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# AN IMPROVED CLASS OF REGRESSION ESTIMATORS USING THE AUXILIARY INFORMATION

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***Abstract.*** *Using the auxiliary information, the paper introduces an enhanced class of regression type estimators. Seven improved regression type estimators are provided, and their biases and MSEs are estimated up to the first degree of approximates. Also presented the conditions under which the proposed estimators lead to superior results compared with other existing estimators. The empirical study in support of the findings is performed to test the efficiency of the proposed estimators.*

***Keywords:*** *regression estimator; PRE; bias; MSE; auxiliary information.*

## 1. INTRODUCTION

In practice, it is difficult to collect the complete information about an object under study and hence predictions and decision making studies are conducted on the basis of sample. Sampling is an art to measure the reliability of available information by making the use of probability theory. In sampling, simple random sampling (SRS) is the most common and easiest method to select sample with equal probability at each selection without concentration of the auxiliary information. In real circumstances with the variable of interest (Y), we collect some additional information (X) which is positively or negatively correlated to the variable of interest. If we incorporate the additional information in classical estimators, this would leads to the flexible results. Many researchers are now working to utilize additional information in away to improved the flexibility of the existing estimators. For example, Kadilar and Cingi [1] worked on the regression type estimators, Yan and Tian [2], Subramani and Kumarapandiyan [3-5], and Jeelani et al. [6].

The notations will be circulated throughout the paper as described below

$N$: population size,

$n$: sample size,

$Y$: study variable,

$X$: auxiliary variable

$\bar{Y}$, $\bar{X}$: population means of study and auxiliary variable,

$\bar{y}$, $\bar{x}$: sample means of study and auxiliary variable,

$S_{yx}$: population covariance between study $(Y)$ and auxiliary $(X)$ variable,

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$S_y, S_x$ : population standard deviation of the study variable $(Y)$ and auxiliary variable $(X)$,

$C_y, C_x$ : coefficient of variation of the study variable $(Y)$ and auxiliary variable $(X)$,

$M_d$ : median of the auxiliary variable $(X)$,

$\rho_{yx}$ : population correlation coefficient between the study $(Y)$ and auxiliary variable $(X)$,

$\beta_{1(x)}, \beta_{2(x)}$ : coefficient of skewness and kurtosis of an auxiliary variable $(X)$,

$$\beta_{1(x)} = \frac{N\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^3}{(N-1)(N-2)S_x^3},$$

$$\beta_{2(x)} = \frac{N(N+1)\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^4}{(N-1)(N-2)(N-3)S_x^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$$

$QD = \frac{Q_3 - Q_1}{2}$ : quartile deviation,

$b_{yx} = \frac{s_{yx}}{s_x^2}$ : regression coefficient of $y$ on $x$.

The usual unbiased estimator $\bar{y}$ incase of simple random sampling can be used to estimate the population mean $\bar{Y}$ with a following variance

$$Var(t_0) = \frac{1-f}{n}\bar{Y}^2 C_y^2 \tag{1}$$

By making the use of the auxiliary information Cochran [7] defined the ratio estimator $t_r$ so that to estimate the population mean $\bar{Y}$ for the study variable $Y$ and is given below

$$t_r = \bar{y}\frac{\bar{X}}{\bar{x}}$$

The bias and Mean square error (MSE) of the above estimator is given by

$$B(t_r) = \frac{1-f}{n}\bar{Y}\left(C_x^2 - \rho_{yx}C_yC_x\right)$$

$$MSE(t_r) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right] \tag{2}$$

Another application of the auxiliary information was presented by Kadilar and Cingi [8] and they discussed the following regression type estimators

$$t_1 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\bar{x}} \bar{X}$$
$$t_2 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x} + C_x\right)}\left(\bar{X} + C_x\right)$$
$$t_3 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x} + \beta_{2(x)}\right)}\left(\bar{X} + \beta_{2(x)}\right)$$
$$t_4 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}\beta_{2(x)} + C_x\right)}\left(\bar{X}\beta_{2(x)} + C_x\right)$$
$$t_5 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}C_x + \beta_{2(x)}\right)}\left(\bar{X}C_x + \beta_{2(x)}\right)$$

The related bias and MSEs of $t_j$ is as under

$$Bias\left(t_j\right) = \frac{1-f}{n}\bar{Y}R_j^2 C_x^2 , \quad j = 1,\ 2, ..., 5$$

$$MSE\left(t_j\right) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2\left(1-\rho_{yx}^2\right) + R_j^2 C_x^2\right] \tag{3}$$

where $R_1 = \frac{\bar{X}}{\bar{X}}$, $R_2 = \frac{\bar{X}}{\bar{X} + C_x}$, $R_3 = \frac{\bar{X}}{\bar{X} + \beta_{2(x)}}$, $R_4 = \frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)} + C_x}$, $R_5 = \frac{\bar{X}C_x}{\bar{X}C_x + \beta_{2(x)}}$.

To estimate the populations mean $\bar{Y}$ more efficiently, several regression type estimators suggested by Kadilar and Cingi [1] and is defined as follows

$$t_6 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x} + \rho\right)}\left(\bar{X} + \rho\right)$$

$$t_7 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}C_x + \rho\right)}\left(\bar{X}C_x + \rho\right)$$

$$t_8 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}\rho + C_x\right)}\left(\bar{X}\rho + C_x\right)$$

$$t_9 = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}\beta_2 + \rho\right)}\left(\bar{X}\beta_2 + \rho\right)$$

$$t_{10} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}\rho + \beta_2\right)}\left(\bar{X}\rho + \beta_2\right)$$

The bias and MSEs of $t_j$ is described below

$$Bias\left(t_j\right) = \frac{1-f}{n}\bar{Y}R_j^2C_x^2\ , \quad j = 6,\ 7,\ 8, 9, 10$$

$$MSE\left(t_j\right) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2\left(1-\rho^2\right) + R_j^2C_x^2\right] \tag{4}$$

where:

$$R_6 = \frac{\bar{X}}{\bar{X} + \rho_{yx}},$$

$$R_7 = \frac{\bar{X}C_x}{\bar{X}C_x + \rho_{yx}},$$

$$R_8 = \frac{\bar{X}\rho_{yx}}{\bar{X}\rho_{yx} + C_x},$$

$$R_9 = \frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)} + \rho_{yx}},$$

$$R_{10} = \frac{\bar{X}\rho_{yx}}{\bar{X}\rho_{yx} + \beta_{2(x)}}.$$

Yan and Tian [2] introduced two more efficient ratio and regression type estimators of the population mean $\bar{Y}$ by using the auxiliary information which are as under

$$t_{11} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x} + \beta_1\right)}\left(\bar{X} + \beta_1\right)$$

$$t_{12} = \frac{\bar{y} + b\left(\bar{X} - \bar{x}\right)}{\left(\bar{x}\beta_1 + \beta_2\right)}\left(\bar{X}\beta_1 + \beta_2\right)$$

The bias and MSE of $t_j$ is described below

$$Bias\left(t_j\right) = \frac{1-f}{n}\bar{Y}R_j^2C_x^2\ , \quad j = 11,\ 12$$

$$MSE\left(t_{11}\right)=\frac{1-f}{n}\bar{Y}^{2}\left[C_{y}^{2}\left(1-\rho^{2}\right)+R_{11}^{2}C_{x}^{2}\right] \quad (5)$$

where, $R_{11}=\dfrac{\bar{X}}{\bar{X}+\beta_{1(x)}}$, $R_{12}=\dfrac{\bar{X}\beta_{1(x)}}{\bar{X}\beta_{1(x)}+\beta_{2(x)}}$.

Subramani and Kumarapandiyan [3,4,5] recommended the following regression type estimators for estimating the population mean $\bar{Y}$ is defined by

$$t_{13}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}+M_{d}\right)}\left(\bar{X}+M_{d}\right)$$

$$t_{14}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}C_{x}+M_{d}\right)}\left(\bar{X}C_{x}+M_{d}\right)$$

$$t_{15}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}\beta_{1}+M_{d}\right)}\left(\bar{X}\beta_{1}+M_{d}\right)$$

$$t_{16}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}\beta_{2}+M_{d}\right)}\left(\bar{X}\beta_{2}+M_{d}\right)$$

The bias and MSE of $t_j$ is given as

$$Bias\left(t_{j}\right)=\frac{1-f}{n}\bar{Y}R_{j}^{2}C_{x}^{2}, \quad j=13,\ 14,\ 15,\ 16$$

$$MSE\left(t_{j}\right)=\frac{1-f}{n}\bar{Y}^{2}\left[C_{y}^{2}\left(1-\rho^{2}\right)+R_{j}^{2}C_{x}^{2}\right] \quad (6)$$

where, $R_{13}=\dfrac{\bar{X}}{\bar{X}+M_{d}}$, $R_{14}=\dfrac{\bar{X}C_{x}}{\bar{X}C_{x}+M_{d}}$, $R_{15}=\dfrac{\bar{X}\beta_{1(x)}}{\bar{X}\beta_{1(x)}+M_{d}}$, $R_{16}=\dfrac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)}+M_{d}}$.

Jeelani et al. [6] recommended another contribution to the estimators of population mean using auxiliary variable which is defined by

$$t_{17}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\bar{x}\beta_{1}+QD\right)}\left(\bar{X}\beta_{1}+QD\right)$$

The bias and MSE of $t_{17}$ as follows

$$Bias\left(t_{17}\right)=\frac{1-f}{n}\bar{Y}R_{17}^{2}C_{x}^{2}$$

$$MSE\left(t_{17}\right)=\frac{1-f}{n}\bar{Y}^{2}\left[C_{y}^{2}\left(1-\rho^{2}\right)+R_{17}^{2}C_{x}^{2}\right] \tag{7}$$

where, $R_{17}=\dfrac{\bar{X}\beta_{1(x)}}{\bar{X}\beta_{1(x)}+QD}$ .

The motive of this paper is to introduce regression type estimators so that to increase the efficiency of estimation of population characteristic on the basis of information obtained from samples. In this paper, we suggest seven new regression type estimators by using auxiliary information which improved the results as compared to the existing estimators.

## 2. ESTIMATORS AND COMPARISIONS

### *2.1. SUGGESTED ESTIMATORS*

This section presents regression type estimators for estimating the populations mean $\bar{Y}$ using the auxiliary information in the forms of Median, Population standard deviation, Coefficient of variation and quartile deviation. The estimators, Bias and their MSEs are described below

$$t_{pr1}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left\{\delta\bar{x}+\left(Q.D\right)\left(C_{x}\right)\right\}}\left\{\delta\bar{X}+\left(Q.D\right)\left(C_{x}\right)\right\}$$

$$t_{pr2}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\delta\bar{x}+\bar{X}C_{x}\right)}\left(\delta\bar{X}+\bar{X}C_{x}\right)$$

$$t_{pr3}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\delta\bar{x}+\bar{X}\right)}\left(\delta\bar{X}+\bar{X}\right)$$

$$t_{pr4}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left\{\delta\bar{x}+\left(Q.D\right)\left(M_{d}\right)\right\}}\left\{\delta\bar{X}+\left(Q.D\right)\left(M_{d}\right)\right\}$$

$$t_{pr5}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\delta\bar{x}+M_{d}\right)}\left(\delta\bar{X}+M_{d}\right)$$

$$t_{pr6}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left(\delta\bar{x}+Q.D\right)}\left(\delta\bar{X}+Q.D\right)$$

$$t_{pr7}=\frac{\bar{y}+b\left(\bar{X}-\bar{x}\right)}{\left\{\delta\bar{x}+\left(M_{d}\right)\left(C_{x}\right)\right\}}\left\{\delta\bar{X}+\left(M_{d}\right)\left(C_{x}\right)\right\}$$

where $\delta = \dfrac{Q.D}{S_x}$.

The bias and MSEs of the generalized estimator $t_{pri}$ is given as

$$Bias\left(t_{pri}\right) = \frac{1-f}{n}\bar{Y}\theta_{pri}^2 C_x^2 \qquad i = 1,\ 2, \dots, 7$$

$$MSE\left(t_{pri}\right) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2\left(1-\rho_{yx}^2\right) + \theta_{pri}^2 C_x^2\right] \tag{8}$$

where, $\theta_{pr1} = \dfrac{\delta\bar{X}}{\left\{\delta\bar{X} + (Q.D)(C_x)\right\}}$, $\theta_{pr2} = \dfrac{\delta\bar{X}}{\left(\delta\bar{X} + \bar{X}C_x\right)}$, $\theta_{pr3} = \dfrac{\delta\bar{X}}{\left(\delta\bar{X} + \bar{X}\right)}$,
$\theta_{pr4} = \dfrac{\delta\bar{X}}{\left\{\delta\bar{X} + (Q.D)(M_d)\right\}}$, $\theta_{pr5} = \dfrac{\delta\bar{X}}{\left(\delta\bar{X} + M_d\right)}$, $\theta_{pr6} = \dfrac{\delta\bar{X}}{\left(\delta\bar{X} + Q.D\right)}$, $\theta_{pr7} = \dfrac{\delta\bar{X}}{\left\{\delta\bar{X} + (M_d)(C_x)\right\}}$.

### *2.2. EFFICIENCY COMPARISONS*

In this section, theoretical conditions are derived so that to assess the performance of the proposed estimators as compared to the existing estimators as follows

The MSE of the proposed regression estimator given in Eq (8) with the usual mean estimator given in Eq (1) can be compared in the following way

$$MSE\left(t_{pri}\right) < Var\left(t_0\right)$$

$$\frac{1-f}{n}\bar{Y}^2\left[C_y^2\left(1-\rho_{yx}^2\right) + \theta_{pri}^2 C_x^2\right] < \frac{1-f}{n}\bar{Y}^2 C_y^2$$

$$C_y^2 - C_y^2\rho_{yx}^2 + \theta_{pri}^2 C_x^2 < C_y^2$$

$$\theta_{pri}^2 < \frac{C_y^2\rho_{yx}^2}{C_x^2},\ i = 1, 2, \dots, 7 \tag{9}$$

The MSE of the proposed regression estimator given in Eq(8) with the estimators given in Eq(2) can be compared as

$$MSE\left(t_{pri}\right) < MSE\left(t_r\right)$$

$$\frac{1-f}{n}\bar{Y}^2\left[C_y^2\left(1-\rho_{yx}^2\right) + \theta_{pri}^2 C_x^2\right] < \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_y C_x\right]$$

$$C_y^2 - C_y^2\rho_{yx}^2 + \theta_{pri}^2 C_x^2 < C_y^2 + C_x^2 - 2\rho_{yx}C_y C_x$$

$$\theta_{pri}^2 < \frac{\left(\rho_{yx}C_y - 2C_x\right)\rho_{yx}C_y + C_x^2}{C_x^2}, \; i = 1,2,\ldots,7 \tag{10}$$

The comparison of the MSE of the proposed regression estimator given in Eq (8) with other estimators given in Eq (3-7) can be defined as

$$MSE\left(t_{pri}\right) < MSE\left(t_j\right)$$

$$\frac{1-f}{n}\bar{Y}^2\left[C_y^2\left(1-\rho_{yx}^2\right) + \theta_{pri}^2 C_x^2\right] < \frac{1-f}{n}\bar{Y}^2\left[C_y^2\left(1-\rho_{yx}^2\right) + R_j^2 C_x^2\right]$$

$$C_y^2\left(1-\rho_{yx}^2\right) + \theta_{pri}^2 C_x^2 < C_y^2\left(1-\rho_{yx}^2\right) + R_j^2 C_x^2$$

$$\theta_{pri}^2 < R_j^2, \; i = 1,2,\ldots,7 \text{ and } j = 1,2,\ldots 17 \tag{11}$$

## 3. RESULTS AND DISCUSSION

To assess the performance of the proposed estimators with those existing estimators, we considered a real data set which describes the production of apples and the number of trees at various villages of Aegean region in turkey 1999. The data set is taken from Kadilar and Cingi [8] with the statistics given below in Table 1.

**Table 1. Data set**

| | | | | |
|---|---|---|---|---|
| $N = 106$ | $n = 20$ | $\bar{Y} = 2212.59$ | $\bar{X} = 27421.7$ | $\rho_{yx} = 0.86$ |
| $C_y = 5.22$ | $C_x = 2.10$ | $S_x = 57460.61$ | $S_y = 11551.53$ | $S_{xy} = 568176176.1$ |
| $\beta_{1(x)} = 2.12$ | $\beta_{2(x)} = 34.5$ | $QD = 12156.25$ | $M_d = 7297.5$ | $\delta = 0.211558$ |

Table 2 reflects the numerical illustration of the efficiency comparison conditions of the proposed estimators with that of existing estimators. If these conditions hold then we can easily claim that the proposed estimators have a less Mean Square Error (MSE) and a bigger value for Percentage Relative Efficiency (PRE).

In Table 2, the numerical results are evaluated in support of the theoretical conditions of the proposed estimators with other existing ratio and regression estimators. The numerical results of the existing estimators are compared with the results of seven new estimators presented in row 1. The values given in row 1 of Table 2 clearly show smaller values as compared to other existing estimator values.

**Table 2. Efficiency Comparison results of the existing ratio and regression estimators with proposed estimator**

| Existing Estimators | Proposed Estimators | | | | | | |
|---|---|---|---|---|---|---|---|
| | $t_{p1}$ | $t_{p2}$ | $t_{p3}$ | $t_{p4}$ | $t_{p5}$ | $t_{p6}$ | $t_{p7}$ |
| $\theta^2_{pri}$ | **0.0000000042760** | **0.00003650837** | **0.008376242** | **0.03049091** | **0.03428809** | **0.07540702** | **0.19614890** |
| $t_0$ | 4.5698 | 4.5698 | 4.5698 | 4.5698 | 4.5698 | 4.5698 | 4.5698 |
| $t_r$ | 1.2943 | 1.2943 | 1.2943 | 1.2943 | 1.2943 | 1.2943 | 1.2943 |
| $t_1$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $t_2$ | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| $t_3$ | 0.9974 | 0.9974 | 0.9974 | 0.9974 | 0.9974 | 0.9974 | 0.9974 |
| $t_4$ | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| $t_5$ | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 |
| $t_6$ | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| $t_7$ | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| $t_8$ | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| $t_9$ | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| $t_{10}$ | 0.9970 | 0.9970 | 0.9970 | 0.9970 | 0.9970 | 0.9970 | 0.9970 |
| $t_{11}$ | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| $t_{12}$ | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9988 |
| $t_{13}$ | 0.6238 | 0.6238 | 0.6238 | 0.6238 | 0.6238 | 0.6238 | 0.6238 |
| $t_{14}$ | 0.7877 | 0.7877 | 0.7877 | 0.7877 | 0.7877 | 0.7877 | 0.7877 |
| $t_{15}$ | 0.7895 | 0.7895 | 0.7895 | 0.7895 | 0.7895 | 0.7895 | 0.7895 |
| $t_{16}$ | 0.9847 | 0.9847 | 0.9847 | 0.9847 | 0.9847 | 0.9847 | 0.9847 |
| $t_{17}$ | 0.6842 | 0.6842 | 0.6842 | 0.6842 | 0.6842 | 0.6842 | 0.6842 |

Table 3 presents the biass, MSEs and PREs of the proposed and other existing estimators. We see that the PRE is bigger for the proposed as compared to the existing estimators while MSE of the proposed estimators are less than that of the existing estimators. For example, the MSE of the proposed estimators $t_{pr1}$, $t_{pr2}$, $t_{pr3}$, $t_{pr4}$, $t_{pr5}$, $t_{pr6}$, and $t_{pr7}$ are less than the MSE of the classical estimator $t_0$. This is the expected results because the condition given in Eq(9) is satisfied as follows

$$\theta^2_{pri} < \frac{C_y^2 \rho_{yx}^2}{C_x^2}$$

0.0000000042760<4.5698,

0.00003650837<4.5698,

0.008376242< 4.5698,

$$0.03049091 < 4.5698,$$

$$0.03428809 < 4.5698,$$

$$0.07540702 < 4.5698,$$

$$0.19614890 < 4.5698.$$

**Table 3. The comparison of Bias, MSE's and PRE's of the proposed estimators with other estimators**

| Estimators | Constants | Bias | MSE | PRE |
|---|---|---|---|---|
| $t_0$ | NA | 0 | 5411348 | 100 |
| $t_r$ | NA | -450.3346 | 2542740 | 212.8156 |
| $t_1$ | NA | 395.8240 | 2284911 | 236.8297 |
| $t_2$ | 0.9999234 | 395.7634 | 2284777 | 236.8436 |
| $t_3$ | 0.9987409 | 394.8279 | 2282707 | 237.0584 |
| $t_4$ | 0.9999978 | 395.8222 | 2284907 | 236.8301 |
| $t_5$ | 0.9994000 | 395.3492 | 2283861 | 236.9386 |
| $t_6$ | 0.9999686 | 395.7992 | 2284856 | 236.8354 |
| $t_7$ | 0.9999851 | 395.8122 | 2284885 | 236.8324 |
| $t_8$ | 0.9999110 | 395.7535 | 2284755 | 236.8459 |
| $t_9$ | 0.9999991 | 395.8233 | 2284910 | 236.8298 |
| $\mathrm{t}_{10}$ | 0.9985362 | 394.6661 | 2282349 | 237.0955 |
| $t_{11}$ | 0.9999226 | 395.7627 | 2284776 | 236.8437 |
| $t_{12}$ | 0.9994063 | 395.3541 | 2283872 | 236.9375 |
| $t_{13}$ | 0.7898137 | 246.9173 | 1955442 | 276.7328 |
| $t_{14}$ | 0.8875284 | 311.7932 | 2098986 | 257.8078 |
| $t_{15}$ | 0.8885646 | 312.5216 | 2100597 | 257.6100 |
| $t_{16}$ | 0.9923608 | 389.7995 | 2271582 | 238.2194 |
| $t_{17}$ | 0.8271912 | 270.8407 | 2008375 | 269.4392 |
| $t_{pr1}$ | **0.0000653** | **0.000001692** | **1409115** | **384.0246** |
| $t_{pr2}$ | **0.0060422** | **0.01445089** | **1409147** | **384.0159** |
| $t_{pr3}$ | **0.0915219** | **3.31551800** | **1416451** | **382.0357** |
| $t_{pr4}$ | **0.1746165** | **12.0690300** | **1435819** | **376.8824** |
| $t_{pr5}$ | **0.1851704** | **13.5720500** | **1439144** | **376.0115** |
| $t_{pr6}$ | **0.2746034** | **29.8479100** | **1475156** | **366.8322** |
| $t_{pr7}$ | **0.4428870** | **77.6404400** | **1580902** | **342.2951** |

Similarly, the MSE of the proposed estimators $t_{pr1}$, $t_{pr2}$, $t_{pr3}$, $t_{pr4}$, $t_{pr5}$, $t_{pr6}$, and $t_{pr7}$ are fever than the MSE of the estimator $t_r$. This results is also expected as the condition given in Eq(10) holds

$$\theta_{pri}^2 < \frac{\left(\rho_{yx}C_y - 2C_x\right)\rho_{yx}C_y + C_x^2}{C_x^2}$$

If we look at the results column wise for the proposed and row wise for the existing estimators started from $t_r$ in Table 2. It is declared the above conditions hold for all the proposed estimators. Again the MSE of the proposed class of estimators is less than that the MSE of the estimators from $t_1$ to $t_{17}$. This relationship of MSEs between the proposed class of estimators and existing is due to condition (10) as following

$$\theta_{pri}^2 < R_j^2$$

The values given in Table 2 clarify the above condition, for illustration see the results row wise for existing estimators from $t_1$ to $t_{17}$ while the results for the proposed estimators can be found column wise. Base on the above discussion, it is evident that the proposed class of regression estimators leads us to more efficient and better results than the existing estimators.

## 4. CONCLUSIONS

This paper examined seven new improved regression type estimators by using the median, population standard deviation, coefficient of variation and quartile deviation as auxiliary information. To assess the model performfance, we derived theoretical conditions and use an applied data in support of the results.

Moreover, the results for the bias, mean square error, and percentage relative efficiency (PRF) are evaluated for the proposed as well as other estimators. The PRF of the proposed estimators is found bigger while the MSE is smaller than others.

Thus, it is evident that the suggested estimators in this paper lead to a better result. Furthermore, among the proposed estimators $t_{pr1}$ provides a better result than $t_{pr2}$, $t_{pr3}$, $t_{pr4}$, $t_{pr5}$, $t_{pr6}$, and $t_{pr7}$.

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