*ORIGINAL PAPER*

# ENERGY OF THE FERMI-WALKER DERIVATIVES OF MAGNETIC CURVES ACCORDING TO THE BISHOP FRAME IN THE SPACE

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***Abstract.*** *Fermi-Walker derivative and the energy of magnetic curves have an important place in physics and differential geometry. In this study, we calculate the Fermi-Walker derivatives of* $\mathbf{T}$, $\mathbf{N}_1$, $\mathbf{N}_2$ *magnetic curves according to the Bishop frame in the space. Moreover, we obtain the energy of the Fermi-Walker derivative of magnetic curves according to the Bishop frame in space. Finally, we have energy relations of some vector fields associated with Bishop frame in the space.*

***Keywords:*** *magnetic curve; Lorentz force; Bishop frame; Fermi-Walker derivative; energy.*

## 1. INTRODUCTION

The closed 2-form $\mathbf{F}$ on Riemannian manifold $(M,g)$ is called a magnetic field. Magnetic curves on a Riemannian manifold $(M,g)$ is a trajectory characterized by a charged particle moving in under the influence of a magnetic field $\mathbf{F}$. If these charged particles enter the magnetic field they are exposed to a force called a Lorentz force. The Lorentz force is an (1, 1)-type tensor field $\Phi$ on Riemannian manifold $(M,g)$ and it satisfies that $g(\Phi(X),Y)=\mathbf{F}(X,Y), \quad \forall X,Y\in\chi(M)$. Lorentz force equation is expressed by $\Phi(X)=\mathbf{V}\times X$. Morever the magnetic trajectories of the magnetic field $\mathbf{F}$ is given by

$$\nabla_{\mathbf{T}}\mathbf{T}=\Phi(\mathbf{T})=\mathbf{V}\times\mathbf{T}.$$

Generalized Lorentz equation obtained from the geodesics of $M$ is given by $\nabla_{\mathbf{T}}\mathbf{T}=0$ [1-3].

A charged particle moves along a curve in the magnetic vector field then it is exposed to the magnetic field. The researchers have examined the trajectories of charged particles moving in an area modeled by the homogeneous space $S^2\times\mathbb{R}$ [4]. The notions of $\mathbf{T}$-magnetic, $\mathbf{N}_1$-magnetic and $\mathbf{N}_2$-magnetic curves and some characterizations for them in the semi-Riemannian manifolds have been determined by some researchers [5-9].

The local theory of the curves has been investigated by some researchers by considering Serret-Frenet laws. Bishop frame, which is also called an alternative or a parallel frame of the curves by means of parallel vector fields. The Serret-Frenet and Bishop frames have one thing in common i.e. their tangent vector field. Recently, many studies have been done on the Bishop frames in the Euclidean space [10-12].

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On the other hand, several methods in the research of magnetic curves for a given magnetic field on the constant energy level have been invesitgated by Muntenau in [1]. Also, many researchers have identified energy related studies using different methods [13-21].

In this study, we calculate the Fermi-Walker derivatives of $\mathbf{T}$, $\mathbf{N}_1$, $\mathbf{N}_2$ magnetic curves according to the Bishop frame in the space. Moreover, we obtain the energy of the Fermi-Walker derivative of magnetic curves according to the Bishop frame in space. Finally, we have energy relations of some vector fields associated with Bishop frame in the space.

## 2. MATERIALS AND METHODS

At this stage, some basic concepts about curves in space are given.

The Euclidean 3-space supplied with the standard straight metric given by

$$\langle , \rangle = dx_1^2 + dx_2^2 + dx_3^2.$$

Here $(x_1, x_2, x_3)$ is a coordinate system of the Euclidean 3-space.

Considering that $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ is the Serret-Frenet frame of $\alpha$ that the following Frenet-Serret equations can be given.

$$\begin{pmatrix} \mathbf{T}' \\ \mathbf{N}' \\ \mathbf{B}' \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}, \tag{2.1}$$

where $\kappa$ and $\tau$ are the curvature function and torsion of $\alpha$, respectively and

$$\begin{aligned} \langle \mathbf{T}, \mathbf{T} \rangle &= \langle \mathbf{N}, \mathbf{N} \rangle = \langle \mathbf{B}, \mathbf{B} \rangle = 1, \\ \langle \mathbf{T}, \mathbf{N} \rangle &= \langle \mathbf{T}, \mathbf{B} \rangle = \langle \mathbf{N}, \mathbf{B} \rangle = 0. \end{aligned}$$

The Bishop frame, which is referred to as the alternative or parallel frame of the curves depending to parallel vector fields, was introduced by L.R. Bishop in 1975. It is a well-defined alternative approach even in the absence of the second derivative of the curve [22]. The Bishop frame is explained as

$$\begin{pmatrix} \mathbf{T}' \\ \mathbf{N}_1' \\ \mathbf{N}_2' \end{pmatrix} = \begin{pmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & 0 \\ -k_2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N}_1 \\ \mathbf{N}_2 \end{pmatrix} \tag{2.2}$$

from here

$$\begin{aligned} \langle \mathbf{T}, \mathbf{T} \rangle &= \langle \mathbf{N}_1, \mathbf{N}_1 \rangle = \langle \mathbf{N}_2, \mathbf{N}_2 \rangle = 1, \\ \langle \mathbf{T}, \mathbf{N}_1 \rangle &= \langle \mathbf{T}, \mathbf{N}_2 \rangle = \langle \mathbf{N}_1, \mathbf{N}_2 \rangle = 0. \end{aligned}$$

Here, $\{\mathbf{T}, \mathbf{N}_1, \mathbf{N}_2\}$ is called Bishop trihedra and $k_1$ and $k_2$ are called Bishop curvatures of the curve and the connection between Frenet and the Bishop frame is expressed as follows

$$\begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta(s) & \sin\theta(s) \\ 0 & -\sin\theta(s) & \cos\theta(s) \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N}_1 \\ \mathbf{N}_2 \end{pmatrix},$$

where $\theta(s) = \arctan\frac{k_2}{k_1}$, $\tau(s) = \theta^{'}(s)$ and $\kappa(s) = \sqrt{(k_1)^2 + (k_2)^2}$. Bishop curvatures are defined by $k_1 = \kappa\cos\theta(s)$ and $k_2 = \kappa\sin\theta(s)$ [12].

## 3. FERMI-WALKER DERIVATIVE

In this section, we give the definitions of $\mathbf{T}$, $\mathbf{N}_1$, $\mathbf{N}_2$ magnetic curves and Fermi-Walker derivative [23-25].

**Definition 3.1.** *Let $\alpha : I \subset \mathbb{R} \to \mathbb{R}^3$ be a curve with Bishop frame in Euclidean 3-space and $\mathbf{F_V}$ be a magnetic field in $\mathbb{R}^3$. If the tangent vector field $\mathbf{T}$ of the Bishop frame satisfies the Lorentz force equation $\nabla_{\alpha'}\mathbf{T} = \Phi(\mathbf{T}) = \mathbf{V}\times\mathbf{T}$, then the curve $\alpha$ is called a $\mathbf{T}$-magnetic curve according to Bishop frame [23].*

**Proposition 3.2**. *Let $\alpha$ be a unit speed $\mathbf{T}$-magnetic curve according to Bishop frame in Euclidean 3-space. Then, the Lorentz force according to the Bishop frame is obtained as*

$$\begin{pmatrix} \Phi(\mathbf{T}) \\ \Phi(\mathbf{N}_1) \\ \Phi(\mathbf{N}_2) \end{pmatrix} = \begin{pmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & \rho \\ -k_2 & -\rho & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N}_1 \\ \mathbf{N}_2 \end{pmatrix}, \tag{3.1}$$

*where $\rho$ is a certain function defined by $\rho = g(\Phi\mathbf{N}_1, \mathbf{N}_2)$, [15].*

**Definition 3.3.** *Let $\alpha : I \subset \mathbb{R} \to \mathbb{R}^3$ be a curve with Bishop frame in Euclidean 3-space and $\mathbf{F_V}$ be a magnetic field in $\mathbb{R}^3$. If the vector field $\mathbf{N}_1$ of the Bishop frame satisfies the Lorentz force equation $\nabla_{\alpha'}\mathbf{N}_1 = \Phi(\mathbf{N}_1) = \mathbf{V}\times\mathbf{N}_1$, then the curve $\alpha$ is called a $\mathbf{N}_1$-magnetic curve according to Bishop frame [23].*

**Proposition 3.4**. *Let $\alpha$ be a unit speed $\mathbf{N}_1$-magnetic curve according to Bishop frame in Euclidean 3-space. Then, the Lorentz force according to the Bishop frame is obtained as*

$$\begin{pmatrix} \Phi(\mathbf{T}) \\ \Phi(\mathbf{N}_1) \\ \Phi(\mathbf{N}_2) \end{pmatrix} = \begin{pmatrix} 0 & k_1 & \eta \\ -k_1 & 0 & 0 \\ -\eta & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N}_1 \\ \mathbf{N}_2 \end{pmatrix}, \tag{3.2}$$

*where $\eta$ is a certain function defined by $\eta = g(\Phi\mathbf{T}, \mathbf{N}_2)$ [23].*

**Definition 3.5.** *Let* $\alpha : I \subset \mathbb{R} \to \mathbb{R}^3$ *be a curve with Bishop frame in Euclidean 3-space and* $\mathbf{F_V}$ *be a magnetic field in* $\mathbb{R}^3$. *If the vector field* $\mathbf{N}_2$ *of the Bishop frame satisfies the Lorentz force equation* $\nabla_{\alpha'} \mathbf{N}_2 = \Phi(\mathbf{N}_2) = \mathbf{V} \times \mathbf{N}_2$, *then the curve* $\alpha$ *is called a* $\mathbf{N}_2$*-magnetic curve according to Bishop frame [23].*

**Proposition 3.6**. *Let* $\alpha$ *be a unit speed* $\mathbf{N}_2$*-magnetic curve according to Bishop frame in Euclidean 3-space. Then, the Lorentz force according to the Bishop frame is obtained as*

$$\begin{pmatrix} \Phi(\mathbf{T}) \\ \Phi(\mathbf{N}_1) \\ \Phi(\mathbf{N}_2) \end{pmatrix} = \begin{pmatrix} 0 & \gamma & k_2 \\ -\gamma & 0 & 0 \\ -k_2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N}_1 \\ \mathbf{N}_2 \end{pmatrix}, \tag{3.3}$$

*where* $\gamma$ *is a certain function defined by* $\gamma = g(\Phi(\mathbf{T}), \mathbf{N}_1)$ *[23].*

**Definition 3.7.** *$X$ is any vector field and $\alpha(s)$ is unit-speed any curve in space.*

$$\frac{\tilde{\nabla} X}{\tilde{\nabla} s} = \frac{dX}{ds} - \langle \mathbf{T}, X \rangle A + \langle A, X \rangle \mathbf{T} \tag{3.4}$$

*defined as* $\frac{\tilde{\nabla} X}{\tilde{\nabla} s}$ *derivative is called Fermi-Walker derivative. Here* $\mathbf{T} = \frac{d\alpha}{ds}$, $A = \frac{d\mathbf{T}}{ds}$ *[26].*

**Definition 3.8.** *$X$ is any vector field along the $\alpha(s)$ space curve. If the Fermi-Walker derivative of the vector field $X$*

$$\frac{\tilde{\nabla} X}{\tilde{\nabla} s} = 0, \tag{3.5}$$

*the vector field $X$ along the curve, parallel to the Fermi-Walker terms, is called [26].*

**Theorem 3.9.***Let* $\alpha$ *be a* $\mathbf{T}$*-magnetic curve with Bishop frame. Then, Fermi Walker derivatives of Lorentz forces* $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ *are given by*

$$\begin{aligned} \tilde{\nabla}_{\mathbf{T}} \Phi(\mathbf{T}) &= k_1' \mathbf{N}_1 + k_2' \mathbf{N}_2, \\ \tilde{\nabla}_{\mathbf{T}} \Phi(\mathbf{N}_1) &= -k_1' \mathbf{T} + \rho' \mathbf{N}_2, \\ \tilde{\nabla}_{\mathbf{T}} \Phi(\mathbf{N}_2) &= -k_2' \mathbf{T} - \rho' \mathbf{N}_1, \end{aligned} \tag{3.6}$$

*where* $\rho = g(\Phi \mathbf{N}_1, \mathbf{N}_2)$.

**Corollary 3.10.** *Lorentz forces* $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ *are parellel according to Fermi Walker if*

$$\begin{aligned} k_1 &= \textit{constant}, \\ k_2 &= \textit{constant}, \\ \rho &= \textit{constant}. \end{aligned}$$

*Proof.* Assume that $\alpha$ is a $\mathbf{T}$-magnetic curve with Bishop frame. By using Fermi Walker derivatives, we have

$$\begin{aligned} \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T}) &= k_1^{'}\mathbf{N}_1 + k_2^{'}\mathbf{N}_2, \\ \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1) &= -k_1^{'}\mathbf{T} + \rho^{'}\mathbf{N}_2, \\ \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2) &= -k_2^{'}\mathbf{T} - \rho^{'}\mathbf{N}_1. \end{aligned}$$

Also, Lorentz forces $\Phi(\mathbf{T}), \Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ are parallel to the Fermi--Walker terms, then

$$k_1^{'} = 0, k_2^{'} = 0 and \rho^{'} = 0.$$

Therefore, $k_1 = constant$, $k_2 = constant$ and $\rho = constant$ is obtained.

**Theorem 3.11.** *Let $\alpha$ be a $\mathbf{N}_1$-magnetic curve with Bishop frame. Then, Fermi Walker derivatives of Lorentz forces $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ are given by*

$$\begin{aligned} \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T}) &= -\eta k_2\mathbf{T} + k_1^{'}\mathbf{N}_1 + \eta^{'}\mathbf{N}_2, \\ \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1) &= -k_1^{'}\mathbf{T} - k_1 k_2\mathbf{N}_2, \\ \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2) &= -\eta^{'}\mathbf{T} - \eta k_2\mathbf{N}_2, \end{aligned} \tag{3.7}$$

*where* $\eta = g(\Phi\mathbf{T}, \mathbf{N}_2)$.

**Corollary 3.12.** *Lorentz forces $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ are parellel according to Fermi Walker if*

$$\begin{aligned} k_1 &= constant, \\ k_2 &= 0, \\ \eta &= constant. \end{aligned}$$

*Proof.* It is clear with Theorem 3.11.

**Theorem 3.13.** *Let $\alpha$ be a $\mathbf{N}_2$-magnetic curve with Bishop frame. Then, Fermi Walker derivatives of Lorentz forces $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ are given by*

$$\begin{aligned} \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T}) &= -\gamma k_1\mathbf{T} + \gamma^{'}\mathbf{N}_1 + k_2^{'}\mathbf{N}_2, \\ \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1) &= -\gamma^{'}\mathbf{T} - \gamma k_1\mathbf{N}_1, \\ \tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2) &= -k_2^{'}\mathbf{T} - k_1 k_2\mathbf{N}_1, \end{aligned} \tag{3.8}$$

*where* $\gamma = g(\Phi\mathbf{T}, \mathbf{N}_1)$.

**Corollary 3.14.** *Lorentz forces $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ are parellel according to Fermi Walker if*

$$\begin{aligned} k_1 &= 0, \\ k_2 &= constant, \\ \gamma &= constant. \end{aligned}$$

*Proof.* It is clear with Theorem 3.13.

## 4. RESULTS AND DISCUSSION

In our this part, we define Fermi-Walker derivative and energy with *Sasaki metric* [27].

**Definition 4.1.** *For two Riemannian manifolds $(M,\rho)$ and $(N,\mathrm{H})$ energy of a differentiable map $f:(M,\rho)\to(N,\mathrm{H})$ is defined by*

$$\varepsilon(f)=\frac{1}{2}\int_M\sum_{a=1}^{n}\mathrm{H}\left(df(\mathbf{b}_a),df(\mathbf{b}_a)\right)v, \tag{4.1}$$

*where $v$ is the canonical volume form on $M$ [27,28]. Sasaki metric defined as*

$$\rho_S(\varsigma_1,\varsigma_2)=\rho\left(d\omega(\varsigma_1),d\omega(\varsigma_2)\right)+\rho\left(Q(\varsigma_1),Q(\varsigma_2)\right).$$

Now, we study relationship between Fermi-Walker derivative and Frenet fields of curves. Fermi transport and derivative have the following theories.

Fermi-Walker transport is defined by

$$\nabla_{\mathbf{T}}^{FW}\mathbf{X}=\nabla_{\mathbf{T}}\mathbf{X}+\mathbf{T}<\mathbf{X},\nabla_{\mathbf{T}}\mathbf{T}>-\nabla_{\mathbf{T}}\mathbf{T}<\mathbf{X},\mathbf{T}>=0.$$

$\nabla_{\mathbf{T}}^{FW}\mathbf{X}$ is called Fermi Walker derivativefor $\mathbf{X}$ by $\mathbf{T}$ along the particle [29].

**Theorem 4.2.** *Let $\alpha$ be a unit speed $\mathbf{T}$-magnetic curve according to Bishop frame in Euclidean 3-space [30]. Then, energy of $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ with Sasakian metric are given by*

$$\begin{aligned} \varepsilon(\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha(1+(k_1^2+k_2^2)^2+(k_1^{'})^2+(k_2^{'})^2)ds, \\ \varepsilon(\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha(1+(k_1^{'}+k_2\rho)^2+k_1^4+(-k_1k_2+\rho^{'})^2)ds, \\ \varepsilon(\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha(1+(k_1\rho-k_2^{'})^2+(k_1k_2+\rho^{'})^2+k_2^4)ds. \end{aligned} \tag{4.2}$$

*Proof.* Let $\alpha$ be a unit speed $\mathbf{T}$-magnetic curve according to Bishop frame in Euclidean 3-space. By using $\nabla_{\mathbf{T}}\Phi(\mathbf{T})=(-k_1^2-k_2^2)\mathbf{T}+k_1^{'}\mathbf{N}_1+k_2^{'}\mathbf{N}_2$ equation and from (4.1) energy formula, we get

$$\begin{aligned}\varepsilon(\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_\mathbf{T}\Phi(\mathbf{T}), \nabla_\mathbf{T}\Phi(\mathbf{T})\rangle ds \\ &= \frac{1}{2}\int_\alpha (1+(k_1^2+k_2^2)^2+(k_1')^2+(k_2')^2)ds.\end{aligned}$$

Also, using $\nabla_\mathbf{T}\Phi(\mathbf{N}_1)$ equation and from energy formula, we have

$$\begin{aligned}\varepsilon(\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_\mathbf{T}\Phi(\mathbf{N}_1), \nabla_\mathbf{T}\Phi(\mathbf{N}_1)\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(k_1'+k_2\rho)^2+k_1^4+(-k_1k_2+\rho')^2)ds.\end{aligned}$$

Similarly by using $\nabla_\mathbf{T}\Phi(\mathbf{N}_2)=(k_1\rho-k_2')\mathbf{T}-(k_1k_2+\rho')\mathbf{N}_1-k_2^2\mathbf{N}_2$ and from energy formula, we get

$$\begin{aligned}\varepsilon(\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_\mathbf{T}\Phi(\mathbf{N}_2), \nabla_\mathbf{T}\Phi(\mathbf{N}_2)\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(k_1\rho-k_2')^2+(k_1k_2+\rho')^2+k_2^4)ds.\end{aligned}$$

**Theorem 4.3.** *Let $\alpha$ be a unit speed $\mathbf{N}_1$-magnetic curve according to Bishop frame in Euclidean 3-space. Then,energy of* $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ *with Sasakian metric are given by*

$$\begin{aligned}\varepsilon(\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha (1+(k_1^2+\eta k_2)^2+(k_1')^2+(\eta')^2)ds, \\ \varepsilon(\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha (1+(k_1')^2+k_1^4+(k_1k_2)^2)ds, \\ \varepsilon(\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha (1+(\eta')^2+(\eta k_1)^2+(\eta k_2)^2)ds.\end{aligned} \tag{4.3}$$

*Proof*: Let $\alpha$ be a unit speed $\mathbf{N}_1$-magnetic curve according to Bishop frame in Euclidean 3-space. Then, by using $\nabla_\mathbf{T}\Phi(\mathbf{T})=(-k_1^2-\eta k_2)\mathbf{T}+k_1'\mathbf{N}_1+\eta'\mathbf{N}_2$ equation and from energy formula

$$\begin{aligned}\varepsilon(\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_\mathbf{T}\Phi(\mathbf{T}), \nabla_\mathbf{T}\Phi(\mathbf{T})\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(k_1^2+\eta k_2)^2+(k_1')^2+(\eta')^2)ds\end{aligned}$$

is obtained.

Similarly by using $\nabla_\mathbf{T}\Phi(\mathbf{N}_1)=-k_1'\mathbf{T}-k_1^2\mathbf{N}_1-k_1k_2\mathbf{N}_2$ equation and from energy formula, we get

$$\begin{aligned}\varepsilon(\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}\Phi(\mathbf{N}_1), \nabla_{\mathbf{T}}\Phi(\mathbf{N}_1)\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(k_1^{'})^2 + k_1^{4} + (k_1k_2)^2))ds.\end{aligned}$$

On the other hand, using $\nabla_{\mathbf{T}}\Phi(\mathbf{N}_2) = -\eta^{'}\mathbf{T} - \eta k_1\mathbf{N}_1 - \eta k_2\mathbf{N}_2$ and from energy formula, we have

$$\begin{aligned}\varepsilon(\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}\Phi(\mathbf{N}_2), \nabla_{\mathbf{T}}\Phi(\mathbf{N}_2)\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(\eta^{'})^2 + (\eta k_1)^2 + (\eta k_2)^2)ds.\end{aligned}$$

**Theorem 4.4.** *Let $\alpha$ be a unit speed $\mathbf{N}_2$-magnetic curve according to Bishop frame in Euclidean 3-space. Then,energy of $\Phi(\mathbf{T})$, $\Phi(\mathbf{N}_1)$, $\Phi(\mathbf{N}_2)$ with Sasakian metric are given by*

$$\begin{aligned}\varepsilon(\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha (1+(\gamma k_1 + k_2^2)^2 + (\gamma^{'})^2 + (k_2^{'})^2)ds, \\ \varepsilon(\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha (1+(\gamma^{'})^2 + (\gamma k_1)^2 + (\gamma k_2)^2)ds, \\ \varepsilon(\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha (1+(k_2^{'})^2 + (k_1k_2)^2 + k_2^4)ds.\end{aligned} \tag{4.4}$$

*Proof.* Let $\alpha$ be a unit speed $\mathbf{N}_2$-magnetic curve according to Bishop frame in Euclidean 3-space. Then, by using $\nabla_{\mathbf{T}}\Phi(\mathbf{T}) = (-\gamma k_1 - k_2^2)\mathbf{T} + \gamma^{'}\mathbf{N}_1 + k_2^{'}\mathbf{N}_2$ and from energy formula

$$\begin{aligned}\varepsilon(\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}\Phi(\mathbf{T}), \nabla_{\mathbf{T}}\Phi(\mathbf{T})\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(\gamma k_1 + k_2^2)^2 + (\gamma^{'})^2 + (k_2^{'})^2)ds,\end{aligned}$$

is obtained.

Then, using $\nabla_{\mathbf{T}}\Phi(\mathbf{N}_1) = -\gamma^{'}\mathbf{T} - \gamma k_1\mathbf{N}_1 - \gamma k_2\mathbf{N}_2$ and from energy formula we have

$$\begin{aligned}\varepsilon(\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}\Phi(\mathbf{N}_1), \nabla_{\mathbf{T}}\Phi(\mathbf{N}_1)\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(\gamma^{'})^2 + (\gamma k_1)^2 + (\gamma k_2)^2)ds.\end{aligned}$$

Similarly, by using $\nabla_{\mathbf{T}}\Phi(\mathbf{N}_2) = -k_2^{'}\mathbf{T} - k_1k_2\mathbf{N}_1 - k_2^2\mathbf{N}_2$ and from energy formula, we get

$$\begin{aligned}\varepsilon(\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}\Phi(\mathbf{N}_2), \nabla_{\mathbf{T}}\Phi(\mathbf{N}_2)\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(k_2^{'})^2 + (k_1k_2)^2 + k_2^4))ds.\end{aligned}$$

**Theorem 4.5.** *Energy of* $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)$ *with Sasakian metric are presented*

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})) &= \frac{1}{2}\int_{\alpha}(1+(k_1^{'}k_1+k_2^{'}k_2)^2+(k_1^{''})^2+(k_2^{''})^2)ds,\\
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_{\alpha}(1+(k_1^{''}+k_2\rho^{'})^2+(k_1^{'}k_1)^2+(\rho^{''}-k_1^{'}k_2)^2)ds,\\
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_{\alpha}(1+(\rho^{'}k_1-k_2^{''})^2+(k_2^{'}k_1+\rho^{''})^2+(k_2^{'}k_2)^2)ds.
\end{aligned} \tag{4.5}$$

*Proof.*Let $\alpha$ be a unit speed $\mathbf{T}$-magnetic curve according to Bishop frame in Euclidean 3-space. When Eq. $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})=k_1^{'}\mathbf{N}_1+k_2^{'}\mathbf{N}_2$ and (2.2) are written in the energy formula

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})) &= \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T}))\rangle)ds\\
&= \frac{1}{2}\int_{\alpha}(1+(k_1^{'}k_1+k_2^{'}k_2)^2+(k_1^{''})^2+(k_2^{''})^2)ds
\end{aligned}$$

is obtained.

Similarly by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)=-k_1^{'}\mathbf{T}+\rho^{'}\mathbf{N}_2$ equation and from energy formula we have

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_{\alpha}[1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1))\rangle]ds\\
&= \frac{1}{2}\int_{\alpha}[1+(k_1^{''}+k_2\rho^{'})^2+(k_1^{'}k_1)^2+(\rho^{''}-k_1^{'}k_2)^2]ds.
\end{aligned}$$

Then by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)=-k_2^{'}\mathbf{T}-\rho^{'}\mathbf{N}_1$ equation and from energy formula we have

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2))\rangle)ds\\
&= \frac{1}{2}\int_{\alpha}(1+(\rho^{'}k_1-k_2^{''})^2+(k_2^{'}k_1+\rho^{''})^2+(k_2^{'}k_2)^2)ds.
\end{aligned}$$

**Theorem 4.6.** *Energy of* $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)$ *with Sasakian metric are presented*

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})) &= \frac{1}{2}\int_{\alpha}(1+((\eta k_2)^{'}+k_1^{'}k_1+\eta^{'}k_2)^2\\
&\quad+(k_1^{''}-\eta k_1k_2)^2+(\eta^{''}-\eta k_2^2)^2)ds,\\
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_{\alpha}(1+(k_1k_2^2-k_1^{''})^2+(k_1^{'}k_1)^2\\
&\quad+(k_1^{'}k_2+(k_1k_2)^{'})^2)ds,\\
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_{\alpha}(1+(\eta k_2^2-\eta^{''})^2+(\eta^{'}k_1)^2\\
&\quad+((\eta^{'}k_2+(\eta k_2)^{'})^2)ds.
\end{aligned} \tag{4.6}$$

*Proof.*Let $\alpha$ be a unit speed $\mathbf{N}_1$-magnetic curve according to Bishop frame in Euclidean 3-space. When Eq. $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T}) = -\eta k_2\mathbf{T} + k_1^{'}\mathbf{N}_1 + \eta^{'}\mathbf{N}_2$ and (2.2) are written in the energy formula

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})), \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T}))\rangle)ds \\
&= \frac{1}{2}\int_\alpha (1+((\eta k_2)^{'} + \eta^{'} k_2 + k_1^{'} k_1)^2 \\
&\quad + (k_1^{''} - \eta k_1 k_2)^2 + (\eta^{''} - \eta k_2^2)^2)ds,
\end{aligned}$$

is obtained.

Similarly by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1) = -k_1^{'}\mathbf{T} - k_1k_2\mathbf{N}_2$ equation and from energy formula we have

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)), \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1))\rangle)ds \\
&= \frac{1}{2}\int_\alpha (1+(k_1k_2^2 - k_1^{''})^2 + (k_1^{'}k_1)^2 + (k_1^{'}k_2 + (k_1k_2)^{'})^2)ds
\end{aligned}$$

Then, by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2) = -\eta^{'}\mathbf{T} - \eta k_2\mathbf{N}_2$ equation and from energy formula, we obtain

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)), \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2))\rangle)ds \\
&= \frac{1}{2}\int_\alpha (1+(\rho^{'} k_1 - k_2^{''})^2 + (k_2^{'}k_1 + \rho^{''})^2 + (k_2^{'}k_2)^2)ds.
\end{aligned}$$

**Theorem 4.7.** *Energy of* $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)$ *with Sasakian metric are presented*

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha (1+((\gamma k_1)^{'} + k_2^{'}k_2 + \gamma^{'} k_1)^2 + (\gamma^{''} - \gamma k_1^2)^2 + (k_2^{''} - \gamma k_1 k_2)^2)ds, \\
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha (1+(\gamma k_1^2 - \gamma^{''})^2 + (\gamma^{'} k_1 + (\gamma k_1)^{'})^2 + (\gamma^{'} k_2)^2)ds, \qquad (4.7) \\
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha (1+(k_1^2 k_2 - k_2^{''})^2 + (k_2^{'}k_1 + (k_1k_2)^{'})^2 + (k_2^{'}k_2)^2)ds.
\end{aligned}$$

*Proof.*Let $\alpha$ be a unit speed $\mathbf{N}_2$-magnetic curve according to Bishop frame in Euclidean 3-space. When Eq. $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T}) = -\gamma k_1\mathbf{T} + \gamma^{'}\mathbf{N}_1 + k_2^{'}\mathbf{N}_2$ and (2.2) are written in the energy formula

$$\begin{aligned}
\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T})), \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{T}))\rangle)ds \\
&= \frac{1}{2}\int_\alpha (1+((\gamma k_1)^{'} + k_2^{'}k_2 + \gamma^{'} k_1)^2 + (\gamma^{''} - \gamma k_1^2)^2 + (k_2^{''} - \gamma k_1 k_2)^2)ds,
\end{aligned}$$

is obtained.

Similarly by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1) = -\gamma'\mathbf{T} - \gamma k_1\mathbf{N}_1$ equation and from energy formula we get

$$\begin{aligned}\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1)), \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_1))\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(\gamma k_1^2-\gamma'')^2+(\gamma' k_1+(\gamma k_1)')^2+(\gamma' k_2)^2)ds.\end{aligned}$$

Then by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2) = -k_2'\mathbf{T} - k_1k_2\mathbf{N}_1$ equation and from energy formula we have

$$\begin{aligned}\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)) &= \frac{1}{2}\int_\alpha (1+\langle \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2)), \nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{N}_2))\rangle)ds \\ &= \frac{1}{2}\int_\alpha (1+(k_1^2k_2-k_2'')^2+(k_2'k_1+(k_1k_2)')^2+(k_2k_2')^2)ds.\end{aligned}$$

## CONCLUSIONS

The study of computing an energy of given vector field depending on the structure of the geometrical spaces has becoming more popular research area recently.

In this paper, it was calculated the Fermi-Walker derivatives of some magnetic curves according to the Bishop frame in the space. Moreover, it was obtained the energy of the Fermi-Walker derivative of magnetic curves according to the Bishop frame in space. Finally, the energy relations of some vector fields associated with Bishop frame in the space was obtained.

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