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ANALYSIS MODEL FOR THE PROPAGATION OF THE EFFECT OF THE GOVERNMENTAL ACQUISITIONS MODIFICATION IN THE ECONOMY

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Abstract. *In the present work, the authors aim to present a model analyzing the propagation of the effect of the governmental acquisitions modification in the economy using a "lag" equation. The authors approached several aspects: a) determining the effects of the governmental acquisitions modification on the revenue at balance; b) calculating the dynamic multipliers; c) calculating the static efficiency; d) determining the number of economic cycles needed for the dynamic efficiency to represent a certain percentage of the static efficiency; e) determining the number of economic cycles needed for the governmental acquisitions modification to no longer generate effects in the economy.*

Keywords: *governmental acquisitions, lag, revenue at balance, dynamic multiplier, static multiplier.*

1. INTRODUCTION

The effects of the application of different economic policies do not emerge instantaneously, but after a shorter or longer period of time known as "lag" (delay). This lag is the time needed for the economic policy to influence the variables desired.

2. ECONOMIC POLICY EFFECT LAG

One can identify two types of lag: *internal lag* and *external lag*.

The internal lag contains three sub-periods: the period of recognition, the decision period, and the purely administrative period. To collect, process and analyze the data concerning the economic variables it takes a certain period of time. Practice highlights the existence of a lag between the moment of emergence of the need for a political decision and the moment when the authorities understand that the respective political decision is necessary [1].

After having analyzed the data, the political decision-makers need to determine if the disturbance is temporary and relatively minor or can determine necessary political changes, meant to bring the economic variables back to their previous values. Then, a period of implementation of the decision follows, which represents the *administrative lag.*

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The internal lag length is variable, due to the action of certain factors. Depending on the legislation, in some countries the administrative lag associated to the monetary policy is smaller than the administrative lag associated to the fiscal policy. Financial information, inflation and unemployment data are relatively rapidly available.

The external lag can be divided into two sub-periods [1]: the period of time needed to influence the intermediate variables (for instance the period of time between tax modification and the modification of the aggregate demand under the effect of the taxes) and the period of time between the modification of the intermediate variables and the modification of the goal variables (for example the period of time between the modification of the aggregate demand and the modification of the employment level).

The length of the external lag is variable, being dependent on the type of applied policy, the state of the economy at the moment of the application of the economic policy, the way the anticipations were formed.

The existance of this lag can give the stabilization policy a destabilizing character (for example the case of temporary disturbances or the case when the disturbance is a component of the normal cyclic movement).

3. ANALYSIS OF THE PROPAGATION OF THE GOVERNAMENTAL ACQUISITIONS MODIFICATIONS EFFECT

The phenomenon of the economic policy effect delay can be studied using a “lag” equation of the production at balance (y) [2]. The analysis of the propagation of the governmental acquisitions modification effect has as a premise the equation of the revenue at balance (revenue under the conditions of equality between the aggregate offer and the aggregate demand) [3]:

$$y_n = A + c' \cdot (1-t) \cdot y \tag{1}$$

The effect of the macroeconomic policy does not manifest itself instantaneously but within several successive economic cycles, by propagation, from "cycle 1" to "cycle n", according to the equation below [3]:

$$y_n = A + \alpha \cdot y_{n-1} \ , \ \alpha = c' \cdot (1-t) \tag{2}$$

The symbols used in the “lag” equation have the following meaning:

A	autonomous expenses;
y_n	revenue (production) at balance in the economic cycle "n";
y_{n-1}	revenue (production) at balance in the economic cycle "n-1",
c'	marginal propension[3] to consumption, $0 \le c' \le 1$;
t	taxation rate, $0 \le t \le 1$.

[3] Natural tendency, trend, natural inclination towards something.

The analysis of the effect propagation is realized in the hypothesis that the values of the marginal propension to consumption (c') and fiscality rate (t) is known, and governmental acquisitions (AG) constitute the instrumental variable of the macroeconomic policy, which influences the level of the autonomous expenses (A) [3]:

$$A = C_0 + AG + \bar{I} + c' \cdot TR - c \cdot T_0 \tag{3}$$

The autonomous expenses (A) have the following components: autonomous consumption (C_0), governamental acquisitions (AG), autonomous investments ($\bar{I}$), a part of the transferable expenses (c'·TR), a part of the autonomous taxes (c'·T_0).

Governmental acquisitions get changed during the first cycle, and the level of the governmental acquisitions modification is ΔAG, which is found integrally in the modification of the autonomous expenses (ΔA):

$$\Delta A = \Delta AG \tag{4}$$

By the analysis of the propagation of the effect generated by the governmental acquisitions modification, the authors aimed to highlight the following aspects:

a) the effect of the governmental acquisitions modification on the production at balance for "n" consecutive economic cycles;

b) dynamic multipliers k_{d1}, k_{d2}, k_{d3} ... k_{dn} pertainaing to "n" economic cycles;

c) static efficiency (k_s) pertaining to the governmental acquisitions modification (ΔAG);

d) number of economic cycles needed for the dynamic efficiency (k_{dn}) to represent a percentage β of the static efficiency (k_s);

e) number of economic cycles needed for the modification of the governmental acquisitions to no longer generate effects in the economy.

The calculations needed to characterize the propagation of the effect of the governmental acquisitions modification are presented below.

a) The effect of the governmental acquisitions modification on the production at balance for "n" consecutive economic cycles [3]

Starting from the lag equation $y_n = A + \alpha \cdot y_{n-1}$ one determines the general equation with absolute variations for the economic cycle "n":

$\Delta y_n = \Delta A + \alpha \cdot \Delta y_{n-1}$, where "n" takes whole positive values: n=1, 2, 3, ….,+∞

α is positive and below 1 and is established using the following relation:

$$\alpha = c' \cdot (1 - t) \rightarrow 0 \le \alpha \le 1 \tag{5}$$

For the first economic cycle, n=1 and the equation with absolute variations is:

$$\Delta y_1 = \Delta A + \alpha \cdot \Delta y_0 \tag{6}$$

According to the hypothesis, in the first cycle, the autonomous expenses change due to the governmental acquisitions modification: ΔA=ΔAG

Due to the fact that the revenue of the reference period y_0 is considered constant, it results that $\Delta y_0=0$

Replacing the values in the absolute variation equation pertaining to the first economic cycle, one obtains:

$$\Delta y_1 = \Delta AG + \alpha \cdot 0 \tag{7}$$

$$\Delta y_1 = \Delta AG \tag{8}$$

For the second economic cycle, n=2 and the absolute variation equation is:

$$\Delta y_2 = \Delta A + \alpha \cdot \Delta y_1 \tag{9}$$

Since in the second cycle the variable AG has not changed anymore, it results that $\Delta AG=0$, hence $\Delta A=\Delta AG=0$.

The absolute variation equation becomes:

$$\Delta y_2 = \alpha \cdot \Delta y_1 \tag{10}$$

On the first cycle, we know that $\Delta y_1 = \Delta AG$

Replacing the values in the absolute variation equation pertaining to the second economic cycle, we obtain:

$$\Delta y_2 = \alpha \cdot \Delta y_1, \, iar \quad \Delta y_1 = \Delta AG \tag{11}$$

$$\Delta y_2 = \alpha \cdot \Delta AG \tag{12}$$

For the third economic cycle, n=3 and the absolute variation equation is:

$$\Delta y_3 = \Delta A + \alpha \cdot \Delta y_2 \tag{13}$$

Since in the second cycle the variable AG has not changed anymore, it results that $\Delta AG=0$, hence $\Delta A=\Delta AG=0$.

The absolute variation equation becomes:

$$\Delta y_3 = \alpha \cdot \Delta y_2 \tag{14}$$

On the second cycle we know that $\Delta y_2 = \alpha \cdot \Delta AG$, a relation that we replace in the absolute variation equation pertaining to the third economic cycle:

$$\Delta y_3 = \alpha^2 \cdot \Delta AG \tag{15}$$

Out of the above presentation, one can easily deduce that for the cycle “n” the modification of the revenue (production) is:

$$\Delta yn = \alpha^{n-1} \cdot \Delta AG \tag{16}$$

b) Dynamic multipliers k_{d1}, k_{d2}, k_{d3} ... k_{dn} corresponding to "n" cycles [3]

For the first economic cycle we use the absolute variation equation $\Delta y_1 = \Delta AG$ using which we calculate the dynamic efficiency for the 1st economic cycle:

$$k_{d1} = \frac{\Delta y_1}{\Delta AG} = \frac{\Delta AG}{\Delta AG} = 1 \tag{17}$$

$$k_{d1} = \frac{\Delta y_1}{\Delta AG} = 1 \tag{18}$$

For the second economic cycle we use the absolute variation equation $\Delta y_2 = \alpha \cdot \Delta AG$ using which we calculate the dynamic efficiency for the 2nd economic cycle:

$$k_{d2} = \frac{\Delta y_2}{\Delta AG} = \frac{\alpha \cdot \Delta AG}{\Delta AG} = \alpha \tag{19}$$

$$k_{d2} = \frac{\Delta y_2}{\Delta AG} = \alpha \tag{20}$$

For the third economic cycle we use the absolute variation equation $\Delta y_3 = \alpha^2 \cdot \Delta AG$ using which we calculate the dynamic efficiency for the 3rd economic cycle:

$$k_{d3} = \frac{\Delta y_3}{\Delta AG} = \frac{\alpha^2 \cdot \Delta AG}{\Delta AG} = \alpha^2 \tag{21}$$

$$k_{d3} = \frac{\Delta y_3}{\Delta AG} = \alpha^2 \tag{22}$$

For the "n"th economic cycle we use the absolute variation equation $\Delta y_n = \alpha^{n-1} \cdot \Delta AG$ using which we calculate the dynamic efficiency for the "n"th economic cycle:

$$k_{dn} = \frac{\Delta y_n}{\Delta AG} = \alpha^{n-1} \tag{23}$$

c) Static efficiency (k_s) pertaining to the governmental acquisitions modifications (ΔAG) [3]

Efficiency is measured using multipliers.

To determine the static efficiency, one sums up the effects recorded in all the economic cycles.

For cycle "i", there is the relation:

$$\Delta y_i = y_i - y_{i-1} \tag{24}$$

The counter "i" takes whole positive values: i=1, 2, 3, …,n

For "n" economic cycles, one obtains the relation:

$$\sum_{i=1}^{n}\Delta y_i = \Delta y_1 + \Delta y_2 + \Delta y_3 + ... + \Delta y_n = y_n - y_0 \tag{25}$$

The sum of the absolute increases based on this chain ($\sum_{i=1}^{n}\Delta y_i$) equals the difference between the last term (y_n) and the reference term (y_0).

The effects of the governmental acquisitions modification on the production at balance in the economic cycles are presented below:

– in cycle 1 $\rightarrow \Delta y_1 = \Delta AG$;

– in cycle 2 $\rightarrow \Delta y_2 = \alpha \cdot \Delta AG$

– in cycle 3 $\rightarrow \Delta y_3 = \alpha^2 \cdot \Delta AG$

– in cycle 4 $\rightarrow \Delta y_4 = \alpha^3 \cdot \Delta AG$

– in cycle n $\rightarrow \Delta y_n = \alpha^{n-1} \cdot \Delta AG$

Summing up the effects in all the economic cycles one obtains:

$$\Delta y_1 + \Delta y_2 + \Delta y_3 + ... + \Delta y_n = \Delta AG + \alpha \cdot \Delta AG + \alpha^2 \cdot \Delta AG + ... + \alpha^{n-1} \cdot \Delta AG \tag{26}$$

Using the concise writing of the sum of the terms on the left and using ΔAG as common factor on the right of the relation one obtains:

$$\sum_{i=1}^{n}\Delta y_i = \left(1 + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots + \alpha^{n-1}\right)\cdot \Delta AG \tag{27}$$

The sum $\sum_{i=1}^{n}\Delta y_i = \left(1 + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots + \alpha^{n-1}\right)\cdot \Delta AG$ represents the total effect of the governmental acquisitions modification (ΔAG) on the revenue after “n” economic cycles.

Dividing the sum calculated above by ΔAG one obtains the sum of the terms of a geometric progression with an infinite number of positive terms below one (n→ ∞, $0 \langle \alpha \langle 1$):

$$\frac{\sum_{i=1}^{n}\Delta y_i}{\Delta AG} = \left(1 + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots + \alpha^{n-1}\right) \tag{28}$$

The ratio $\frac{\sum_{i=1}^{n}\Delta y_i}{\Delta AG}$ represents the value of the static multiplier k_s:

$$k_s = \frac{\sum_{i=1}^{n}\Delta y_i}{\Delta AG} = \left(1 + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots + \alpha^{n-1}\right) \tag{29}$$

$$k_s = \left(1 + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots + \alpha^{n-1}\right) \tag{30}$$

Taking into account that $\sum_{i=1}^{n} \Delta y_i = y_n - y_0$, one obtains another calculation relation of the static multiplier if one knows y_n and y_0:

$$k_s = \frac{y_n - y_0}{\Delta AG} \tag{31}$$

In the calculation formula of the static multiplier $k_s = \left(1 + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots + \alpha^{n-1}\right)$, the number of terms of the sum is considered infinite, and the value of "α" is positive and under one ($0 \langle \alpha \langle 1$). Analyzing the calculation formula, one can observe that as the exponent increases, the value of the power term decreases continually. The value of the power term tends to zero when the exponent "n" tends to plus infinite, hence, the relation: $\lim_{n \to \infty} \alpha^n = 0$

The level of the static multiplier (k_s) is calculated as a limit:

$$k_s = \lim_{n \to \infty} \left(1 + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots + \alpha^{n-1}\right) = \lim_{n \to \infty} \frac{1 - \alpha^n}{1 - \alpha} = \frac{1 - \lim_{n \to \infty} \alpha^n}{1 - \alpha} = \frac{1 - 0}{1 - \alpha} = \frac{1}{1 - \alpha} \tag{32}$$

For an infinite number of economic cycles one obtains:

$$k_s = \frac{1}{1 - \alpha}$$

Out of the results of the calculations realized, one can easily deduce:

$$k_{d1}=1,\ k_{d2}=\alpha,\ k_{d3}=\alpha^2,\ k_{d4}=\alpha^3, \ldots\ldots, k_{dn}=\alpha^{n-1} \tag{33}$$

$$k_s = 1 + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots + \alpha^{n-1}, \text{ unde } 0 \langle \alpha \langle 1 \tag{34}$$

$$k_s = k_{d1} + k_{d2} + k_{d3} + k_{d4} + \ldots\ldots\ldots\ldots + k_{dn} \tag{35}$$

$$k_s = \sum_{i=1}^{n} k_{di} \tag{36}$$

Summing up the dynamic multipliers of a period one obtains the static multiplier of the period.

For example, the static multiplier for three consecutive economic cycles is obtained using the relation below:

$$k_{s\,(1-3)} = k_{d1} + k_{d2} + k_{d3} \tag{37}$$

$$k_{s(1-3)} = 1 + \alpha + \alpha^2 \tag{38}$$

d) The number of economic cycles necessary (t) for the dynamic efficiency (k_{dn}) to represent a percentage β of the static efficiency (k_s) [3]

In general, for the cycle "n", the dynamic multiplier pertaining to it is: $k_{dn} = \alpha^{n-1}$

For "t" cycles, the condition enounced above must be met: $k_{dt}=\beta\cdot k_s$, $0< \beta <1$

In the general case, for known values of the parameters n, k_{dn}, k_s, β, the value of "t" is determined using the following calculations:

n……………..k_{dn}

t…………… $\beta \cdot k_s$ $\quad\rightarrow\quad$ $t = \dfrac{n\cdot\beta\cdot k_s}{k_{dn}}$ (39)

e) The number of economic cycles needed (t) for the modification of the governmental acquisitions to no longer generate effects in the economy [3]

In general, when the effects of the modification of the instrumental variable AG have ceased, for n, k_{dn}, k_s, the value of "t" is determined using the calculations below:

n……………..k_{dn}

t…………… k_s $\quad\rightarrow\quad$ $t = \dfrac{n\cdot k_s}{k_{dn}}$ (40)

4. CONCLUSIONS

The effect of the governmental acquisitions modification on the revenue at balance (Δy_n) and the level of the dynamic multiplier (k_{dn}) decrease as the number of economic cycles increases (n): $\Delta yn = \alpha^{n-1}\cdot\Delta AG$, $\quad k_{dn} = \dfrac{\Delta y_n}{\Delta AG} = \alpha^{n-1}$, $\quad 0\le\alpha\le 1$.

The static efficiency $k_s = \left(1+\alpha+\alpha^2+\alpha^3+\cdots\cdot+\alpha^{n-1}\right)$ increases less and less as the number of the economic cycles grows and tends after a very large number of cycles to the level $k_s = \dfrac{1}{1-\alpha}$.

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