(E^3)$.

Theorem 3.3.

$$\frac{\partial \mathbf{\Pi}_1}{\partial t} = [p_1-\cos A(\frac{\partial b_1}{\partial s}+b_3\varepsilon_1+\frac{\partial A}{\partial s}\cos A)]\mathbf{\Pi}_2+p_2\mathbf{B},$$

$$\frac{\partial \mathbf{\Pi}_2}{\partial t} = [p_3+(\frac{\partial b_2}{\partial s}+b_3\varepsilon_2-\frac{\partial A}{\partial s}\sin A)\sin A]\mathbf{\Pi}_1+p_4\mathbf{B},$$

$$\frac{\partial \mathbf{B}}{\partial t} = [p_5+(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\sin A]\mathbf{\Pi}_1 +[p_6-(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\cos A]\mathbf{\Pi}_2,$$

where $p_1, p_2, p_3, p_4, p_5, p_6 \in C^{\infty}(E^3)$.

Theorem 3.4.

$$\frac{\partial \mathbf{T}^{\phi}}{\partial t} = [p_5+(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\sin A]\mathbf{\Pi}_1 +[p_6-(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\cos A]\mathbf{\Pi}_2,$$

$$\begin{aligned}\frac{\partial \mathbf{N}^{\phi}}{\partial t} = & -[\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[p_3+(\frac{\partial b_2}{\partial s}+b_3\varepsilon_2-\frac{\partial A}{\partial s}\sin A)\sin A]]\mathbf{\Pi}_1 \\ & -[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathbf{\Pi}_2+(\frac{1}{\kappa^{\phi}})[p_1-\cos A(\frac{\partial b_1}{\partial s}+b_3\varepsilon_1+\frac{\partial A}{\partial s}\cos A)]]\mathbf{\Pi}_2 \\ & -[(\frac{1}{\kappa^{\phi}})p_2+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})p_4]\mathbf{B},\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{B}^{\phi}}{\partial t} = & [\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\frac{1}{\kappa^{\phi}})[p_3+(\frac{\partial b_2}{\partial s}+b_3\varepsilon_2-\frac{\partial A}{\partial s}\sin A)\sin A]]\mathbf{\Pi}_1 \\ & +[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[p_1-\cos A(\frac{\partial b_1}{\partial s}+b_3\varepsilon_1+\frac{\partial A}{\partial s}\cos A)]-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})]\mathbf{\Pi}_2 \\ & +[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})p_2-(\frac{1}{\kappa^{\phi}})p_4]\mathbf{B},\end{aligned}$$

where $p_1, p_2, p_3, p_4, p_5, p_6, b_1, b_2, b_3$ *are smooth functions of time and arc length.*

Proof: Using definition of ϕ, we have

$$\frac{\partial \mathbf{T}^{\phi}}{\partial t}=\frac{\partial \mathbf{B}}{\partial t}=[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]\mathbf{\Pi}_1+[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]\mathbf{\Pi}_2.$$

Using the (2.3) equation, we have

$$\begin{aligned}\frac{\partial \mathbf{N}^{\phi}}{\partial t} &= -\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})\mathbf{\Pi}_1-\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathbf{\Pi}_2\\ &-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]\mathbf{\Pi}_2-(\frac{1}{\kappa^{\phi}})\mathrm{p}_2\mathbf{B}\\ &-(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]\mathbf{\Pi}_1-(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4\mathbf{B}\end{aligned}$$

This implies:

$$\begin{aligned}\frac{\partial \mathbf{B}^{\phi}}{\partial t} &= \frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathbf{\Pi}_1-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})\mathbf{\Pi}_2\\ &+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]\mathbf{\Pi}_2+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_2\mathbf{B}\\ &-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]\mathbf{\Pi}_1-(\frac{1}{\kappa^{\phi}})\mathrm{p}_4\mathbf{B}.\end{aligned}$$

Then, we obtain the theorem. So, theorem is proved.

Theorem 3.5. *Let* $\frac{\partial \alpha}{\partial t}$ *be inextensible according to new type-2 Bishop frame. If* ϕ *is spherical image of* α, *then,*

$$\begin{aligned}\frac{\partial}{\partial s}[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}] &= [-\frac{\partial}{\partial t}(\varepsilon_1\kappa^{\phi})(\frac{1}{\kappa^{\phi}})-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})\\ &-(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]],\end{aligned}$$

where $\mathrm{p}_1,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Proof. Then we can easily see that

$$\begin{aligned}\frac{\partial}{\partial s}\frac{\partial}{\partial t}\mathbf{T}^{\phi} &= \frac{\partial}{\partial s}[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]\mathbf{\Pi}_1+\frac{\partial}{\partial s}[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]\mathbf{\Pi}_2\\ &-[\varepsilon_1[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]+\varepsilon_2[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]]\mathbf{B}.\end{aligned}$$

Also, we have the following

$$\frac{\partial}{\partial t}\frac{\partial}{\partial s}\mathbf{T}^{\phi} = [-\frac{\partial}{\partial t}(\varepsilon_1\kappa^{\phi})(\frac{1}{\kappa^{\phi}})-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})-(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]]\mathbf{\Pi}_1$$
$$+[-\frac{\partial}{\partial t}(\varepsilon_1\kappa^{\phi})(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]]\mathbf{\Pi}_2$$
$$-[(\frac{1}{\kappa^{\phi}})\mathrm{p}_2+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4]\mathbf{B}.$$

Thus, we obtain the theorem.

In the light of Theorem 3.5, we express the following corollary without proof:

Corollary 3.6.

$$\frac{\partial}{\partial s}[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]=[-\frac{\partial}{\partial t}(\varepsilon_1\kappa^{\phi})(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})$$
$$-\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Corollary 3.7.

$$(\frac{1}{\kappa^{\phi}})\mathrm{p}_2+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4] = [\varepsilon_1[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]$$
$$+\varepsilon_2[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Theorem 3.8. *Let* $\frac{\partial\alpha}{\partial t}$ *be inextensible according to new type-2 Bishop frame. If* ϕ *is spherical image of* α, *then,*

$$-\frac{\partial}{\partial s}\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})-\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]]$$
$$-[(\frac{1}{\kappa^{\phi}})\mathrm{p}_2]\varepsilon_1-[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4]\varepsilon_1=[\frac{\partial}{\partial t}(\varepsilon_1\tau^{\phi})(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\varepsilon_1\kappa^{\phi})[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1$$
$$-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]+(\varepsilon_1\tau^{\phi})[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Proof. We can write

$$\frac{\partial}{\partial s}\frac{\partial \mathbf{N}^{\phi}}{\partial t} = [-\frac{\partial}{\partial s}\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})-\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathsf{p}_3+(\frac{\partial \mathsf{b}_2}{\partial s}+\mathsf{b}_3\varepsilon_2$$

$$-\frac{\partial \mathsf{A}}{\partial s}\sin\mathsf{A})\sin\mathsf{A}]]-[(\frac{1}{\kappa^{\phi}})\mathsf{p}_2]\varepsilon_1-[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathsf{p}_4]\varepsilon_1]\mathbf{\Pi}_1$$

$$+[-\frac{\partial}{\partial s}\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-\frac{\partial}{\partial s}[(\frac{1}{\kappa^{\phi}})[\mathsf{p}_1-\cos\mathsf{A}(\frac{\partial \mathsf{b}_1}{\partial s}+\mathsf{b}_3\varepsilon_1$$

$$+\frac{\partial \mathsf{A}}{\partial s}\cos\mathsf{A})]]-[(\frac{1}{\kappa^{\phi}})\mathsf{p}_2]\varepsilon_2-[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathsf{p}_4]\varepsilon_2]\mathbf{\Pi}_2$$

$$+[\varepsilon_1\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})+\varepsilon_2\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-\frac{\partial}{\partial s}[(\frac{1}{\kappa^{\phi}})\mathsf{p}_2]$$

$$+\varepsilon_2[(\frac{1}{\kappa^{\phi}})[\mathsf{p}_1-\cos\mathsf{A}(\frac{\partial \mathsf{b}_1}{\partial s}+\mathsf{b}_3\varepsilon_1+\frac{\partial \mathsf{A}}{\partial s}\cos\mathsf{A})]]$$

$$-\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathsf{p}_4]+\varepsilon_1[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathsf{p}_3+(\frac{\partial \mathsf{b}_2}{\partial s}+\mathsf{b}_3\varepsilon_2$$

$$-\frac{\partial \mathsf{A}}{\partial s}\sin\mathsf{A})\sin\mathsf{A}]]]\,\mathbf{B}.$$

By a direct computation, we have:

$$\frac{\partial}{\partial t}\frac{\partial}{\partial s}\mathbf{N}^{\phi} = [\frac{\partial}{\partial t}(\varepsilon_1\tau^{\phi})(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\varepsilon_1\kappa^{\phi})[\mathsf{p}_5+(\frac{\partial \mathsf{b}_3}{\partial s}-\mathsf{b}_1\varepsilon_1-\mathsf{b}_2\varepsilon_2)\sin\mathsf{A}]$$

$$+(\varepsilon_1\tau^{\phi})[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\frac{1}{\kappa^{\phi}})[\mathsf{p}_3+(\frac{\partial \mathsf{b}_2}{\partial s}+\mathsf{b}_3\varepsilon_2-\frac{\partial \mathsf{A}}{\partial s}\sin\mathsf{A})\sin\mathsf{A}]]\mathbf{\Pi}_1$$

$$+[(\varepsilon_1\tau^{\phi})[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathsf{p}_1-\cos\mathsf{A}(\frac{\partial \mathsf{b}_1}{\partial s}+\mathsf{b}_3\varepsilon_1+\frac{\partial \mathsf{A}}{\partial s}\cos\mathsf{A})]-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})]$$

$$-(\varepsilon_1\kappa^{\phi})[\mathsf{p}_6-(\frac{\partial \mathsf{b}_3}{\partial s}-\mathsf{b}_1\varepsilon_1-\mathsf{b}_2\varepsilon_2)\cos\mathsf{A}]-\frac{\partial}{\partial t}(\varepsilon_1\tau^{\phi})(\frac{1}{\kappa^{\phi}})]]\mathbf{\Pi}_2$$

$$+[(\varepsilon_1\tau^{\phi})[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathsf{p}_2-(\frac{1}{\kappa^{\phi}})\mathsf{p}_4]-\frac{\partial}{\partial t}(\varepsilon_1\kappa^{\phi})]\mathbf{B}$$

Combining above equations, we have theorem. Hence the proof is completed.
In the light of Theorem 3.8, we express the following corollary without proof:

Corollary 3.9.

$$-\frac{\partial}{\partial s}\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-\frac{\partial}{\partial s}[(\frac{1}{\kappa^{\phi}})[\mathrm{p}_1-\cos \mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos \mathrm{A})]]-[(\frac{1}{\kappa^{\phi}})\mathrm{p}_2]\varepsilon_2$$

$$-[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4]\varepsilon_2]=[(\varepsilon_1\tau^{\phi})[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_1-\cos \mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos \mathrm{A})]$$

$$-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})]-(\varepsilon_1\kappa^{\phi})[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos \mathrm{A}]-\frac{\partial}{\partial t}(\varepsilon_1\tau^{\phi})(\frac{1}{\kappa^{\phi}})],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Corollary 3.10.

$$+[\varepsilon_1\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})+\varepsilon_2\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-\frac{\partial}{\partial s}[(\frac{1}{\kappa^{\phi}})\mathrm{p}_2]+\varepsilon_2[(\frac{1}{\kappa^{\phi}})[\mathrm{p}_1-\cos \mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}$$

$$+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos \mathrm{A})]]-\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4]+\varepsilon_1[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2$$

$$+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos \mathrm{A})]]-\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4]+\varepsilon_1[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Theorem 3.11. *Let* $\frac{\partial \alpha}{\partial t}$ *be inextensible according to new type-2 Bishop frame. If* ϕ *is spherical image of* α, *then:*

$$\frac{\partial}{\partial s}[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin \mathrm{A})\sin \mathrm{A}]]$$

$$+\varepsilon_1[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_2-(\frac{1}{\kappa^{\phi}})\mathrm{p}_4]]=[\frac{\partial}{\partial t}(\varepsilon_1\tau^{\phi})(\frac{1}{\kappa^{\phi}})+(\varepsilon_1\tau^{\phi})[\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})$$

$$+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin \mathrm{A})\sin \mathrm{A}]]],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

*Proof.*Using Theorem 3.4, we have

$$\frac{\partial}{\partial s}\frac{\partial \mathbf{B}^{\phi}}{\partial t} = [\frac{\partial}{\partial s}[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2$$

$$-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]]+\varepsilon_1[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_2-(\frac{1}{\kappa^{\phi}})\mathrm{p}_4]]\mathbf{\Pi}_1$$

$$+[\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]$$

$$-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})]+\varepsilon_2[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_2-(\frac{1}{\kappa^{\phi}})\mathrm{p}_4]]\mathbf{\Pi}_2$$

$$+[\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_2-(\frac{1}{\kappa^{\phi}})\mathrm{p}_4]-\varepsilon_1[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})$$

$$-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]]$$

$$-\varepsilon_2[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]$$

$$-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})]]\mathbf{B}$$

or, equivalently:

$$\frac{\partial}{\partial t}\frac{\partial}{\partial s}\mathbf{B}^{\phi} = [\frac{\partial}{\partial t}(\varepsilon_1\tau^{\phi})(\frac{1}{\kappa^{\phi}})+(\varepsilon_1\tau^{\phi})[\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})$$

$$\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]]]\mathbf{\Pi}_1$$

$$+[\frac{\partial}{\partial t}(\varepsilon_1\tau^{\phi})(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})+(\varepsilon_1\tau^{\phi})[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})$$

$$+(\frac{1}{\kappa^{\phi}})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]]]\mathbf{\Pi}_2$$

$$+(\varepsilon_1\tau^{\phi})[(\frac{1}{\kappa^{\phi}})\mathrm{p}_2+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4]\mathbf{B}.$$

Thus, we obtain the theorem.

4. RESULTS AND DISCUSSION

In the light of Theorem 3.11, we express the following corollary without proof:

Corollary 4.1.

$$\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})]$$

$$+\varepsilon_2[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_2-(\frac{1}{\kappa^{\phi}})\mathrm{p}_4]]=[\frac{\partial}{\partial t}(\varepsilon_1\tau^{\phi})(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})+(\varepsilon_1\tau^{\phi})$$

$$\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})+(\frac{1}{\kappa^{\phi}})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]]],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Corollary 4.2.

$$[\frac{\partial}{\partial s}[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_2-(\frac{1}{\kappa^{\phi}})\mathrm{p}_4]-\varepsilon_1[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})-(\frac{1}{\kappa^{\phi}})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}$$

$$+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]]-\varepsilon_2[(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1$$

$$+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\phi}})]]=(\varepsilon_1\tau^{\phi})[(\frac{1}{\kappa^{\phi}})\mathrm{p}_2+(\frac{\varepsilon_2}{\kappa^{\phi}\varepsilon_1})\mathrm{p}_4],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

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