NUMERAL SYSTEMS OF GREAT ANCIENT HUMAN CIVILIZATIONS

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Abstract. *We present here a systematic study of numeral systems of world's renowned ancient human civilizations. We discuss their important properties regarding number of different symbols, base, positional or place-value character, presence of zero, fractions, representiblity limits and influence on each other. Also we give examples by representing common numbers in these systems. Most of these systems are out of use today, but they have gradually led us to where we are today, the acceptance of most effective Indian decimal place-value system with zero. So, their importance should receive due credit from point of view of preserving our historical path of development.*

Keywords: *Numeral Systems, Zero, Digital Place Value.*

1. INTRODUCTION

Human race is the most intelligent existing known race on the earth. Humans have created marvels with the aid of the superior brainpower that they have received. As such, there is sufficient progress in all disciplines; efficient fulfillments of day-to-day needs, business, administration of complex systems, overall management, different forms of arts, are just few to mention. But above all, the field, the progress in which has yielded most fruitful results, is that of technology. At the base of development of technology lie the basic sciences and to their base the "Queen of the Sciences", viz., mathematics. So, it is worthwhile to delve into the gradual advancements in the very beginning of mathematics, which are number systems, by various great civilizations of humans those flourished on different parts of the earth during ancient periods.

2. EGYPTIAN NUMERAL SYSTEM

Egyptian civilization was well-advanced in quite ancient period. In fact, in as early as 3000 BC, they had good techniques to do calculations. The Great Pyramid at Giza, which is one of the wonders of human creations, is an excellent application of the mathematical knowledge that Egyptians had. It is also argued that Egyptians had information with quite precision about standard mathematical constants like π and the golden ratio, which appear in more or less approximations in their giant constructions. Their calendar creation formed the basis of modern Julian and Gregorian calendars.

Egyptians had two number systems, of which ancient one was based on Hieroglyphs and the later was of hieratic numerals.

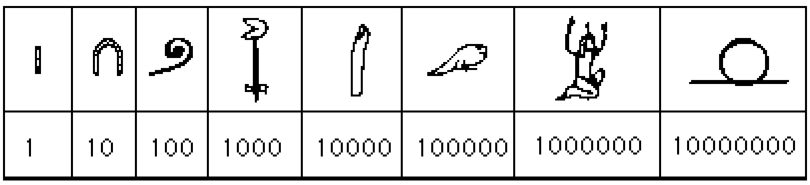
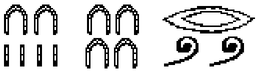


Fig. 1. Egyptian Hieroglyphical Numeral Symbols.

Hieroglyphs are picture symbols. Hieroglyphical numeral system used base 10 with no symbol for zero (Fig. 1). That is, there were symbols for one unit, one ten, one hundred, one thousand, one ten thousand, one lakh, one million, and one crore, i.e., ten million. It was so designed that there was no requirement of a symbol for zero.

A number was represented using required numerical symbols in rows and columns decreasing from right to left, from top to bottom. For example, 108 would be , and 256 as . All symbols have absolute values and there is no place value. Still owing to the sequence of putting various symbols, it can be called quasi-positional. There were 8 different symbols. Clearly, all symbols up to 10,00,000 could not be used more than 9 times in a number, rather it was just not necessary as separate symbol for next power of 10 existed. If that rule is to be applied for 10^7, the largest representable number would have been $10^8 - 1 = 9,99,99,999$. But if largest symbol for 10^7 is given exemption from this rule, no limit to largest representable number would apply as the largest known symbol could be then used required any number of times. Whether Egyptians ever faced some situation demanding this is not known exactly.

Egyptians heavily used fractions from ancient time as is seen in the construction of Pyramids. But they favoured fractions with 1 in numerator, which are now popularly known as Egyptian fractions. If the denominator symbol was enough small, an open mouth symbol , mentioning fraction and consequently 1 in numerator, was placed above all of denominator notation. For larger length denominator symbol, open mouth fraction symbol was placed above the largest 10-power part of denominator. For example, $\frac{1}{32}$ is  and $\frac{1}{264}$ is .

The second Egyptian numeral system came widely in use after invention of papyrus writing. This system had hieratic numerals, i.e., numerals having priestly symbols. The drawback of the earlier system of vary lengthy representations was overcome by compactifying them, but at the cost of increased number of different symbols to be used. There were almost 36 different symbols as shown in Fig. 2.

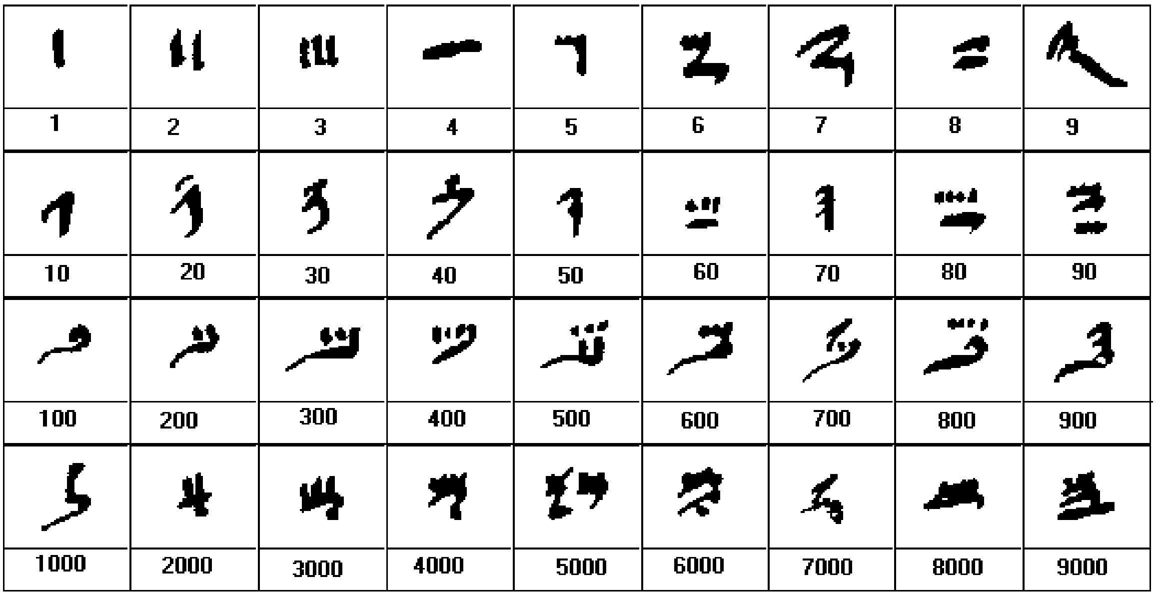


Fig. 2. Egyptian Hieratic Numeral Symbols.

This system was not purely decimal but used bases 10 with no symbol for zero, which was actually not required as such. But this was clearly not positional as the earlier system but was additive. 108 can be written as or as , and 256 as any of , , , , , and . Fractions could also be considered as in earlier system.

The way the different symbols have been assigned to various numbers in this system, it is obvious that no symbol was required to be used more than once in a number up to 9999. For larger numbers, the repetition was must and under allowed repetition, there was virtually no limit to largest representable number.

In both Egyptian numeral systems, addition and subtraction operations were very easy to handle. Multiplication was essentially achieved from doubling and addition, so also the division. What division required extra was to tackle fractions in case of imperfect division and for this, the Egyptians very well had the technique credited to their own name.

3. BABYLONIAN NUMERAL SYSTEM

Around 2000 BC, Babylonian civilization took over from Sumerian civilization in Mesopotamia. They had excellent tables for squares and reciprocals of numbers, and could solve some equations in unknowns up to third degree using these tables.

Sumerians, which were overthrown by Babylonians, are known to have used a base 60, i.e., sexagesimal, number system quite ago. Babylonians also continued with a sexagesimal number system. But they made it a positional system, which lead to real advancements. There was no number 0. If repetitions or combinations are not treated as different symbols, despite having largest used base 60, the system had only two distinct numerical symbols, for 1 and another for 10, and using these every required number could be generated. The first 59 numbers in this system were as shown in Fig. 3.

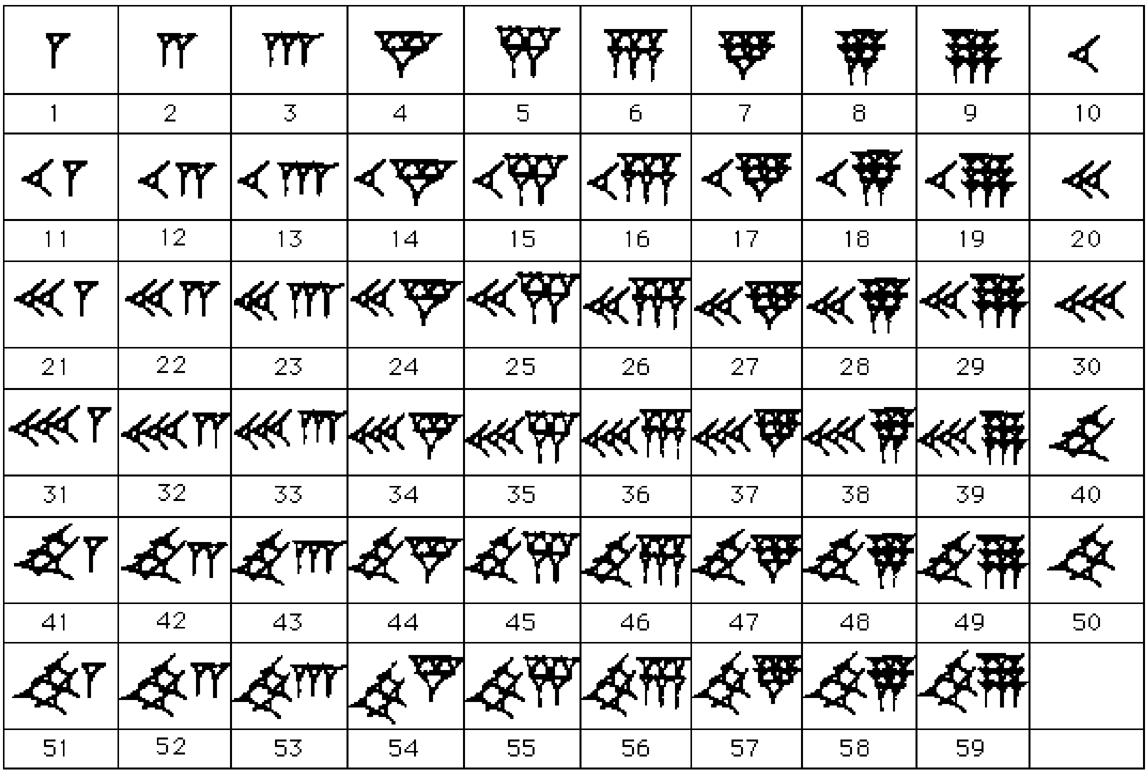
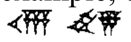


Fig. 3. Babylonian Sexagesimal Numeral Symbols in range 1 – 59.

To represent any number between 1 and 9, the symbol for 1 was marked that many number of times with at most three in a row and starting second row immediately below it, if required. To represent multiple of 10 between 10 and 50, the symbol for 10 was marked that many number of times in linear way up to 30 and in a horizontal triangular way starting from upper side for higher. For number between 11 and 59, the required numbers of 10's symbols were immediately followed by required numbers of 1's symbols obeying their respective appearance rules.

Now for numbers higher than 59, the positional pattern was followed. It was a left to right magnitude decreasing system like our present day decimal system. The rightmost position was for units $60^0 \times n = n$ (between 1 to 59), the position one left to it after small space was for sexagesimal unit that is $60^1 \times n$, and so on. It is just like present decimal system with 10 replaced by 60. But the fundamental difference arises when it comes to zero as an in-between digit in a number. For, there was no symbol for zero in Babylonian system. Then they had adopted the convention of keeping a large empty space to denote no digits, i.e., zero, occurring in-between. For example, the decimal number 1008 is sexagesimal 16, 48, which is denoted in Babylonian by  , and decimal 14422, which is sexagesimal 4, 0, 22, by .

Clearly, there was no theoretical limit and this system could virtually denote a number of as large magnitude as needed.

Operations of addition and subtraction could easily be done using usual carry technique. There were ready reference tables for squares and reciprocals. For multiplication, Babylonians used square tables and for division, the reciprocal tables. Fractions were denoted like in decimal system; only the absence of symbol for 'sexagesimal' point complicated the matters, as did the absence of zero.

There are lot of arguments and counter-arguments about the huge base 60. Whatever may be the reasons for this figure 60; this is still found in use in some of our time measurement systems, making 60 seconds a minute and 60 minutes an hour.

Arab civilization had one of the number systems having base 60. It is believed to have come from Babylonian.

4. GREEK NUMERAL SYSTEM

Greeks seem to have used numerals in as early as 2nd millennium BC. Greek mathematicians Pythagoras, Euclid, Archimedes, Apollonius, Ptolemy had great contributions, particularly in geometry, laying ground for developments in the centuries to come. Other civilizations, like Arabic, translating and gaining their knowledge, inspired from Greek works.

There were minute variations in Greek numeral systems in different states. Still, we analyze the standard ones. There were two major number systems in use, viz., acrophonic and alphabetical.

I	Γ	Δ	H	X	M
1	Pente	Deka	Hekaton	Khilioi	Murioi
	Πεντε	Δεκα	Ηεκατον	Χιλιοι	Μυριοι
	5	10	100	1000	10000

Fig. 4. Greek Acrophonic Numeral Symbols.

In acrophonic system, 1 was represented by simple vertical bar |. 10 and its powers were assigned symbols of the first letter of their names, and hence the name of the system. It was also called as Attic or Herodianic system.

5 also had an acrophonic symbol although it may not seem to be so at first glance. The symbol for 5 is not Π and the probable reason is that may be the Greek alphabets underwent change after numeral symbols were fixed. Combination of 5's symbol with symbols for multiples of 10 powers gave compound symbols for corresponding products. They were as shown in Fig. 5.

There were other combination rules for 50, 500, 5000, 50000 varying more or less.

Γ × Δ = 𐅄	Γ × Η = 𐅅	Γ × Χ = 𐅆	Γ × Μ = 𐅇
5 × 10 = 50	5 × 100 = 500	5 × 1000 = 5000	5 × 10000 = 50000

Fig. 5. Greek Acrophonic Numeral Symbols for 10 power multiples of 5.

Acrophonic system had 6 distinct numeric symbols as mentioned above. Except for 10 power multiples of 5, symbols for other numbers were obtained by combining these by the additive rule. First ten natural numbers were as shown in Fig. 6.

Ι	ΙΙ	ΙΙΙ	ΙΙΙΙ	Γ	ΓΙ	ΓΙΙ	ΓΙΙΙ	ΓΙΙΙΙ	Δ
1	2	3	4	5	6	7	8	9	10

Fig. 6. Greek Acrophonic Numeral Symbols in range 1 – 10.

Acrophonic numeral system was not place value system, nor a decimal system. But since symbols were assigns for 5 and powers of 10, it is said to have 10 as primary base and 5 as the secondary base. Zero didn't find a place, but its absence was not hurdle.

This was positional system in which numeric symbols were used in left to right decreasing magnitude order. 108 is ΗΓΙΙΙ and 256 is ΗΗ𐅄ΓΙ.

Symbol of any power of 10 was necessary to be used at most 4 times in any number, and of any 10 power multiples of 5 at most once. Had this restriction been applied to biggest symbol 50000 also, the largest representable number would have been 99999, but if 50000 were exempted from this rule, and then any large number could be represented by this system.

The second number system of Greeks was alphabetical system. This system had more number of numerical symbols as compared to acrophonic. In fact, all available 27 alphabets were assigned to various numbers and there was one extra for larger number 10000, thus totaling to 28. Their upper case and lower case versions were as in Fig. 7.

Α	Β	Γ	Δ	Ε	Ϛ	Ζ	Η	Θ
α	β	γ	δ	ε	ϛ	ζ	η	θ
1	2	3	4	5	6	7	8	9
Ι	Κ	Λ	Μ	Ν	Ξ	Ο	Π	Ϙ
ι	κ	λ	μ	ν	ξ	ο	π	ϙ
10	20	30	40	50	60	70	80	90
Ρ	Σ	Τ	Υ	Φ	Χ	Ψ	Ω	Ϡ
ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
100	200	300	400	500	600	700	800	900

Fig. 7. Greek Alphabetical Numeral Symbols.

All these alphabets are very well known today except three obsolete, viz., digamma for 6, koppa for 90, and san for 900.

Like Acrophonic system, this is also not place value system. There is no base. Zero is not present. Although, positional nature is not strictly necessary as every symbol has absolute value, owing to their writing style, Greeks used this system to represent a number, using symbols in left to right decreasing order magnitude. 108 is PH or ρη and 256 is ΣΝϚ or σνς.

For numbers above 999, composite symbols were designed. An iota ι prefixed in subscript or superscript of any numerical symbol from 1 to 9 would mean 1000 multiple of that number.

$^{\iota}$Α, $_{\iota}$Α, $^{\iota}$α, $_{\iota}$α	$^{\iota}$Β, $_{\iota}$Β, $^{\iota}$β, $_{\iota}$β	$^{\iota}$Γ, $_{\iota}$Γ, $^{\iota}$γ, $_{\iota}$γ	$^{\iota}$Δ, $_{\iota}$Δ, $^{\iota}$δ, $_{\iota}$δ	$^{\iota}$Ε, $_{\iota}$Ε, $^{\iota}$ε, $_{\iota}$ε	$^{\iota}$Ϛ, $_{\iota}$Ϛ, $^{\iota}$ς, $_{\iota}$ς	$^{\iota}$Ζ, $_{\iota}$Ζ, $^{\iota}$ζ, $_{\iota}$ζ	$^{\iota}$Η, $_{\iota}$Η, $^{\iota}$η, $_{\iota}$η	$^{\iota}$Θ, $_{\iota}$Θ, $^{\iota}$θ, $_{\iota}$θ
1000	2000	3000	4000	5000	6000	7000	8000	9000

Fig. 8. Greek Alphabetical Numeral Symbols for 1000 multiples.

This raised the representability limit to 9999. For the next, i.e., 10000, a myriad symbol M was used. This symbol had two forms of use. To mean 10000 multiple of any number, that number was prefixed to myriad. 90000 is ΘM and combining all rules 12345678 would be $^{\iota}$ΑΛΣΔM$^{\iota}$ΕΧΟΗ. The second form of use of myriad was putting a number above myriad and had two different meanings, one as the first form, that is, 10000 multiple of number above it, and the second as that power of myriad. Thus, $\overset{\beta}{\mathrm{M}}$ would either mean β multiple of myriad, i.e., $2 \times 10000 = 20000$ or β power of myriad, i.e., $10000^2 = 100000000$. Apollonius proposed the second use for allowing bigger numbers to be written in compact form. Power meaning usage had different style in that the number to be multiplied by it was not prefixed but postfixed to it and its multiplier was separated from the rest part of the number by additive symbol χαι. 1234567890123456 would be written as

$\overset{\gamma}{\mathrm{M}}$ $^{\iota}$ΑΣΛΔ χαι $\overset{\beta}{\mathrm{M}}$ $^{\iota}$ΕΧΟΗ χαι $\overset{\alpha}{\mathrm{M}}$ $^{\iota}$ΘΙΒ χαι $^{\iota}$ΓΥΝϚ

$= 10000^3 \cdot 1234 + 10000^2 \cdot 5678 + 10000 \cdot 9012 + 3456.$

Thus a number of as large a magnitude as needed could be symbolized in this system.

Addition, subtraction, multiplication could be done comfortably. Greeks used all of division, powers, roots and fractions while dealing with Geometrical objects.

5. CHINESE NUMERAL SYSTEM

Although exact dates cannot be stated with accuracy, it is inferred from available evidences that Chinese civilization had begun using sufficient level mathematics before beginning of Christ Era. In the very early period, they had invented first of the counting boards. Up to end of 1st Millennium, they had lot of things to their credit. They could construct calendar, tried to predict positions of heavenly bodies, studied geometry of regular polygons determining value of π to some accuracy, had developed techniques to obtain cube roots and solve simultaneous equations.

There were three Chinese number systems. All were based on the decimal system and did not have number zero; interestingly, they did not have a need for zero.

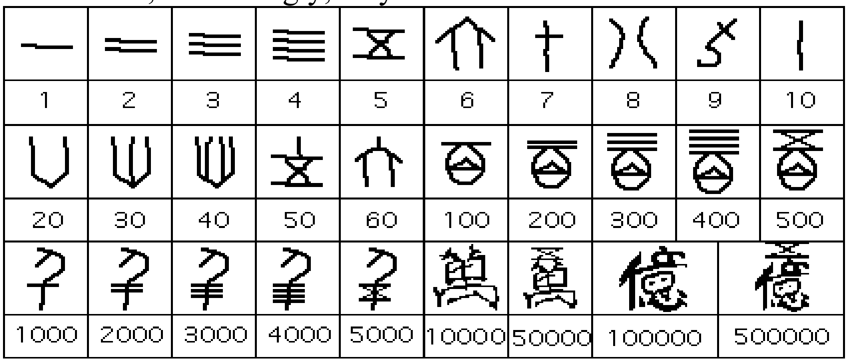
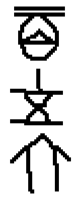


Fig. 9. Chinese Numeral Symbols.

The first system was not positional but was both additive and multiplicative in nature. Symbols for some of the numbers are shown in following Fig. 9.

This system was multiplicative in the sense that the symbols for smaller factor numbers are combined to form larger numbers. For example, the symbol for factors 4 and 100 combined together give symbol for 400. The system was additive in the sense that for very large numbers, the symbols for small numbers were put below symbols for large numbers to denote that they are to be added to form the larger number. For example, the numbers 108 and 256 would be denoted in traditional Chinese respectively by

 and .

This system is still not strictly positional because even if there were this order of occurrence, it is just customary and if it is not followed, there would result no confusion at all.

This system consists the symbol for the largest number 500000. Numbers greater than this have not been found in use. If we omit repetitive occurrence or combination of symbols, the total number of distinct symbols was 14.

Chinese had another numeral system. The numerals are known as bamboo or counting-rod numerals. It was decimal system and also didn't contain zero. The significant change as compared to earlier system was that it was a place value system in which magnitude decreased from left to right. There were two versions of it. If repetitions or combinations of symbols are not treated as different, this system had 2 numerical symbols, viz., a vertical bar and a horizontal bar, which were interchangeably used in the parallel versions. The reason for choice of these symbols is obvious as they were meant for counting board. Following is the construction of first nine numbers in both these notations.

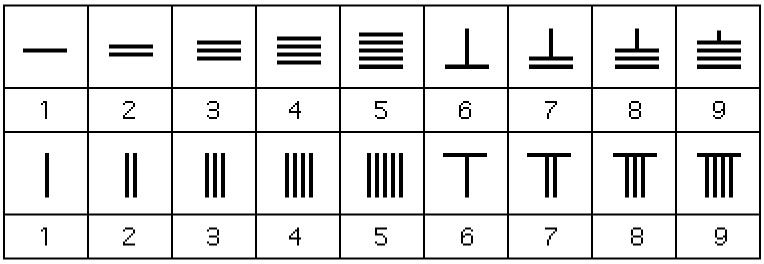
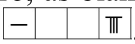


Fig. 10. Chinese Bamboo Numeral Symbols in range 1 – 9.

In use of any one of these forms of numerals, there was a possible scope of ambiguity in representation. For example, in the lower system, what does |||| mean? It is clear from table that it could mean 4; if three bars on right side are considered to be in units place and left most bar in tens place, it would mean 13; and with all such possible combinations of this kind it could equally well mean any of 22, 31, 121, 211 or 1111. Then there was a way out to resolve any such ambiguities. Individual digit was put in a separate box, and to further clarify the matters, both systems were used in combination in alternate digit position, starting with lower system in units place, upper in tens place and continuing this way. This also eliminated the necessity of zero, as blank box would serve the purpose. Thus, 256 would have the form , and 1008 .

Arithmetical operations of addition, subtraction, and multiplication were possible but division, powers and roots were not considered in detail until 13th century. The decimal fractions could have been dealt with provided there was the symbol for decimal point. There is no account of largest number represented by Chinese, but this numeral system could theoretically handle any large number.

The third Chinese counting system came in wide use in 14th Century. It was the famous abacus for which most people know Chinese mathematics. Abacus system was essentially place value oriented, the base was 10, zero was not required and it used minimum number of, i.e., only one, symbol. In fact, it was practiced on a physical device abacus which had straight strings tied at both ends and five or more beads weaved in each string. They were separated by a cross line in two parts one containing minimum 1 bead and the other minimum 4. The successive strings from rightmost to left stood for units place, tens place etc. To indicate a digit in a specific place, the required number of beads in corresponding string were brought close to the middle cross line. For digits 1 to 4 the beads in the lower part were taken up. For digit 5, one bead in upper part was slided to middle line. For digit higher than 5, a combination of upper bead and lower beads was used. No bead brought close to middle line meant zero in that place. Fig. 11 gives position of abacus showing number 256.

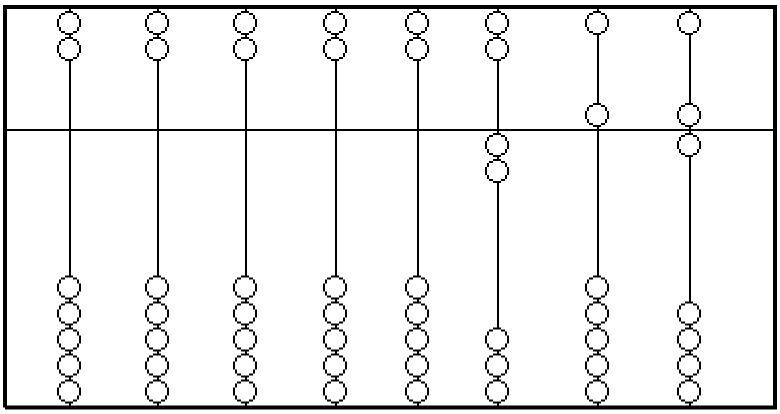


Fig. 11. Chinese Abacus Numeral System showing number 256.

Each string has to represent a digit through 1 to 9. So, it is really enough for each string to have four beads in bottom part and one in top part. But sometimes, there was provision for extra beads to increase storage capacity of strings and to temporarily hold carries in arithmetical operations.

The operations of addition and subtraction could be done with great ease on abacus. Multiplications and division were not as easy, while powers and roots, although not impossible but still, were cumbersome. The number of strings that were attached to abacus only limited largest representable number. As each string stood for a higher ten multiple place, as the number of strings were increased, the range increased. Here also had a symbol of decimal point been inserted, fractions could have been represented. The same abacus is considered to be the first computing machine, the ancestor of today’s hi-end computers, the super machines invented by humans.

6. INDIAN NUMERAL SYSTEM

Available records confirm that Indian civilization was well equipped in mathematical techniques in even 3rd Millennium BC. If we pay heed to beliefs, this date goes back to 17000 years, but is doubted by many, in the absence of firm proofs. All early developments, including mathematical, had religious origin and base. Nonetheless, there is uniform agreement that it is at least as ancient as any most early civilization on any part of the earth. More records are available for era after Christ. The major contributions were Sulbasutras, containing lot of formulae. The study areas covered zero, negative integers and the arithmetic involving them, geometry, astrology and astronomy, solving equations in unknowns.

Three major recognized number systems in India are Brahmi, Gupta and Nagari. Some believe that pre-Brahmi system existed, which consisted simply of tally marks.

Brahmi numeral system had many variants. Not being place-value system, it had lot of symbols each one for 1, 2, · · · , 10, 20, 30, · · · , 100, 200, 300, · · · , 1000, · · · . Zero was not there. We present here few of them in Fig. 12.

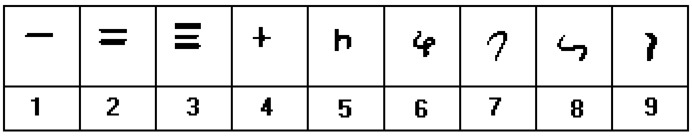


Fig. 12. Indian Brahmi Numeral Symbols in range 1 – 9

From Brahmi numerals evolved Gupta numerals, which were widely used in the Gupta empire. To correlate Brahmi numerals with Guptas’, same Gupta numbers as earlier Brahmi ones are presented in Fig. 13.

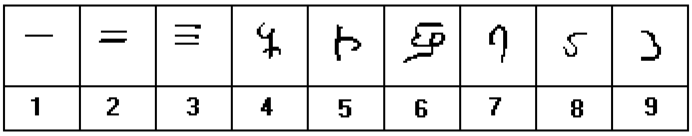


Fig. 13. Indian Gupta Numeral Symbols in range 1 – 9.

Gupta numerals led to Nagari numerals, which is the main system in India today and now worldwide accepted. Nagari, or Devanagari as it is called, system contains 10 symbols, for numbers 1 to 9 and a symbol for zero.

०	१	२	३	४	५	६	७	८	९
0	1	2	3	4	5	6	7	8	9

Fig. 14. Indian Nagari Numeral Symbols in range 0 – 9.

This system is world's first place value decimal system with 0. Value of a symbol depends on its place. If ८ is present in tens place, its value is 80, if in hundreds place, its value is 800, and so on. Numbers are written in left to right decreasing magnitude order. Common example numbers 108 and 256 have very straightforward representations १०८ and २५६. There is no limit to largest representable number. All arithmetical simple and complex operations of addition, subtraction, multiplication, division, taking powers, roots, can be done with most ease. Fractions are very clearly representable with a decimal point.

The Indian system of numerals is adopted now by the whole world. This is the system the world admires like anything, for it has all essential features present in most suitable form.

7. MAYAN NUMERAL SYSTEM

Mayan civilization existed from 2000 BC and had reached peak in advancements in around 2nd Century. Maya people had done good work in astronomical observations, built farming and irrigation systems, big cities, buildings, and quite precise calendar of the year.

The Mayan numeral system had 3 distinct symbols, but they combined to give a total of 20 symbols. It was a partial base 20, i.e., vigesimal, system with numbers from 0 to 19 allowed in units place and next places. Importantly, there was zero in the system and the symbols for numbers from 0 to 19 were as shown in the figure below.

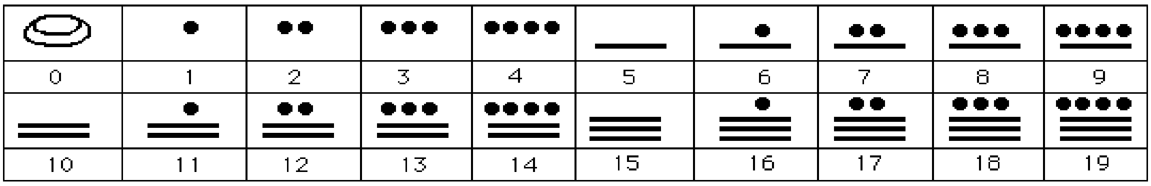
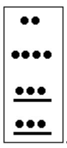


Fig. 15. Mayan vigesimal Numeral Symbols in range 0 – 19.

The reason for choice of base 20 seems the total number of fingers on human hands and feet. There are three symbols, for 0, 1 and 5. 5 is number of fingers on any one hand or foot. The symbols representing numbers larger than 19 were arranged in a vertical column with those in each position, moving upward, multiplied by successive powers of 20 or appropriate number which is explained below. The number $108 = 5 \times 20 + 8$ would be  and $256 = 12 \times 20 + 16$ as .

This was place-value system but was not truly base-20 system. The reason is that although the digit in units place was multiplied by 1, and in next 20's place was multiplied by 20, the digits in next places after that were not multiplied by next power of 20. In fact, one of the 20 was replaced by 18 in 400's place and onwards. Thus, a number like 16008 would be split for writing in Mayan as $16008 = \underline{2} \times 18 \times 20 \times 20 + \underline{4} \times 18 \times 20 + \underline{8} \times 20 + \underline{8}$. So, it is

.

The largest number that could be written by using this system was not limited. Mayans didn't seem to have handled fractions by their system.

8. ARABIC NUMERAL SYSTEM

It is believed that the significant mathematical development of Arabic civilization began around 8^{th} century. Its key features are translations of Indian and Greek mathematical texts as part of research, and the beginning of the algebra instead of Greek exclusive geometrical approach.

By 10^{th} Century, the Arabs had three major arithmetic systems. The one popular amongst businessmen & traders was "Finger Reckoning arithmetic". It used counting on fingers with the names given to numbers. It hardly involved any symbols and didn't contribute to advancement of the subject as such. But there is recorded literature, which uses this system.

In the second system, the numbers were denoted by Arabic alphabets. It had base 60, i.e., was sexagesimal, as mentioned earlier, was probably borrowed from Babylonians, as the base suggests, and was used particularly for large calculations required in astronomical studies.

The significant number system of Arabs was a decimal, i.e., base 10, place-value system with 0 in which the numerals were initially same as that of Indian ones. Later, in different parts in different time periods, these symbols varied to more or less extent. Fig. 16 depicts forms of these numbers in 10^{th} and 11^{th} centuries respectively.

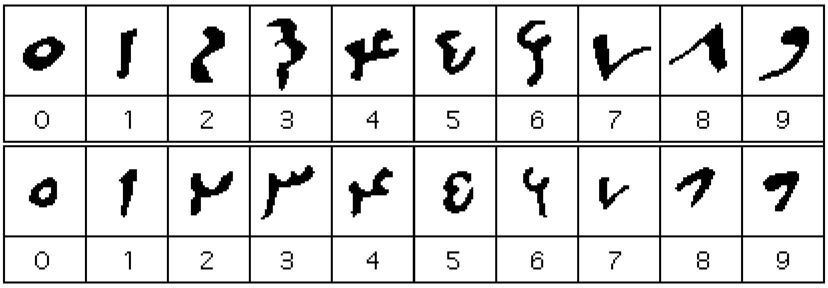


Fig. 16. Arabic Decimal Numeral Symbols in range 0 – 9.

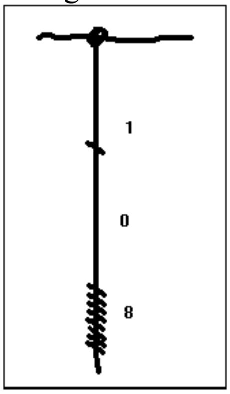
Rotations seems to be the only minute change as far as their variation is concerned. This was the number system used by mathematicians and using which most of their progress was achieved. This was purely based on Indian number system. Thus it had only 10 numerical symbols including the one for 0. Clearly, the arithmetical operations of addition, subtraction, multiplication, division, exponentiation and taking roots could be dealt with quite easily using this system as also the fractions.

Arabic mathematics had deep influence of contemporary Hindu mathematics. In fact, the sizable credit of their growth is due to their contact with Indian mathematicians. Some part of share also goes to the Greek work as ancient Arabic literature shows evidence of Greek translations, which enriched its concepts.

9. INCA NUMERAL SYSTEM

Incas formed an empire in South American parts and had flourished very well before Spanish invaded them in 1532. They had engineering skills, constructed stone temples, public buildings of great size and developed a vast network of roads and bridges. These, including their systematic administration, couldn't have been possible without use of numerical calculations. But surprisingly, Incas are not known to use any form of writing.

In the absence of writing art, Incas dealt with numbers, both for their storage and operations, using knotted strings called quipus. A quipu was a cord or a string on which knots were formed to represent numbers. One string could represent one number at a time and it was the type and number of knots and their position that decided what that number was. Quipu was a positional and place value decimal system. Knots representing the units place were placed nearest the bottom, knots representing the tens place above it and so on. Absence of any knot at a place would signify zero at that place. But then this imparted the necessarily of being highly regular in positioning the knots at various places to avoid confusion. Here are two examples of quipus showing numbers 108 and 256, respectively (Fig. 17).



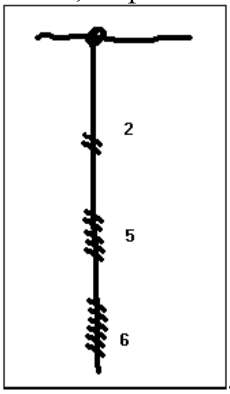
and


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Fig. 17. Inca Quipu Numeral System representing numbers 108 and 256.

Quipu was top to bottom decreasing magnitude system. Here knot can be considered as symbol. Although only one type of knot is enough at required places, two types of knots were in use. Virtually, any large number could be stored on quipu by increasing its length sufficiently to hold higher and higher places.

In India, some illiterate or tribal communities are also known to use this string-knot number system. The connection between those and Incas is unestablished.

10. CONCLUSIONS

All these numeral systems can be compared on the basis of key points. Table 1 shows a comparative summary of all the important characters of these systems.

Indian decimal place-value system with zero is the only indigenous system rich in all features. Today, this numeral system gifted to the world by Indian civilization is used so commonly and frequently, that its importance in simplifying all algebraic operations is ignored without notice. But it is due to this simplification that we could advance our mathematical knowledge very rapidly to great heights within very short time span. Had the world remained stuck to any other complicated numeral system in usage, may be it would have taken centuries more to be at the stage where we are proudly today. Numeral systems could seem to some too small a topic to study, but the same has led us gradually to huge heights. It’s an example of what a very little looking thing in mathematics can contribute to development, if studied in proper perspective.

Table 1. Comparative Chart of All Significant Ancient Numeral Systems

Civilization	Numeral System	Base	Zero	No. of Symbols	Place Value Character	Positional Character	Direction (Decreasing)	Fraction Support	Largest Number
Egyptian	Hieroglyphs	10	No	8	No	Quasi	(<┘) Top to Bottom, Right to Left	Yes	99999999
	Hieratic	On 10	No	36	No	No	Not Specific	Yes	9999
Babylonian	Sexagesimal	60	No	2	Yes	Yes	Left to Right (→)	Yes	No limit
Greek	Attic or Herodianic or Acrophonic	Primary 10 Secondary 5	No	6	No	No	Left to Right (→)	Yes	99999
	Alphabetical	No base	No	28	No	Partial	Left to Right (→)	Yes	No limit
Chinese	Symbolic	10	No	14	No	Quasi	Top to bottom (↓)	No	999999
	Counting Board	10	No	2	Yes	Yes	Left to Right (→)	Conditional	No limit
	Abacus	10	No	1	Yes	Yes	Left to Right (→)	Conditional	No limit
Indian	Nagari or Devanagari	10	Yes	10	Yes	Yes	Left to Right (→)	Yes	No limit
Mayan	Vigesimal	Quasi 20	Yes	3	Yes	Yes	Top to Bottom (↓)	No	No limit
Arabic	Finger Reckoning	Probably 10	No	0	Not Applicable	Not Applicable	Not Applicable	No	Unknown
	Sexagesimal	60	No	Unknown	Unknown	Unknown	Right to Left	No	Unknown
	Decimal	10	Yes	10	Yes	Yes	Right to Left (←)	Yes	No limit
Inca	Quipu	10	No	2	Yes	Yes	Top to Bottom (↓)	No	No limit

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