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ORIGINAL PAPER

NEW CLASS OF CLOSED SETS IN NANO TOPOLOGICAL SPACES

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Abstract.*Theaim of this paper is to introduce a new class of closed sets namelyNanog**-closed sets in Nano topological spaces and study some of their properties.The relationship between these sets with other types of closed sets was also discussed in this paper.*

Keywords:*Nano topology;Nano closed set;Nano interior;Nano closure; Nano open set;Nano generalized closed set; Nano generalized star closed set.*

1. INTRODUCTION

Generalized closed sets were introduced by N. Levine [1] as a super class of closed sets in 1970.Using these closed sets many authors introduced new concepts in topological spaces.Nano topological spaces were introduced by Lellis Thivagar [2].The elements of aNano topological space are called Nano open sets. Nano closed sets, Nano interior and Nano closure were also introduced [3]. Bhuvaneswari [4-6] introduced Nano generalized closedset,Nano α generalized closed set, Nano gα closed set, Nano gr closed set and Nano rg closed set in Nano topological spaces. V.Rajendran et al. [7, 8] introduced and studied Nano generalized star closed sets in Nano topological spaces. With this inspiration, new class of closed sets namely Nano g**-closed sets were introduced in Nano topological spaces and their properties, relationship with other types of closed sets were studied in this paper.

2. PRELIMINARIES

Definition 2.1. [1]A subset A of a topological space (X,τ) is calleda generalized closed set (briefly g-closed)if $cl(A)\subseteq V$ whenever $A\subseteq V$ and V is open in (X,τ).The complement of a g-closed set is a g-open set.

Definition 2.2. [9] A subset A of a topological space (X,τ) is calleda generalized star closed set (brieflyg*-closed) if $cl(A)\subseteq V$ whenever $A\subseteq V$ and V is g-open in (X,τ). The complement of a g*-closed set is a g*-open set.

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Definition 2.3. [2]Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space.

Let $X \subseteq U$. Then,

(i) The lower approximation of X with respect to R is the set of all objects denoted by $L_R(X)$.It is defined as $L_R(X)=\bigcup_{x\in U}\{R(x):R(x)\subseteq X\}$ where $R(x)$ denotes the equivalence class determined by $x\in U$.

(ii) The upper approximation of X with respect to R is the set of all objects denoted by $U_R(X)$.It is defined as $U_R(X)=\bigcup_{x\in U}\{R(x):R(x)\cap X\neq\varnothing\}$ where $R(x)$ denotes the equivalence class determined by $x\in U$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.It is defined as $B_R(X)=U_R(X)-L_R(X)$.

Property 2.4.[2] If (U,R) is an approximation space and $X,Y\subseteq U$, then

1. $L_R(X)\subseteq X\subseteq U_R(X))$
2. $L_R(\varnothing)=U_R(\varnothing)=\varnothing$
3. $L_R(U)=U_R(U)=U$
4. $U_R(X\cup Y)=U_R(X)\cup U_R(Y)$
5. $U_R(X\cap Y)\subseteq U_R(X)\cap U_R(Y)$
6. $L_R(X\cup Y)\supseteq L_R(X)\cup L_R(Y)$
7. $L_R(X\cap Y)=L_R(X)\cap L_R(Y)$
8. If $X\subseteq Y$ then, $L_R(X)\subseteq L_R(Y)$ and $U_R(X)\subseteq U_R(Y)$
9. $U_R(X^c)=[L_R(X)]^c$ and $L_R(X^c)=[U_R(X)]^c$
10. $U_R(U_R(X))=L_R(U_R(X))=U_R(X)$
11. $L_R(L_R(X))=U_R(L_R(X))=L_R(X)$

Definition 2.5.[2] Let U be the universe, R be an equivalence relation on U, $X\subseteq U$ and $\tau_R(X)=\{U,\varnothing,U_R(X),L_R(X),B_R(X)\}$.Then, $\tau_R(X)$ satisfies the following.

(i) $U,\varnothing\in\tau_R(X)$

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

The set $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U,\tau_R(X))$ as a Nano topological space.The elements of $\tau_R(X)$ are called Nano open sets. The complements of Nano open sets are called Nanoclosed sets.

Definition 2.6.[2] Let $(U,\tau_R(X))$ be a Nano topological space and $A \subseteq U$, then
(i) The Nano interior of A is defined as the union of all Nano open subsets of A and is denoted by $N\operatorname{int}(A)$. That is, $N\operatorname{int}(A)$ is the largest Nano open subset of A.
(ii) The Nano closure of A is defined as the intersection of all Nano closed sets containing A and is denoted by $Ncl(A)$. That is, $Ncl(A)$ is the smallest Nano closed set containing A.

Definition 2.7. [4] A subset A of a Nano topological $(U,\tau_R(X))$ is called,

(i) Nano generalized closed set (briefly Ng-closed) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is a Nano open set.

(ii) Nano g*-closed set (briefly Ng*-closed) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is a Nano generalized open set.

3. NANO g**-CLOSED SETS

Throughout this paper $(U,\tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U and U/R denotes the family of equivalence classes of U by R.

We define the following closed set in a Nano topological space $(U,\tau_R(X))$.

Definition 3.1. A subset A of a Nano topological space $(U,\tau_R(X))$ is called a Nano g**-closed set (briefly $\text{Ng}**-\text{closed set}$) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\text{Ng}*-\text{open set}$.

Example 3.2. Consider the set $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b,d\},\{c\}\}$. Let $X=\{a,b\}$ and $\tau_R(X)=\{U,\emptyset,\{a\},\{b,d\},\{a,b,d\}\}$.Then, $(U,\tau_R(X))$ is a Nano topological space.

Consider the subset $A=\{a,b,c\}$.The Ng*-open set containing A is only U and $Ncl(A)=U$. Hence $A=\{a,b,c\}$ is a $\text{Ng}**-\text{closed set}$.

Theorem 3.3. In a Nano Topological space $(U,\tau_R(X))$ if A is $\text{Ng}**-\text{closed set}$, then $Ncl(A)-A$ contains no nonempty $\text{Ng}*-\text{closed set}$.

Proof: Suppose A is a $\mathrm{Ng}^{**}-$closed set in the Nano Topological space $\left(U,\tau_R(X)\right)$.

Let Z be a $\mathrm{Ng}^{*}-$closed set contained in $NCl(A)-A$. Then, $Z\subseteq Ncl(A)$ and $Z\not\subset A$.

This implies $A\subseteq Z^c$. But Z^c is a $\mathrm{Ng}^{*}-$open set. Since A is a $\mathrm{Ng}^{**}-$closed set, $Ncl(A)\subseteq Z^c$ which implies $Z\subseteq\left[Ncl(A)\right]^c$.

Now, $Z\subseteq Ncl(A)$ and also $Z\subseteq\left[Ncl(A)\right]^c\Rightarrow Z\subseteq Ncl(A)\cap\left[Ncl(A)\right]^c\Rightarrow Z=\varnothing$. This shows that $Ncl(A)-A$ contains no nonempty $\mathrm{Ng}^{*}-$closed set.

Theorem 3.4. If A and B are $\mathrm{Ng}^{**}-$closed sets in a Nano Topological space $\left(U,\tau_R(X)\right)$, then $A\cup B$ is a $\mathrm{Ng}^{**}-$closed set.

Proof: Let V be a $\mathrm{Ng}^{*}-$open set containing $A\cup B$. It is clear that $A\subseteq A\cup B$ and $B\subseteq A\cup B$ and hence $A\subseteq V$, $B\subseteq V$. Since A and B are $\mathrm{Ng}^{**}-$closed sets, $Ncl(A)\subseteq V$ and $Ncl\left(B\right)\subseteq V$. So, $Ncl(A)\cup Ncl(B)\subseteq V$. But, $Ncl\left(A\cup B\right)=Ncl\left(A\right)\cup Ncl\left(B\right)$ and hence $Ncl\left(A\cup B\right)\subseteq V$. This shows that $A\cup B$ is a $\mathrm{Ng}^{**}-$closed set in $\left(U,\tau_R(X)\right)$.

Theorem 3.5. If A is a $\mathrm{Ng}^{**}-$closed set and $A\subseteq B\subseteq Ncl\left(A\right)$ then, B is also a $\mathrm{Ng}^{**}-$closed set.

Proof: Let V be a $\mathrm{Ng}^{*}-$open set containing B. Since, $A\subseteq B$, V is a $\mathrm{Ng}^{*}-$open set containing A also. But, A is a $\mathrm{Ng}^{**}-$closed set and hence, $Ncl\left(A\right)\subseteq V$.

Now, $B\subseteq Ncl\left(A\right)\Rightarrow Ncl\left(B\right)\subseteq Ncl\left(A\right)\Rightarrow Ncl\left(B\right)\subseteq V$.

This proves the theorem.

Theorem 3.6. Every Nano closed set is a $\mathrm{Ng}^{**}-$closed set.

Proof: Let A be a Nano closed set and V be a $\mathrm{Ng}^{*}-$open set containing A. Since A is a Nano closed set, $Ncl(A)=A$. This implies, $Ncl(A)\subseteq V$ proving that A is a $\mathrm{Ng}^{**}-$closed set.

Remark 3.7. The converse of the above theorem need not be true as shown in the following example.

Example 3.8. Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$. Let $X=\{a,b\}$ and $\tau_R\left(X\right)=\{U,\varnothing,\{a\},\{b,d\},\{a,b,d\}\}$. Then, $\left(U,\tau_R\left(X\right)\right)$ is a Nano topological space.

In this space,

- ✓ The Nano closed sets are $\{U,\varnothing,\{b,c,d\},\{a,c\},\{c\}\}$.
- ✓ The Ng*-open sets are $\{U,\varnothing,\{d\},\{b\},\{a\},\{a,b\},\{b,d\},\{a,b,d\},\{a,d\}\}$.

Consider the subset $A=\{a,b,c\}$. The $\mathrm{Ng}^{*}-$open set containing A is only U and $Ncl(A)=U$. Hence $A=\{a,b,c\}$ is a $\mathrm{Ng}^{**}-$closed set. But it is not a Nano closed set.

Theorem 3.9. Every Ng*−closed set is a Ng**−closed set .

Proof: Let A be a Ng*−closed set and V be a Ng*−open set containing A.Since, every Ng*−open set is a Nano generalized open set, V is a Nano generalized open set containing A. But, A is a Ng*−closed set set. So, $Ncl(A) \subseteq V$ .This shows that A is a Ng**−closed set.

Remark 3.10. The converse of the above theorem need not be true as shown in the following example.

Example 3.11. Consider the set $U = \{a,b,c\}$ with $U / R = \{\{a\},\{b,c\}\}$. Let $X = \{a,c\}$. By the definition consider $\tau_R(X) = \{U,\varnothing,\{a\},\{b,d\},\{a,b,d\}\}$.Then, $(U,\tau_R(X))$ is a Nano topological space.

In the space $(U,\tau_R(X))$,

- ✓ The Nano closed sets are $\{U,\varnothing,\{a,c\},\{b,c\},\{a,b\},\{a\},\{b\}\}$
- ✓ The Nano generalized closed sets are $\{U,\varnothing,\{a,c\},\{b,c\},\{a,b\},\{a\},\{b\}\}$
- ✓ The Nano g*- closed sets are $\{U,\varnothing,\{a\},\{b,c\},\{a,b\}\}$
- ✓ The Nano g**- closed sets are $\{U,\varnothing,\{a,c\},\{b,c\},\{a,b\},\{a\},\{b\}\}$

The subset $\{a,c\}$ is a Ng**−closed set , but it is not a $Ng*$−closed set .

Theorem 3.12. For each $a \in U$, either $\{a\}$ is a N$g*$−closed set or $\{a\}^c$ is a Ng**−closed set in $\tau_R(X)$.

Proof: Suppose for each $a \in U$, $\{a\}$ is not a N$g*$−closed set in $\tau_R(X)$.Then, $\{a\}^c$ is not a N$g*$−open set in $\tau_R(X)$.This implies, the only N$g*$−open set containing $\{a\}^c$ is U. Hence, $Ncl(\{a\}^c) \subseteq U$. This shows that $\{a\}^c$ is a Ng**−closed set.

4. CONCLUSION

In the present paper, new class of sets namely Nanog**-closed sets was introduced in a Nano topological space.Some of their properties and the relationship between these sets with other types of closed sets were discussed.

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