ORIGINAL PAPER

ESTIMATION OF RARE SENSITIVE PARAMETER UNDER POISSON APPROXIMATION USING STRATIFIED THREE STAGE RANDOMIZED RESPONSE MODEL

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Abstract. *This study investigates the process for estimating the mean number of individuals having rare sensitive attribute in stratified random sampling for known population using Poisson distribution. The properties of the suggested estimation procedures are deeply examined. Empirical studies are performed to support the theoretical results, which show the dominance of the proposed estimators over well-known existing estimators. The results are interpreted and suitable recommendations have been put forward to the survey practitioners.*

Keywords: *randomized response model; rare sensitive attribute; rare unrelated non-sensitive attribute; stratified random sampling; Poisson distribution.*

1. INTRODUCTION

In human population surveys, direct questions about sensitive character often yield untruthful response or non-response. Sensitive issues in sample surveys may include tax evasion, alcohol, drugs habit and some stigmatized disease such as Aids, etc. In such situations, it is very cumbersome task to get truthful response from the respondents. To beat such circumstances, [1] first introduced an unpretentious indirect survey technique known as randomized response technique (RRT). To enhance the confidence of the respondents [2] proposed unrelated question model or U model. Some noteworthy contributions related to randomized response technique and their importance have been carried out by researchers such as [3-15] and among others.

In [6] it is used the randomization device carrying three types of cards bearing statements: (i) “I belong to sensitive group A1,” (ii) “ I belong to group A2,” and (iii) “ Blank cards,” with corresponding probabilities Q1, Q2, and Q3, respectively, such that $\sum_{i=1}^{3} Q_i = 1$. In case the blank card is drawn by the respondent, he/she will report “no.” The rest of the procedure remains as usual. The probability of “yes” answer is given by

$$\theta_1 = Q_1\pi_1 + Q_2\pi_2 \tag{1}$$

where π_1 and π_2 are the true proportion of the rare sensitive attribute A1 and the rare unrelated attribute A2 in the population, respectively. From the above Equation (1) the estimator of π_1 is as

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$$\hat{\pi}_1 = \frac{\hat{\theta}_1 + P_2\pi_2}{P_1} \tag{2}$$

where $\hat{\theta}_1$ the observed proportion of "yes" answers in the sample. The variance of the estimator is $\hat{\pi}_1$ given as

$$V(\hat{\pi}_1) = \frac{\pi_1(1-\pi_1)}{n} + \frac{\pi_1(1-P_1\pi_1-2P_2\pi_2)}{nP_1} + \frac{P_2\pi_2(1-P_2\pi_2)}{nP_1^2} \tag{3}$$

The study [16] suggested a method for estimating the mean number of individuals wtih a rare sensitive attribute by using the Poisson distribution. In [17, 18], the authors extended the work of [6] for stratified and stratified double sampling using Poisson probability distribution. Taking into account the results presented in [19, 20] and motivated by the above works, in this paper, we have made an attempt to extend [6] unrelated randomized response model to three stage unrelated randomized response procedure for stratified and stratified random double sampling using Poisson distribution to estimate the rare sensitive attribute when the parameter of the rare unrelated question is known and unknown. Empirical studies are carried out and it has been demonstrated that the suggested model performs better over [11, 17-19] models [20]. Results are interpreted and suitable recommendations have been made.

2. PROPOSED MODELS – ESTIMATION OF A RARE SENSITIVE ATTRIBUTE IN STRATIFIED RANDOM SAMPLING WHEN THE PROPORTION OF A RARE UNRELATED NON-SENSITIVE ATTRIBUTE IN THE POPULATION IS KNOWN

Let Ω be a finite population of size N which is composed into L strata of size N_h (h=1,2,3,...,L). Sample of size n_h is drawn by simple random sampling with replacement (SRSWR) from h^{th} stratum. It is assumed that π_{h2} is known. The n_h respondents from h^{th} stratum at provided with following three stage randomization device:

First-stage randomization device R_{1h} consists of two statements

Probability	Statements	Selection
Statement 1:	Are you a member of a rare sensitive Group A_1?	T_h
Statement 2:	Go to randomization device R_{2h}	$(1- T_h)$

Second-stage randomization device R_{2h} consists of two statements

Probability	Statements	Selection
Statement 1:	Are you a member of a rare sensitive Group A_1?	P_h
Statement 2:	Go to randomization device R_{3h}	$(1- P_h)$

The randomization device R_{3h} used three statements which ate same as [6]. The probability of getting answer "yes" in h^{th} stratum from the respondent using the above mentioned randomized response devices is

$$\theta_{h0} = T_h + (1-T_h)\left[P_h\,\pi_{h1} + \left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)\{Q_{1h}\pi_{h1} + Q_{2h}\pi_{h2}\}\right] \tag{4}$$

where π_{h1} and π_{h2} are the true proportion of the rare sensitive attribute A_1 and the rare unrelated non-sensitive attribute A_2 in the population, respectively. Since, A_1 and A_2 are rare attributes, assuming that $n_h \rightarrow \infty$ and $\theta_{h0} \rightarrow 0$ such that $n_h\theta_{h0} = \lambda_{h0}$ is finite and follows Poisson distribution with parameter λ_{h0}.

$$\lambda_{h0} = T_h + (1 - T_h)\left[P_h \lambda_{h1} + \left(\frac{1 - \pi_{h1}}{\pi_{h1}}\right)(1 - P_h)\{Q_{1h}\lambda_{h1} + Q_{2h}\lambda_{h2}\}\right] \tag{5}$$

The likelihood function based on the sample is defined as

$$L = \prod_{i=1}^{n_h} \frac{e^{-\lambda_{h0}}\lambda_{h0}^{y_{hi}}}{y_{hi}!}$$

The natural log-likelihood function is given by

$$logL = -n_h\left[T_h + (1 - T_h)\left[P_h \lambda_{h1} + \left(\tfrac{1-\pi_{h1}}{\pi_{h1}}\right)(1 - P_h)\{Q_{1h}\lambda_{h1} + Q_{2h}\lambda_{h2}\}\right]\right] + \textstyle\sum_{i=1}^{n_h} y_{hi}\, log\left[T_h + (1 - T_h)\left[P_h \lambda_{h1} + \left(\tfrac{1-\pi_{h1}}{\pi_{h1}}\right)(1 - P_h)\{Q_{1h}\lambda_{h1} + Q_{2h}\lambda_{h2}\}\right]\right] - \textstyle\sum_{i=1}^{n_h} log y_{hi} \tag{6}$$

Differentiating equation (6) with respect to λ_{h1} and equating to zero. We have the maximum likelihood estimator of λ_{h1} in the hth stratum is given as

$$\hat{\lambda}_{h1} = \frac{1}{T_h + (1 - T_h)P_h + (1 - T_h)\left(\frac{1 - \pi_{h1}}{\pi_{h1}}\right)(1 - P_h)Q_{1h}} \times \left[\frac{1}{nh}\sum_{i=1}^{n_h} y_{hi} - (1 - T_h)\left(\frac{1 - \pi_{h1}}{\pi_{h1}}\right)(1 - P_h)\, Q_{2h}\lambda_{h2}\right] \tag{7}$$

Therefore, the estimator $\hat{\lambda}_1$ for the mean number of persons in the population with rare sensitive characteristics $\hat{\lambda}_1$ is proposed under stratified population as

$$\hat{\lambda}_1 = \sum_{h=1}^{L} W_h\hat{\lambda}_{h1} \tag{8}$$

where, $W_h = \frac{N_h}{N}$.

2.1. PROPERTIES OF THE ESTIMATOR $\hat{\lambda}_1$

The properties of the proposed estimator $\hat{\lambda}_1$ are discussed in the following theorems:

Theorem 2.1. The proposed estimator $\hat{\lambda}_1$ is an unbiased estimator of the parameter λ_1.

$$E(\hat{\lambda}_1) = E\left(\sum_{h=1}^{L} W_h \hat{\lambda}_{h1}\right) = \lambda_1$$

Proof: Since $y_{h1}, y_{h2}, \dots, y_{hn_h}$ are iid Poisson variate with parameter λ_{h0}, therefore, we have

$$E(\hat{\lambda}_1) = E\left(\sum_{h=1}^{L} W_h \hat{\lambda}_{h1}\right)$$

$$= \sum_{h=1}^{L} \frac{W_h}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]} \times \left[\frac{1}{nh}\sum_{i=1}^{n_h} E(y_{hi}) - (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)\, Q_{2h}\lambda_{h2}\right]$$

$$= \sum_{h=1}^{L} \frac{W_h}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]} \times \left[\frac{1}{nh}\sum_{i=1}^{n_h} \lambda_{h0} - (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)\, Q_{2h}\lambda_{h2}\right] = \sum_{h=1}^{L} W_h \lambda_{h1} = \lambda_1$$

Theorem 2.2. The variance of the unbiased estimator λ_1 is given as

$$V(\hat{\lambda}_1) = \sum_{h=1}^{L} W_h \left[\frac{\lambda_{h1}}{n_h\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]} + \frac{(1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)\, Q_{2h}\lambda_{h2}}{n_h\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]^2}\right] \tag{9}$$

Proof: Since $y_{h1}, y_{h2}, \dots, y_{hn_h}$ are iid Poisson variate with parameter λ_{h0} and samples are drawn independently from different strata, therefore, we have

$$V(\hat{\lambda}_1) = V\left(\sum_{h=1}^{L} W_h \hat{\lambda}_{h1}\right)$$

$$V(\hat{\lambda}_1) = \sum_{h=1}^{L} W_h^2 V(\hat{\lambda}_{h1})$$

$$= \sum_{h=1}^{L} \frac{W_h^2}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]^2} \left[\frac{1}{n_h^2}\sum_{i=1}^{n_h} \lambda_{h0}\right]$$

Hence, after simplification we have

$$V(\hat{\lambda}_1) = \sum_{h=1}^{L} W_h \left[\frac{\lambda_{h1}}{n_h \left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]} + \frac{(1-T_h)(1-P_h)\,Q_{2h}\lambda_{h2}}{n_h \left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]^2} \right]$$

Theorem 2. 3. The unbiased estimate of the variance of the proposed estimator $\hat{\lambda}_1$ is given by

$$V(\hat{\lambda}_1) = \sum_{h=1}^{L} W_h \left[\frac{1}{n_h^2 \left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]^2} \right] \left(\sum_{i=1}^{n_h} \lambda_{hi} \right) \tag{10}$$

Proof: Taking expectation both sides Equation (10). We may easily prove that $\hat{V}(\hat{\lambda}_1)$ is an unbiased estimate of $V(\hat{\lambda}_1)$ by utilizing $E(y_{ij}) = \lambda_{h0}$ as y_{ij} follows $P(\lambda_{h0})$.

2.2. ALLOCATION OF SAMPLE SIZE AND VARIANCE UNDER DIFFERENT SYSTEM OF ALLOCATION

The precision of the proposed estimator under stratified random sampling depends upon the selection of sample size n_h from h^{th} stratum (h=1,2,...,L). The allocation method for selection of sample from different strata is based on the availability of prior information of stratum variance.

(I) **Proportional allocation:** Under the proportional allocation the sample size $n_h = nw_h$, where, $n = \sum_{h=1}^{L} n_h$ is the size of total sample drawn from L strata. The variance of the proposed estimator under proportional allocation is obtained as

$$V(\hat{\lambda}_1)_P = \frac{1}{n} \sum_{h=1}^{L} W_h \left[\frac{\lambda_{h1}}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]} + \frac{(1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)\,Q_{2h}\lambda_{h2}}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]^2} \right] \tag{11}$$

(II) **Optimum allocation:** In this method, the size of the sample to be drawn from the hth stratum is derived using the following cost function

$$C = c_0 + \sum_{h=1}^{L} n_h\, c_h$$

where c_0 denotes the overhead cost and c_h be the survey cost per unit in the hth stratum. Sample size n_h from the hth stratum is as follows

$$n_h = n \frac{W_h \sqrt{\frac{\xi_h}{c_h}}}{\sum_{h=1}^{L} W_h \sqrt{\frac{\xi_h}{c_h}}}$$

Under optimum allocation variance of the proposed estimator is given as

$$V(\hat{\lambda}_1)_P = \frac{1}{n} \sum_{h=1}^{L} W_h \left[\frac{\lambda_{h1}}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]} + \frac{(1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)\, Q_{2h}\lambda_{h2}}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]^2} \right] \tag{12}$$

$$\xi_h = \left[\frac{\lambda_{h1}}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]} + \frac{(1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)\, Q_{2h}\lambda_{h2}}{\left[T_h + (1-T_h)P_h + (1-T_h)\left(\frac{1-\pi_{h1}}{\pi_{h1}}\right)(1-P_h)Q_{1h}\right]^2} \right]$$

3. SIMULATION STUDY

We perform a simulation study to validate the theoretical results of proposed two-stage unrelated question RRT model. The simulated results of proposed estimators and are presented in Table 1. When the proportion of unrelated innocuous attribute is known, the design parameters T and p_1, $p_2 = (1 - p_1)$ and $p_3 = (1 - p_1 - p_2)$ were allowed to vary $P_1 = 0.2$, $P_2 = 0.3$, $P_3 = 0.5$, $T_1 = 0.1$, $T_2 = 0.7$, $T_3 = 0.2$.

Table 1. Percent relative efficiency of the proposed estimator with respective to [11] and [20] estimator, when the proportion of the rare unrelated non-sensitive is known.

W_1	W_2	λ_1	Q_1	Q_2	Q_3	Efficiency				
						λ_2=0.9	λ_2=0.7	λ_2=0.5	λ_2=0.3	λ_2=0.1
0.5	0.5	2.5	0.3	0.3	0.4	113.2093	113.6123	114.0403	114.4958	114.9815
0.5	0.5	2.6	0.3	0.3	0.4	113.2773	113.6684	114.0829	114.523	114.9912
0.5	0.5	2.7	0.3	0.3	0.4	113.3409	113.7208	114.1225	114.5483	115.0002
0.5	0.5	2.8	0.3	0.3	0.4	113.4006	113.7697	114.1595	114.5718	115.0085
0.5	0.5	2.9	0.3	0.3	0.4	113.4566	113.8157	114.1942	114.5938	115.0162
0.5	0.5	2.5	0.2	0.4	0.4	110.0295	110.4842	110.9635	111.4693	112.0039
0.5	0.5	2.6	0.2	0.4	0.4	110.1066	110.5473	111.0109	111.4993	112.0145
0.5	0.5	2.7	0.2	0.4	0.4	110.1785	110.6061	111.0551	111.5272	112.0243
0.5	0.5	2.8	0.2	0.4	0.4	110.2458	110.661	111.0963	111.5532	112.0334
0.5	0.5	2.9	0.2	0.4	0.4	110.309	110.7124	111.1348	111.5774	112.0419
0.6	0.4	2.5	0.3	0.3	0.4	115.6223	116.1722	116.7604	117.3911	118.069

W_1	W_2	λ_1	Q_1	Q_2	Q_3	Efficiency				
						λ_2=0.9	λ_2=0.7	λ_2=0.5	λ_2=0.3	λ_2=0.1
0.6	0.4	2.6	0.3	0.3	0.4	115.7149	116.249	116.8191	117.4288	118.0826
0.6	0.4	2.7	0.3	0.3	0.4	115.8016	116.3208	116.8738	117.464	118.0951
0.6	0.4	2.8	0.3	0.3	0.4	115.8829	116.3881	116.925	117.4967	118.1068
0.6	0.4	2.9	0.3	0.3	0.4	115.9593	116.4511	116.9728	117.5273	118.1177
0.6	0.4	2.5	0.2	0.4	0.4	111.7797	112.3497	112.9533	113.5937	114.2742
0.6	0.4	2.6	0.2	0.4	0.4	111.8761	112.4289	113.0133	113.6318	114.2877
0.6	0.4	2.7	0.2	0.4	0.4	111.9661	112.5028	113.0691	113.6673	114.3003
0.6	0.4	2.8	0.2	0.4	0.4	112.0505	112.572	113.1211	113.7003	114.3119
0.6	0.4	2.9	0.2	0.4	0.4	112.1297	112.6367	113.1698	113.7311	114.3227
0.7	0.3	2.5	0.3	0.3	0.4	116.9891	117.6299	118.3183	119.0596	119.8601
0.7	0.3	2.6	0.3	0.3	0.4	117.0968	117.7197	118.3872	119.1041	119.8762
0.7	0.3	2.7	0.3	0.3	0.4	117.1977	117.8036	118.4514	119.1455	119.891
0.7	0.3	2.8	0.3	0.3	0.4	117.2925	117.8822	118.5114	119.1841	119.9048
0.7	0.3	2.9	0.3	0.3	0.4	117.3815	117.956	118.5676	119.2201	119.9177
0.7	0.3	2.5	0.2	0.4	0.4	112.7445	113.3818	114.0585	114.7785	115.5459
0.7	0.3	2.6	0.2	0.4	0.4	112.8521	113.4705	114.1258	114.8214	115.5612
0.7	0.3	2.7	0.2	0.4	0.4	112.9528	113.5533	114.1885	114.8613	115.5753
0.7	0.3	2.8	0.2	0.4	0.4	113.0471	113.6308	114.247	114.8985	115.5885
0.7	0.3	2.9	0.2	0.4	0.4	113.1356	113.7034	114.3017	114.9332	115.6007
0.8	0.2	2.5	0.3	0.3	0.4	117.6875	118.3771	119.1193	119.9204	120.7876
0.8	0.2	2.6	0.3	0.3	0.4	117.8034	118.4738	119.1937	119.9685	120.805
0.8	0.2	2.7	0.3	0.3	0.4	117.9119	118.5642	119.263	120.0133	120.8211
0.8	0.2	2.8	0.3	0.3	0.4	118.0138	118.649	119.3278	120.0551	120.8361
0.8	0.2	2.9	0.3	0.3	0.4	118.1097	118.7285	119.3885	120.0941	120.8501
0.8	0.2	2.5	0.2	0.4	0.4	113.2304	113.9026	114.6174	115.3789	116.1919
0.8	0.2	2.6	0.2	0.4	0.4	113.3439	113.9963	114.6885	115.4244	116.2081
0.8	0.2	2.7	0.2	0.4	0.4	113.45	114.0837	114.7548	115.4666	116.2231
0.8	0.2	2.8	0.2	0.4	0.4	113.5494	114.1654	114.8166	115.506	116.237
0.8	0.2	2.9	0.2	0.4	0.4	113.6428	114.2421	114.8745	115.5427	116.25
0.9	0.1	2.5	0.3	0.3	0.4	117.9897	118.7008	119.4669	120.2946	121.1915
0.9	0.1	2.6	0.3	0.3	0.4	118.1091	118.8006	119.5437	120.3444	121.2095
0.9	0.1	2.7	0.3	0.3	0.4	118.221	118.8939	119.6153	120.3907	121.2262
0.9	0.1	2.8	0.3	0.3	0.4	118.3261	118.9814	119.6823	120.4338	121.2417
0.9	0.1	2.9	0.3	0.3	0.4	118.425	119.0634	119.745	120.4742	121.2562
0.9	0.1	2.5	0.2	0.4	0.4	113.4391	114.1266	114.8579	115.6376	116.4706
0.9	0.1	2.6	0.2	0.4	0.4	113.5551	114.2224	114.9307	115.6842	116.4872
0.9	0.1	2.7	0.2	0.4	0.4	113.6636	114.3118	114.9986	115.7275	116.5025
0.9	0.1	2.8	0.2	0.4	0.4	113.7653	114.3955	115.0619	115.7678	116.5168
0.9	0.1	2.9	0.2	0.4	0.4	113.8609	114.4739	115.1211	115.8054	116.5301

4. CONCLUSION

From the above interpretation, it may be also concluded that proposed randomized response model is rewarding in terms of percent relative efficiencies and dominates over [11] and [17] randomized response model may be recommended to survey practitioners encouragingly in the real life problems, whenever they intent to deal with stigmatized issues.

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