ORIGINAL PAPER

DECOMPOSITIONS OF NANO $g^{\#}$-CONTINUITY VIA IDEALIZATION

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Abstract. *In this paper we introduce the notions of $\alpha g^{\#}$-$n\mathcal{J}$-open sets, $\eta^{\#}$-$n\mathcal{J}$-open sets, $h^{\#}$-$n\mathcal{J}$-open sets, $g^{\#}{}_{t}$-$n\mathcal{J}$-sets, $g^{\#}{}_{\alpha^*}$-$n\mathcal{J}$-sets and $g^{\#}{}_{S}$-$n\mathcal{J}$-sets in ideal nano topological spaces and investigate some of their properties.*

Keywords: *$\alpha g^{\#}$-$n\mathcal{J}$-open set; $\eta^{\#}$-$n\mathcal{J}$-open set; $h^{\#}$-$n\mathcal{J}$-open set; $g^{\#}{}_{t}$-$n\mathcal{J}$-set; $\alpha g^{\#}$-$n\mathcal{J}$-continuity.*

1. INTRODUCTION AND PRELIMINARIES

In this paper we introduce the notions of $\alpha g^{\#}$-$n\mathcal{J}$-open sets, $\eta^{\#}$-$n\mathcal{J}$-open sets, $h^{\#}$-$n\mathcal{J}$-open sets, $g^{\#}{}_{t}$-$n\mathcal{J}$-sets, $g^{\#}{}_{\alpha^*}$-$n\mathcal{J}$-sets and $g^{\#}{}_{S}$-$n\mathcal{J}$-sets in ideal nano topological spaces and investigate some of their properties.

Definition 1.1. A subset A of a nano topological space $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$ is called

1. a ng-closed [1] if $ncl(A) \subset H$, whenever $A \subset H$ and H is nano open in U.
2. a $n\alpha g$-closed [2] if $ncl(A) \subset H$, whenever $A \subset H$ and H is nano semi-open in U.

The complement of a ng-closed (resp. $n\alpha g$-closed) set is called ng-open (resp. $n\alpha g$-open).

Definition 1.2.[3] A subset A of a topological space $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$ is called

1. a $ng^{\#}$-closed if $ncl(A) \subset H$, whenever $A \subset H$ and H is $n\alpha g$-open in $(U, \tau_{(}X))$.
2. an $n\alpha g^{\#}$-closed [4] if $n\alpha cl(A) \subset H$, whenever $A \subset H$ and H is $n\alpha g$-open in $(U, \tau_R(X))$.
3. a $nh^{\#}$-closed if $nscl(A) \subset H$, whenever $A \subset H$ and H is $n\alpha g$-open in $(U, \tau_R(X))$.
4. a $n\eta^{\#}$-closed if $npcl(A) \subset H$, whenever $A \subset H$ and H is $n\alpha g$-open in $(U, \tau_R(X))$.

The complement of $ng^{\#}$-closed set (resp. $n\alpha g^{\#}$-closed set, $n\eta^{\#}$-closed set, $nh^{\#}$-closed set) is $ng^{\#}$-open (resp. $n\alpha g^{\#}$-open, $n\eta^{\#}$-open, $nh^{\#}$-open).

Definition 1.3. A subset A of a topological space $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$ is called:

1. a nt-set [5] if $nint(A) = nint(ncl(A))$.
2. an $n\alpha^*$-set [5] if $nint(A) = nint(ncl(nint(A)))$.

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3. $ng_t^{\#}$-set [3] if $A = H \cap G$, where H is $ng^{\#}$-open and G is a nt-set in $(U, \tau_R(X))$.
4. $ng_{\alpha^*}^{\#}$-set [3] if $A = H \cap G$, where H is $ng^{\#}$-open and G is an $n\alpha$*-set in $(U, \tau_R(X))$.

The collection of all $ng_t^{\#}$-sets (resp. $ng_{\alpha^*}^{\#}$-sets) of U is denoted by $ng_t^{\#}(U, \tau_R(X))$ (resp. $ng_{\alpha^*}^{\#}(U, \tau_R(X))$).

Definition 1.4. A subset A of an ideal topological space $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}), \mathcal{J})$ is called:
1. pre-$n\mathcal{J}$-open [6] if $A \subset nint(ncl^*(A))$.
2. semi-$n\mathcal{J}$-open [6] if $A \subset ncl^*(nint(A))$.
3. α-$n\mathcal{J}$-open [6] if $A \subset nint(ncl^*(nint(A)))$.
4. t-$n\mathcal{J}$-set [4] if $nint(ncl^*(A)) = nint(A)$.
5. α^*-$n\mathcal{J}$-set [7] if $nint(ncl^*(nint(A))) = nint(A)$.
6. S-$n\mathcal{J}$-set [5] if $ncl^*(nint(A)) = nint(A)$.

2. ON $\alpha g^{\#}$-$n\mathcal{J}$-OPEN SETS, $\eta^{\#}$-$n\mathcal{J}$-OPEN SETS AND $h^{\#}$-$n\mathcal{J}$-OPEN SETS

Definition 2.1. A subset E of an ideal nano topological space $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}), \mathcal{J})$ is called:
1. $\alpha g^{\#}$-$n\mathcal{J}$-open if $G \subset \alpha$-$n\mathcal{J}$-$int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U.
2. $\eta^{\#}$-$n\mathcal{J}$-open if $G \subset p$-$n\mathcal{J}$-$int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U.
3. $h^{\#}$-$n\mathcal{J}$-open if $G \subset s$-$n\mathcal{J}$-$int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U.

The complement of open sets is said to be closed sets.

Proposition 2.2. For a subset of an ideal nano topological space, the following hold:
1. Every $\alpha g^{\#}$-$n\mathcal{J}$-open set is $n\alpha g^{\#}$-open.
2. Every $\eta^{\#}$-$n\mathcal{J}$-open set is $n\eta^{\#}$-open.
3. Every $h^{\#}$-$n\mathcal{J}$-open set is $nh^{\#}$-open.

Proof: 1. Let E be an $\alpha g^{\#}$-$n\mathcal{J}$-open. Then we have, $G \subset \alpha$-$n\mathcal{J}$-$int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U. Now, $G \subset E \cap nint(ncl^*(nint(E))) \subset E \cap nint(ncl(nint(E))) = n\alpha$-$int(E)$. This shows that E is $n\alpha g^{\#}$-open.

2. Let E be an $\eta^{\#}$-$n\mathcal{J}$-open set. Then we have, $G \subset p$-$n\mathcal{J}$-$int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U. Now, $G \subset E \cap nint(ncl^*(E)) \subset E \cap nint(ncl(E)) = np$-$int(E)$. This shows that E is $n\eta^{\#}$-open.

3. Let E be an $h^{\#}$-$n\mathcal{J}$-open set. Then we have, $G \subset s$-$n\mathcal{J}$-$int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U. Now, $G \subset E \cap ncl^*(nint(E)) \subset E \cap ncl(nint(E)) = s$-$int(E)$. This shows that E is $nh^{\#}$-open.

Proposition 2.3. For a subset of an ideal nano topological space, the following hold:
1. Every $ng^{\#}$-open set is $\alpha g^{\#}$-$n\mathcal{J}$-open.
2. Every $\alpha g^{\#}$-$n\mathcal{J}$-open set is $\eta^{\#}$-$n\mathcal{J}$-open.
3. Every $\alpha g^{\#}$-$n\mathcal{J}$-open set is $h^{\#}$-$n\mathcal{J}$-open.

Proof: 1. Let E be a $ng^{\#}$-open set. Then we have, $G \subset int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U. Now, $G \subset nint((nint(E))^*) \cup nint(E) = nint((nint(E))^*) \cup nint(nint(E)) \subset nint[(nint(E))^* \cup nint(E)] = nint(ncl^*(nint(E)))$. That is, $G \subset E \cap nint\left(ncl^*\left(nint(E)\right)\right) = \alpha$-$n\mathcal{J}$-$int(E)$. Hence E is $\alpha g^{\#}$-$n\mathcal{J}$-open.

2. Let E be an $\alpha g^{\#}$-$n\mathcal{J}$-open set. Then we have, $G \subset \alpha$-$n\mathcal{J}$-$int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U. Now, $G \subset E \cap nint(ncl^*(nint(E))) \subset E \cap nint(ncl^*(E)) = p$-$n\mathcal{J}$-$int(E)$. Hence E is $\eta^{\#}$-$n\mathcal{J}$-open.

3. Let E be an $\alpha g^{\#}$-$n\mathcal{J}$-open set. Then we have, $G \subset \alpha$-$n\mathcal{J}$-$int(E)$ whenever $G \subset E$ and G is $n\alpha g$-closed in U. Now, $G \subset E \cap nint(ncl^*(nint(E))) \subset E \cap ncl^*(nint(E)) = s$-$n\mathcal{J}$-$int(E)$. Hence E is $h^{\#}$-$n\mathcal{J}$-open.

Definition 2.4. A subset E of an ideal nano topological space $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}), \mathcal{J})$ is called:
1. $g_t^{\#}$-$n\mathcal{J}$-set if $E = G \cap H$, where G is $ng^{\#}$-open and H is a t-$n\mathcal{J}$-set.
2. $g_{\alpha^*}^{\#}$-$n\mathcal{J}$-set if $E = G \cap H$, where G is $ng^{\#}$-open and H is an α^*-$n\mathcal{J}$-set.
3. $g_S^{\#}$-$n\mathcal{J}$-set if $E = G \cap H$, where G is $ng^{\#}$-open and H is an S-$n\mathcal{J}$-set.

Proposition 2.5. For a subset of an ideal nano topological space, the following hold:
1. Every t-$n\mathcal{J}$-set is $g_t^{\#}$-$n\mathcal{J}$-set.
2. Every α^*-$n\mathcal{J}$-set is $g_{\alpha^*}^{\#}$-$n\mathcal{J}$-set.
3. Every S-$n\mathcal{J}$-set is $g_S^{\#}$-$n\mathcal{J}$-set.
4. Every $ng^{\#}$-open set is $g_t^{\#}$-$n\mathcal{J}$-set.
5. Every $ng^{\#}$-open set is $g_{\alpha^*}^{\#}$-$n\mathcal{J}$-set.
6. Every $ng^{\#}$-open set is $g_S^{\#}$-$n\mathcal{J}$-set.

Proof: The proof is obvious.

Remark 2.6. The converses of Proposition 2.5 need not be true as seen from the following Examples.

Example 2.7. Let $\mathrm{U} = \{a_1, a_2, a_3, a_4\}$ with $\mathrm{U/R} = \{\{a_2\}, \{a_4\}, \{a_1, a_3\}\}$ and $\mathrm{X} = \{a_3, a_4\}$. Then $\mathcal{N} = \{\phi, \{a_4\}, \{a_1, a_3\}, \{a_1, a_3, a_4\}, \mathrm{U}\}$ and $\mathcal{J} = \{\phi, \{a_1\}, \{a_4\}, \{a_1, a_4\}\}$. Then $\mathrm{E} = \{a_1, a_3, a_4\}$ is a $g_t^{\#}$-n$\mathcal{J}$-set, but it is not a t-n$\mathcal{J}$-set.

Example 2.8. Let $\mathrm{U} = \{a_1, a_2, a_3\}$ with $\mathrm{U/R} = \{\{a_1\}, \{a_2\}, \{a_3\}\}$ and $\mathrm{X} = \{a_1\}$. $\mathcal{N} = \{\phi, \{a_1\}, \mathrm{U}\}$ and $\mathcal{J} = \{\phi, \{a_2\}\}$. Then $\mathrm{E} = \{a_1\}$ is a $g_{\alpha^*}^{\#}$-n$\mathcal{J}$-set, but it is not an α^*-n$\mathcal{J}$-set.

Example 2.9. Let $\mathrm{U} = \{a_1, a_2, a_3, a_4\}$ with $\mathrm{U/R} = \{\{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}\}$ and $\mathrm{X} = \{a_1, a_4\}$. $\mathcal{N} = \{\phi, \{a_1\}, \{a_4\}, \{a_1, a_4\}, \mathrm{U}\}$ and $\mathcal{J} = \{\phi, \{a_4\}\}$. Then $\mathrm{E} = \{a_1\}$ is a $g_S^{\#}$-n$\mathcal{J}$-set, but it is not an S-n$\mathcal{J}$-set.

Example 2.10. Let $\mathrm{U} = \{a_1, a_2, a_3\}$ with $\mathrm{U/R} = \{\{a_1\}, \{a_2, a_3\}\}$ and $\mathrm{X} = \{a_1\}$. Then $\mathcal{N} = \{\phi, \{a_1\}, \mathrm{U}\}$ and $\mathcal{J} = \{\phi, \{a_2\}\}$. Then $\mathrm{E} = \{a_2, a_3\}$ is both $g_t^{\#}$-n$\mathcal{J}$-set and $g_{\alpha^*}^{\#}$-n$\mathcal{J}$-set, but it is not a $ng^{\#}$-open set.

Example 2.11. Let $\mathrm{U} = \{a_1, a_2, a_3\}$ with $\mathrm{U/R} = \{\{a_1, a_2\}, \{a_3\}\}$ and $\mathrm{X} = \{a_1, a_2\}$. $\mathcal{N} = \{\phi, \{a_1, a_2\}, \mathrm{U}\}$ and $\mathcal{J} = \{\phi, \{a_3\}\}$. Then $\mathrm{E} = \{a_1, a_3\}$ is a $g_S^{\#}$-n$\mathcal{J}$-set, but it is not a $ng^{\#}$-open set.

Proposition 2.12. A subset E of $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}), \mathcal{J})$ is $ng^{\#}$-open if and only if it is both $\eta^{\#}$-n$\mathcal{J}$-open and $g_t^{\#}$-n$\mathcal{J}$-set.

Proof: Necessity is trivial. We prove the sufficiency. Assume that E is $\eta^{\#}$-$n\mathcal{J}$-open and $g_t^{\#}$-$n\mathcal{J}$-set in U. Let $G \subset E$ and G be $n\alpha g$-closed in U. Since E is a $g_t^{\#}$-$n\mathcal{J}$-set in U, $E = P \cap Q$, where P is $ng^{\#}$-open and Q is a t-$n\mathcal{J}$-set. Now G is $n\alpha g$-closed and P is $ng^{\#}$-open implies $G \subset nint(P)$. Since E is $\eta^{\#}$-$n\mathcal{J}$-open, $G \subset p$-$n\mathcal{J}$-$nint(E) = E \cap nint(ncl^*(E)) = (P \cap Q) \cap nint(ncl^*(P \cap Q)) \subset (P \cap Q) \cap nint(ncl^*(P) \cap ncl^*(Q)) = P \cap Q \cap nint(ncl^*(P)) \cap nint(ncl^*(Q))$. Hence $G \subset nint(ncl^*(Q))$, but Q is a t-$n\mathcal{J}$-set, therefore $nint(Q) =$

$nint(ncl^{*}(Q))$, which implies $G \subset nint(Q)$. Therefore $G \subset nint(P) \cap nint(Q) = nint(P \cap Q) = nint(E)$. Hence E is $ng^{\#}$-open in U.

Proposition 2.13. A subset E of $(U, \tau_R(X), \mathcal{J})$ is $ng^{\#}$-open if and only if it is both $\alpha g^{\#}$-$n\mathcal{J}$-open and $g^{\#}_{\alpha^*}$-$n\mathcal{J}$-set.

Proof: Necessity is trivial. We prove the sufficiency. Assume that E is $\alpha g^{\#}$-$n\mathcal{J}$-open and $g^{\#}_{\alpha^*}$-$n\mathcal{J}$-set in U. Let $G \subset E$ and G be $n\alpha g$-closed in U. Since E is a $g^{\#}_{\alpha^*}$-$n\mathcal{J}$-set in U, $E = P \cap Q$, where P is $ng^{\#}$-open and Q is an $n\alpha^*$-$n\mathcal{J}$-set. Now G is $n\alpha g$-closed and P is $ng^{\#}$-open implies $G \subset nint(P)$. Since E is $\alpha g^{\#}$-$n\mathcal{J}$-open. $G \subset \alpha$-$n\mathcal{J}$-$int(E) = E \cap nint(ncl^{*}(nint(A))) = (P \cap Q) \cap nint(ncl^{*}(nint(P \cap Q))) = (P \cap Q) \cap nint(ncl^{*}(nint(P) \cap nint(Q))) \subset (P \cap Q) \cap nint(ncl^{*}(nint(P)) \cap ncl^{*}(nint(Q)))P \cap Q \cap nint(ncl^{*}(nint(P))) \cap nint(ncl^{*}(nint(Q)))$. Hence $G \subset nint(ncl^{*}(nint(Q)))$. But Q is an α^*-$n\mathcal{J}$-set. Therefore $nint(Q) = nint(ncl^{*}(nint(Q)))$, which implies $G \subset nint(Q)$. Therefore $G \subset nint(P) \cap nint(Q) = nint(P \cap Q) = nint(E)$. Hence E is $ng^{\#}$-open in U.

Proposition 2.14. A subset E of $(U, \tau_R(X), \mathcal{J})$ is $ng^{\#}$-open if and only if it is both $h^{\#}$-$n\mathcal{J}$-open and $g^{\#}_{S}$-$n\mathcal{J}$-set.

Proof: Necessity is trivial. We prove the sufficiency. Assume that E is $h^{\#}$-$n\mathcal{J}$-open and $g^{\#}_{S}$-$n\mathcal{J}$-set in U. Let $G \subset E$ and G be $n\alpha g$-closed in U. Since E is a $g^{\#}_{S}$-$n\mathcal{J}$-set in U, $E = P \cap Q$, where P is $ng^{\#}$-open and Q is an S-$n\mathcal{J}$-set. Now G is $n\alpha g$-closed and P is $ng^{\#}$-open implies $G \subset nint(P)$. Since E is $h^{\#}$-$n\mathcal{J}$-open, $G \subset s$-$n\mathcal{J}$-$int(E) = E \cap ncl^{*}(nint(E)) = (P \cap Q) \cap ncl^{*}(nint(P \cap Q)) \subset ncl^{*}(nint(P \cap Q)) = ncl^{*}(nint(P) \cap nint(Q)) \subset ncl^{*}(nint(P)) \cap ncl^{*}(nint(Q))$. Hence $G \subset ncl^{*}(nint(Q))$. But Q is an S-$n\mathcal{J}$-set, therefore $nint(Q) = ncl^{*}(nint(Q))$, which implies $G \subset nint(Q)$. Therefore $G \subset nint(P) \cap nint(Q) = nint(P \cap Q) = nint(E)$. Hence E is $ng^{\#}$-open in U.

3. CONCLUSION

In this manuscript, we introduced and studied the concepts of $\alpha g^{\#}$-$n\mathcal{J}$-open sets, $\eta^{\#}$-$n\mathcal{J}$-open sets, $h^{\#}$-$n\mathcal{J}$-open sets, $g^{\#}{}_{t}$-$n\mathcal{J}$-sets, $g^{\#}{}_{\alpha^*}$-$n\mathcal{J}$-sets and $g^{\#}{}_{S}$-$n\mathcal{J}$-sets in ideal nano topological spaces. In the future, we can study in the area of an ideal nano functions. And we can learn in various areas of topological spaces with associated applications.

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