ORIGINAL PAPER

ON SOME PROPERTIES OF INCLINES

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Manuscript received: 30.10.2011; Accepted paper: 15.12.2011;

Published online: 10.03.2012.

Abstract. *In this paper we obtained some properties satisfied by the elements of an Incline. We defined an incline with unity and obtained some results related to regular, idempotent elements in an incline and established relations among them. We introduced inverse of an element in an incline with unity and obtained results about invertible and complemented elements.*

Keywords: *Incline; incline with unity, sub-incline, complement of an element, regular element, idempotent element, invertible element.*

1. INTRODUCTION

Inclines are additively idempotent semi rings in which products are less than or equal to either factor. The notion of inclines and their applications are described comprehensively in Cao et al. [1]. Recently, Kim and Roush [2] have surveyed and outlined algebraic properties of inclines.

2. PRELIMINARIES

Definition 1: An incline is a nonempty set R with binary operations addition(+) and multiplication(.) defined on $R.R \to R$ such that for all $x, y, z \in R$,

$$(1)\, x+y = y+x,\, x+(y+z) = (x+y)+z$$
$$(2)\, x(y+z) = xy+xz,\, (y+z)x = yx+zx$$
$$(3)\, x(yz) = (xy)z,\, x+x = x,\, x+xy = x,\, y+xy = y$$

Definition 2: An incline R is said to be an incline with unity if there exists an element '1' in R such that $x.1 = 1.x = x \quad \forall x \in R$.

Remark: In an incline R with unity 1 + x = 1 for all 'x' in R.

Definition 3: In an incline R with unity '1' the complement of an element 'a' is defined as the element 'b' in R such that a + b =1. It is denoted with a'.

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Definition 4: In an incline R with unity an element ‘a’ is said to be left invertible if there exists an element ‘b’ in R such that b. a =1. Similarly if there exists an element ‘c’ in R such that a .c = 1 then ‘c’ is called the right inverse of ‘a’ in R.

If an element ‘a’ has both left and right inverses in R then ‘a’ is said to be invertible.

Definition 5: In an incline R an element ‘a’ $\in R$ is said to be regular if there exists an element ‘ $x \in R$ ’ such that $a.x.a = a$ [3–5].

2.1 RESULTS

Proposition 1: If a, b are idempotent elements in an incline R then ‘a + b’ is idempotent in R.

Proof: a, b are idempotent in R .By the property (3) of an Incline R,

$$\begin{aligned}(a+b)(a+b) &= a.a + a.b + b.a + b.b \\ &= a + a.b + b.a + b \\ &= a + b\end{aligned}$$

Hence the proposition.

Proposition 2: Let a,b,c be elements of an Incline R. If b, c are idempotent, $a + b = a + c$ and b.a = a.c then b = c.

Proof: Given ‘b’ and ‘c’ are idempotent in R.

$$\begin{aligned}& a + b = a + c \\ &\Rightarrow b.(a+b) = b.(a+c) \\ &\Rightarrow b.a + b.b = b.a + b.c \\ &\Rightarrow b.a + b = a.c + b.c \\ &\Rightarrow b + b.a = (a+b).c \\ &\Rightarrow b + b.a = (a+c).c \\ &\Rightarrow b + b.a = c + a.c \\ &\Rightarrow b = c\end{aligned}$$

This proves the proposition.

Proposition 3: If R is an incline with unity and ‘a’ $\in$ R is idempotent then ‘a’ is regular.

Proof: ‘ a’ is idempotent $\Rightarrow a^2 = a \Rightarrow a.a = a \Rightarrow a.1.a = a$

Hence ‘a’ is regular. .

Proposition 4: If R is an incline with unity and ‘a’ $\in$ R is invertible then ‘a’ is regular.

Proof: ‘a’ $\in$ R is invertible. There exists an element ‘b’ $\in$ R such that $a.b = 1$

$$a.b = 1 \Rightarrow a.b.a = a \quad .$$

Hence ‘a’ is regular.

Proposition 5: If ‘a’ is a regular element in an incline R with unity then for some $x \in R$ a.x and x.a are idempotent elements in R.

Proof: ‘a’ is a regular element in R. There exists $x \in R$ such that $a.x.a = a$.

$$a.x.a = a \Rightarrow a.x.a.x = a.x \Rightarrow (a.x).(a.x) = a.x \Rightarrow (a.x)^2 = a.x$$

Similarly, $a.x.a = a \Rightarrow x.a.x.a = x.a \Rightarrow (x.a)(x.a) = x.a \Rightarrow (x.a)^2 = x.a$

Proposition 6: If R is an incline with unity and a,b$\in$R have complements, then
(1) $(a+b)' = a'+b'$ (2) $(a.b)' = a'+a.b'$

Proof: $(a+b)+(a'+b') = (a+a')+(b+b') = 1+1 = 1$
$a.b + a' + a.b' = a.(b+b') + a' = a.1 + a' = a + a' = 1$

Lemma 7: In an incline R with unity the set of all elements whose complement exists form a sub-incline of R.

Proof: Follows from the proposition 6.

Proposition 8: Suppose R is an incline with unity and a, b $\in R$.
(1) If ‘a.b’ is idempotent and ‘b’ is right invertible then ‘a’ is regular.
(2) If ‘a.b’ is idempotent and ‘a’ is left invertible then ‘b’ is regular.

Proof: ‘a.b’ is idempotent in R $\Rightarrow$ $(a.b)(a.b) = a.b$
Since ‘b’ is right invertible there exists an element ‘d’ in R such that b.d = 1.

$$(a.b)(a.b) = a.b$$
$$\Rightarrow (a.b.a).b = a.b$$
$$\Rightarrow (a.b.a).b.d = a.b.d$$
$$\Rightarrow a.b.a = a$$

Since ‘a’ is left invertible there exists ‘c’ in R such that c.a = 1.

$$(a.b)(a.b) = a.b$$
$$\Rightarrow a.(b.a.b) = a.b$$
$$\Rightarrow c.a.(b.a.b) = (c.a).b$$
$$\Rightarrow b.a.b = b$$

Hence , (1) and (2) are proved.

Acknowledgements

I am very much thankful to Dr.M.Anjan Kumar, Principal, B.V.C.C.E, Rajahmundry for giving me good encouragement in preparing the paper.

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