DE-NOISING KIDNEY ULTRASOUND ANALYSIS USING HAAR WAVELETS

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Abstract. *In medical image processing, image de-noising has become a very essential exercise all through diagnose. A novel approach for the kidney ultrasound de-noising at multiple scales using the wavelet frame analysis was presented in this paper. Wavelets give significant information on the evolution of a time series. In particular, due to their localization properties the significant local changes in observed data (both in time and in frequency) can be easily detected by a limited set of their corresponding wavelet coefficients. In this paper we have made some examples by de-noising kidney ultrasound analysis using Haar Wavelets, in the following, showing the effectiveness of this method. We used for de-noising a ultrasound image of healthy kidney and ultrasound image of kidney with Bull's eye disease and extracted any wavelet coefficients of four test images.*

Keywords: *Ultrasound kidney, Haar wavelet, de-noising.*

1. INTRODUCTION

Wavelets are a tool for hierarchically decomposing functions. They allow a function to be described in terms of a coarse overall shape, plus details that range from broad to narrow mathematical. Regardless of whether the function of interest is an image, a curve, or a surface, wavelets offer an elegant technique for representing the levels of detail present.

We set the stage here by first presenting the simplest form of wavelets, the Haar basis. We cover one-dimensional wavelet transforms and basis functions, and show how these tools can be used to compress the representation of a piecewise-constant function. Then we discuss two-dimensional generalizations of the Haar basis, and demonstrate how to apply these wavelets to image compression.

The basic principle of wavelet de-noising is to identify and zero out wavelet coefficients of the signal which are likely to contain mostly noise. By identifying and preserving significant coefficients, wavelet thresholding preserves important high pass features of the signal such as discontinuities. This property is useful, for example, in image de-noising to maintain the sharpness of edges in the image.

The Haar basis is the simplest wavelet basis. We will first discuss how a one-dimensional function can be decomposed using Haar wavelets, and then describe the actual basis functions. Finally, we show how using the Haar wavelet decomposition leads to a straight-forward technique for compressing a one-dimensional function [1].

2. DISCRETE WAVET TRANSFORM

The discrete wavelet transform (DWT) provides a means of decomposing sequences of real numbers in a basis of compactly supported orthonormal sequences each of which is related by being a scaled and shifted version of a single function. As such it provides the possibility of efficiently representing those features of a class of sequences localized in both position and scale. Compactly supported wavelet bases, like complex exponential bases, carry the significant advantage that fast, numerically stable algorithms exist for sequence analysis (decomposition into the wavelet basis coordinates) and synthesis (reconstruction from the coordinates in the wavelet basis) [2].

3. INTRODUCTION IMAGE DE-NOISING AND COMPRESSION

An image is often corrupted by noise in its acquisition or transmission. The goal of de-noising is to remove the noise while retaining as much as possible the important signal features. Traditionally, this is achieved by linear processing such as Wiener filtering. A vast literature has emerged recently on signal de-noising using nonlinear techniques, in the setting of additive white Gaussian noise. The seminal work on signal de-noising via wavelet thresholding or shrinkage of Donoho and Johnstone have shown that various wavelet thresholding schemes for de-noising have near-optimal properties in the mini max sense and perform well in simulation studies of one-dimensional curve estimation. It has been shown to have better rates of convergence than linear methods for approximating functions in Besov spaces. Thresholding is a nonlinear technique, yet it is very simple because it operates on one wavelet coefficient at a time.

On a seemingly unrelated front, loss compression has been proposed for de-noising in several works. Concerns regarding the compression rate were explicitly addressed. This is important because any practical coder must assume a limited resource (such as bits) at its disposal for representing the data. Other works also addressed the connection between compression and de-noising, especially with nonlinear algorithms such as wavelet thresholding in a mathematical framework. However, these latter works were not concerned with quantization and bitrates: compression results from a reduced number of nonzero wavelet coefficients, and not from an explicit design of a coder.

The intuition behind using loss compression for de-noising may be explained as follows. A signal typically has structural correlations that a good coder can exploit to yield a concise representation. White noise, however, does not have structural redundancies and thus is not easily compressible. Hence, a good compression method can provide a suitable model for distinguishing between signal and noise. The discussion will be restricted to wavelet-based coders, though these insights can be extended to other transform-domain coders as well. A concrete connection between loss compression and de-noising can easily be seen when one examines the similarity between thresholding and quantization, the latter of which is a necessary step in a practical loss coder. That is, the quantization of wavelet coefficients with a zero-zone is an approximation to the thresholding function. Thus, provided that the quantization outside of the zero-zone does not introduce significant distortion, it follows that wavelet-based loss compression achieves de-noising [3].

4. HAAR WAVELETS

Haar wavelets [4] have been proposed for the analysis of localized significant changes in observed data Y, with jumps either in time-space or in frequency-space. These changes can be easily detected by analyzing the wavelet coefficients (in general), but more in particular by focusing only on a few of them [5].

It is well known that each time-series might be roughly represented by the composition of a sequence of high and low frequency "small" waves with a trend. These waves have bounded frequencies, and are localized according to the fact that what happen at a given time t, in general, has a negligible influence (correlation) with the other data at time $t^{'} \geq t$. Due to these properties, the local properties of the time-series are well described by a reduced number of wavelet coefficients; one can choose a limited number of the basis (wavelet) functions to locally represent the time-series. As a consequence, the local analysis can be improved by slicing the time-series of length N into a set of small segments of length $\rho(= N/\sigma)$ (at a fixed time intervals) and analyzing each segment.

4.1. TWO-DIMENSIONAL HAAR WAVELET TRANSFORMS

Let $Y \equiv \{Y_i\}$, $\left(i = 0,...,2^M - 1, 2^M = N < \infty, M \in \mathrm{N}\right)$, be a real square sum able de-noised, time- series $Y \in K^N \subset \ell^2$ (where K is real field): $t_i = i/2^M - 1$ is regular equispaced grid of dyadic points on the interval restricted , for convenience and without restriction on [0;1] ; its family of translated and dilated scaling function is introduced as:

$$\begin{cases} \varphi_k^n \equiv 2^{n/2}\varphi\left(2^n t - k\right), \qquad \left(0 \leq n, 0 \leq k \leq 2^n - 1\right) \\ \varphi_k^n\left(2^n t - k\right) = \begin{cases} 1, t \in \Omega_k^n, \Omega_k^n \equiv \left[\dfrac{k}{2^n}, \dfrac{k+1}{2^n}\right) \\ 0, t \notin \Omega_k^n \end{cases} \end{cases} \tag{1}$$

The Haar wavelet family $\left\{\Psi_k^n(t)\right\}$ is an orthonormal basis for $L^2([0.1])$ functions [3]

$$\begin{cases} \Psi_k^n(t) \equiv 2^{n/2}\Psi\left(2^n t - k\right), \quad \left\|\Psi_k^n\right\|_{L^2} = 1 \\ \Psi_k^n(t) = \begin{cases} -2^{-n/2}, & t \in \left[\dfrac{k}{2^n}, \dfrac{k+1/2}{2^n}\right) \\ 2^{-n/2}, & t \in \left[\dfrac{k+1/2}{2^n}, \dfrac{k+1}{2^n}\right), \left(0 \leq n, 0 \leq k, \leq 2^n - 1\right) \\ 0, & elsewhere \end{cases} \end{cases} \tag{2}$$

Although, without loss of generality, we restrict over selves to $0 \leq n, 0 \leq k \leq 2^n \Rightarrow \Omega_k^n \subseteq [0,1]$, for other integer values of k the family Haar scaling functions and wavelets are defined also outside [0,1] making possible to extend the following consideration to any interval of R [3].

There are two ways we can use wavelets to transform the pixel values within an image. Each is a generalization to two dimensions of the one-dimensional wavelet. To obtain the standard decomposition [6] of an image, we first apply the one-dimensional wavelet transform to each row of pixel values. This operation gives us an average value along with detail coefficients for each row. Next, we treat these transformed rows as if they were themselves an image and apply the one-dimensional transform to each column. The resulting values are all detail coefficients except for a single overall average coefficient.

The algorithm below computes the standard decomposition illustrates each step of its operation [7]:

```
procedure StandardDecomposition(C: array [1. . h, 1. . w] of reals)
        for row ←1 to h do
            Decomposition(C[row, 1. . w])
        end for
        for col ←1 to w do
            Decomposition(C[1. . h, col])
        end for
end procedure[5]
```

5. RESULTS

We consider some ultrasound kidney images to perform two dimensional Haar wavelet transforms for each of them. These operations are realized with 2-D wavelet *haar* from Matlab Application.

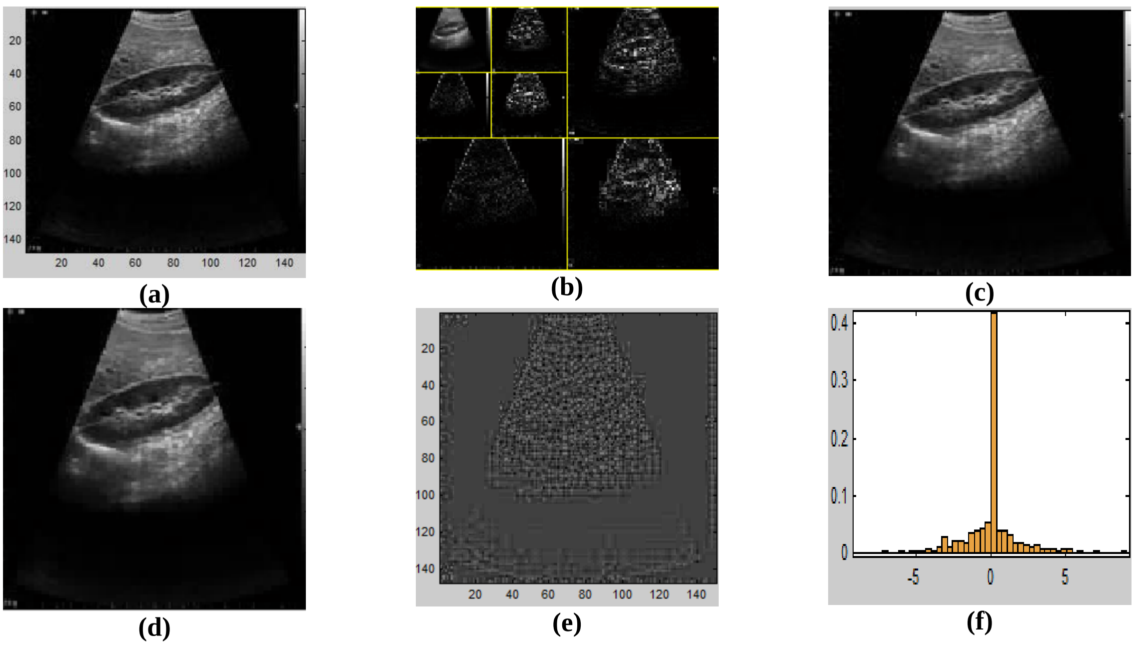

Fig. 1. a) Original ultrasound image of healthy kidney; b) Decomposition image at level 2; c) Synthesized image; d) De-noised image; e) Residuals; f) Histogram of the wavelet.

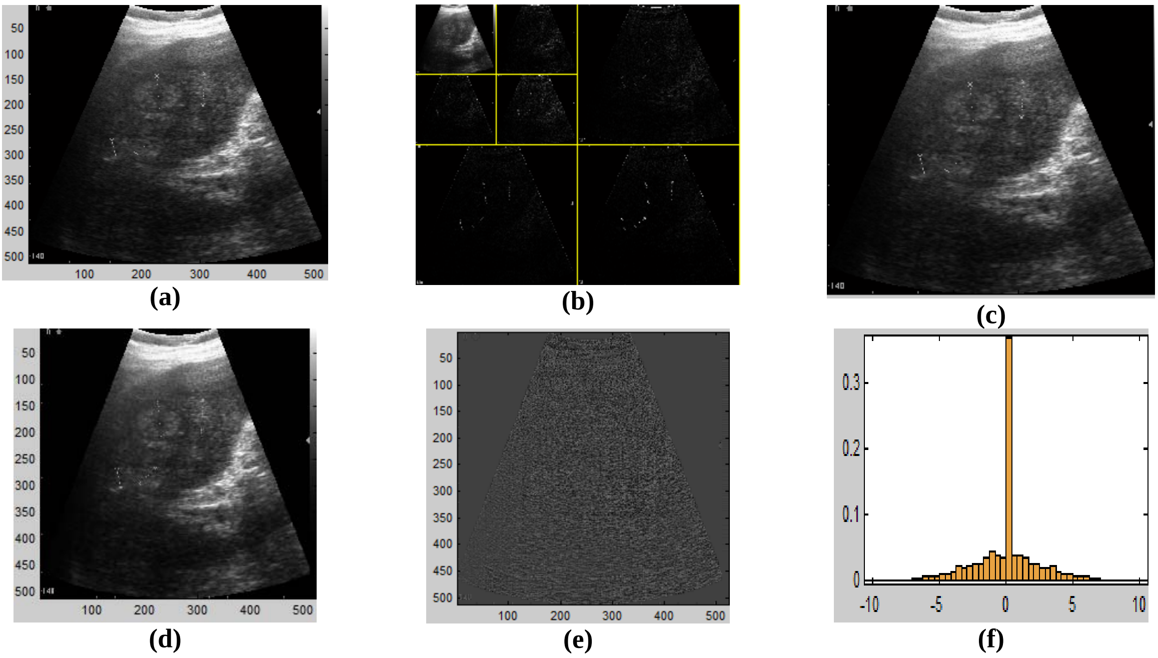

Fig. 2. a) Original ultrasound image of kidney with Bull's eye disease; b) Decomposition at level 2; c) Synthesized Image; d) De-noised image; e) Residuals; f) Histogram of the wavelet.

Table 1. Experimental results

Image	Mean	Deviation standard
Ultrasound image of healthy kidney	-0.001311	2.96
Ultrasound image of kidney with Bull's eye disease	0	2.486

Comparison different kidney ultrasound we observe that deviation standard of ultrasound image of healthy kidney it is greater then deviation standard of ultrasound image of kidney with Bull's eye disease, because first ultrasound image has a higher contrast.

6. CONCLUSIONS

We have presented an efficient wavelet for de-noising ultrasound kidney, called Haar wavelet. The main contribution of our work was processing ultrasound images of healthy kidney and of kidney with Bull's eye disease and we extracted the coefficients for each. However, by visual inspection it is evident that the de-noised image, while removing a substantial amount of noise, suffers practically no degradation in sharpness and details. Experimental results show that our proposed wavelet yields significantly improved visual quality.

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