ORIGINAL PAPER

A STUDY OF NUCLEAR STRUCTURE FOR $^{200}_{80}Hg_{120}$ ISOTOPE BY USING IBM-1 MODEL

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Abstract. *The Hg (A=200) isotope belong to the dynamical symmetry O(6). The (IBM-1.FOR) was used to study the energy levels and their transitions, where it was identified spin and parity for some energy levels is not exactly determined experimentally, as it predicted some of the energy levels and their transitions for some bands, it was assured that spin and parity for energy levels number (2) and it was predicted for energy levels number (6), the probability of electric transitions B (E2) values and reduced matrix elements* $\langle L_f || \hat{T}^{(E_2)} || L_i \rangle$ *for isotope under study was calculated by using the computer code (IBMT), In addition the calculate B(E2) ratio, As it has been found the intrinsic quadrupole moment* (Q_0)*, deformation parameter* β*, the potential energy surface (P.E.S), the square of rotational energy* $\hbar^2\omega^2$ *and the moment of inertia* $\frac{2\vartheta}{\hbar^2}$ *to clarify the shape of the nucleus.*

Keywords: *IBM-1, IBMT, Nuclear structure, Energy levels, The electric transitions, Potential energy surface, Square of rotational energy, Moment of inertia.*

1. INTRODUCTION

To comprehend the constructing of heavy and medium mass nuclei; therefore it must be seek refuge to rounding. Selection the typecasts rounding interdepend on the type of nuclei (its nature) in accordance with the presumption [1]. Arima and Iachello were proposed in 1974 a new nuclear model [2]; is (IBM) Interacting Boson Model of nuclear structure. IBM has been used widely for even-even nuclei to relate the congregational characteristics [3]. It was supposed that in heavy and medium even-even nuclei for low-lying collective states out of closed shells were controlled by the excitement of the valence neutrons and protons "i.e. particles outside the main closed shells at 2, 8, 20, 28, 50, 82, and 126" only; however the closed shell core is inactive [2].

The number of bosons depend on the number of effective nuclear particle (or hole) pairs outside a closed shell, while the total boson number (N) is calculated by adding the partial numbers *i.e.* $(N = N_\pi + N_\nu)$, where N_π and N_ν are the number of proton and neutron bosons respectively [4].

"Mercury isotopes belong to a very interesting but complex region of the periodic table known as the transition region". These cores are characterized with changing shape between deformed and spherical [3]. The nuclear installation for the Mercury region has certain difficult to explain in expressions of the conventional descriptions [5]. In the accounts by the IBM-1; these nuclei have been successfully treated in the O(6) symmetry of this model

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[6]. The Mercury has a large number of isotopes from Hg (A=171–210). These isotopes have 80 protons and neutron number from 91 to 130, respectively.

The aim of this study that; know all devote the nuclear structure of Hg (A=200) isotope to clarify the shape of the isotope under study and its properties by theoretical method and comparison with experimental results.

2. THEORETICAL BASICS

2.1. THE HAMILTONIAN OPERATOR OF THE (IBM-1)

By the IBM-1 Hamiltonian it can be expressed as [2, 7, 8]:

$$"\widehat{H} = \varepsilon_s\,(\hat{S}^{\dagger}.\,\hat{\tilde{S}}) + \varepsilon_d\,\left(\hat{d}^{\dagger}.\hat{\tilde{d}}\right) + \sum_{L=0,2,4}\frac{1}{2}\sqrt{2L+1}\,C_L\,\left[\left[\hat{d}^{\dagger}\times\hat{d}^{\dagger}\right]^{L}\times\left[\hat{d}\times\hat{\tilde{d}}\right]^{L}\right]^{0} + \frac{1}{\sqrt{2}}\,V_2\left[\left[\hat{d}^{\dagger}\times\hat{d}^{\dagger}\right]^{2}\times\left[\hat{\tilde{d}}\times\hat{\tilde{S}}\right]^{2} + \left[\hat{d}^{\dagger}\times\hat{S}^{\dagger}\right]^{2}\times\left[\hat{d}\times\hat{\tilde{d}}\right]^{2}\right]^{0} + \frac{1}{2}\,V_0\left[\left[\hat{d}^{\dagger}\times\hat{d}^{\dagger}\right]^{0}\times\left[\hat{\tilde{S}}\times\hat{\tilde{S}}\right]^{0} + \left[\hat{\tilde{S}}^{\dagger}\times\hat{\tilde{S}}^{\dagger}\right]^{0}\times\left[\hat{\tilde{d}}\times\hat{\tilde{d}}\right]^{0}\right]^{0} + U_2\left[\left[\hat{d}^{\dagger}\times\hat{S}^{\dagger}\right]^{2}\times\left[\hat{\tilde{d}}\times\hat{\tilde{S}}\right]^{2}\right]^{0} + \frac{1}{2}\,U_0\left[\left[\hat{S}^{\dagger}\times\hat{S}^{\dagger}\right]^{0}\times\left[\hat{\tilde{S}}\times\hat{\tilde{S}}\right]^{0}\right]^{0}" \quad (1)$$

The operators annihilation and Creation are ($\hat{\tilde{S}},\ \hat{\tilde{d}}$) and ($\hat{S}^{\dagger}, \hat{d}^{\dagger}$) to d and s- bosons; respectively [9]. Two expressions of one body interactions "ε_s and ε_d" and seven expressions of two-body interactions "$[C_L(L=0,2,4), V_L(L=0,2), U_L(L=0,2)]$" in this Hamiltonian; "where the single-boson energies are ε_s and ε_d; and the two-boson interactions had been described by C_L, V_L and U_L, and so on"; it shows that the number of a fixed boson N; just one of the one body expression and five of the two body expressions are freelance; It can be seen through a note (N) is the total number of bosons. Any way; it is more diffuse to write the Hamiltonian for the IBM-1 as a multipole extension, collected into various boson-boson interactions Equation (2) [7]:

$$"\widehat{H} = \varepsilon\,\hat{n}_d + a_0(\hat{p}.\hat{p}) + a_1\left(\hat{L}.\hat{L}\right) + a_2\left(\hat{Q}.\hat{Q}\right) + a_3\left(\hat{T}_3.\hat{T}_3\right) + a_4\left(\hat{T}_4.\hat{T}_4\right)" \quad (2)$$

Where:

$$"\hat{n}_d = \left(\hat{d}^{\dagger}.\hat{\tilde{d}}\right)" \text{ "is the total number of the } d_{boson} \text{ operator"} \quad (3)$$

$$"\hat{p} = 1/2\left[\left(\hat{\tilde{d}}.\hat{\tilde{d}}\right) - (\hat{\tilde{S}}.\hat{\tilde{S}})\right]" \text{ "is the pairing operator"} \quad (4)$$

$$"\hat{L} = \sqrt{10}\left[\hat{d}^{\dagger}\times\hat{\tilde{d}}\right]^{1}" \text{ "is the angular momentum operator"} \quad (5)$$

$$"\hat{Q} = \left[\hat{d}^{\dagger}\times\hat{\tilde{S}} + \hat{S}^{\dagger}\times\hat{\tilde{d}}\right]^{(2)} + \chi\left[\hat{d}^{\dagger}\times\hat{\tilde{d}}\right]^{(2)}" \text{ "is the quadrupole operator"} \quad (6)$$

"Where χ is the quadrupole structure parameter and take the values 0 and $\pm\frac{\sqrt{7}}{2}$ " [7, 9].

"$\hat{T}_r = \left[\hat{d}^\dagger \times \hat{\tilde{d}}\right]^{(r)}$ " "is the octoupole ($r = 3$) and hexadecapole ($r = 4$) operator" (7)

And

"$\varepsilon = \varepsilon_d - \varepsilon_s$" "is the boson energy". (8)

The parameters a_0, a_1, a_2, a_3 and a_4 designated the strength of the pairing, angular momentum, quadrupole, octupole and hexadecapole interaction between the bosons.

2.2. ELECTROMAGNETIC TRANSITIONS

Electromagnetic transitions rates had been described by IBM as well, besides excitation energy spectra. One has to specify the transition operators in terms of the boson operators, in order to do so [10]. In lowest order, the transition operators supposedly it will contain only one-body terms. Clearly, that the most general form of such an operator in IBM-1 can be given by [7, 9, 10]:

"$T_m^l = \alpha_2 \delta_{12}\left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]_m^2 + \beta_l\left[d^\dagger \times \tilde{d}\right]_m^l + \gamma_0 \delta_{10}\delta_{m0}[s^\dagger \times \tilde{s}]_0^0$" (9)

"It is clear that the first term can be presented only in the case of $l = 2$ transitions, while the last term can be presented only in the case of $l = 0$ transitions". "This is assured by Kronecker delta (δ) accompanying them". "In the special cases of electric monopole; quadrupole and hexadecapole transitions; the specific form of the transition operator is respectively; γ_0, α_2 and $\beta_l (l = 0, 1, 2, 3, 4)$ which are parameters specifying the various terms in the corresponding operators. Then the electric quadrupole transition is":

"$T_m^{E2} = \alpha_2\left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]_m^2 + \beta_2\left[d^\dagger \times \tilde{d}\right]_m^2 = \alpha_2\left(\left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]_m^2 + \chi\left[d^\dagger \times \tilde{d}\right]_m^2\right) = e_B \hat{Q}$" (10)

The α_2 and β_2 are two parameters. The "$\beta_2 = \chi\alpha_2, \alpha_2 = e_B$" "effective charge of boson" "and the quadrupole operator $\hat{Q}$" show in equation (6).

It can calculate the electromagnetic transition average in the usual method; by taking the reduced matrix element between the initial and the final state for the corresponding transition operator. "The symbol for this reduced matrix element is $\langle L_f || T^l || L_i \rangle$" [11, 12]. Then, by definition, the $B(El)$ values are:

"$B\left((El); L_i \to L_f\right) = \frac{1}{2L_i+1}\left|\langle L_f || T^{(El)} || L_i \rangle\right|^2$" (11)

Where
L_i is the initial angular momentum.
L_f is the final angular momentum.

"In the general case, the calculation has been done numerically. However, as in the case of the excitation energies, analytical expression can be found in the cases of the three dynamical symmetries".

2.3. B(E2) RATIO

The (Branching ratios) of the reduced transition probabilities for electric quadrupole transition of the dynamical symmetry U(5), SU(3) and O(6) obeys the following relations [13]:

$$R = \frac{B(E2;4_1^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)}, R' = \frac{B(E2;2_2^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)}, R'' = \frac{B(E2;0_2^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)} \quad (12)$$

In addition, they can be written in terms of the total boson number [13]:

$$R = R' = R'' = \frac{2(N-1)}{N} \to 2_{N\to\infty} \quad \text{In U(5) limit} \quad (13)$$

$$R = \frac{10}{7}\frac{(N-1)(2N+5)}{N(2N+3)} \to_{N\to\infty} \frac{10}{7};\ R' = R'' = 0 \quad \text{In SU(3) limit} \quad (14)$$

$$R = R' = \frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)} \to_{N\to\infty} \frac{10}{7};\ R'' = 0 \quad \text{In O(6) limit} \quad (15)$$

where N is the total boson number.

2.4. BACKBENDING CURVE

"The collective rotation motion for nucleus depends on the valence nucleons motion with nucleus motion, which causes a rotation numeral of nucleons around axis different from the nuclear symmetry axis". "In some nuclei occurs a sudden change in the value of the moment of inertia at the high angular momentum relatively that leads to landing in the rotational energy of the nuclei". "These sudden changes lead to occur (curvatures) like letter Z inverses" [14]. The formula for calculating all the moment of inertia and the square of rotational energy are [15, 16]:

$$"\frac{2\vartheta}{\hbar^2} = \frac{4L-2}{E(L)-E(L-2)} = \frac{4L-2}{E_\gamma}(MeV)^{-1}" \quad (16)$$

"The ϑ is the moment of inertia and E_γ is the transition energy".

$$"(\hbar\omega)^2 = \left(\frac{E(L)-(E(L-2)}{\sqrt{L(L+1)}-\sqrt{(L-2)(L-1)}}\right)^2 (MeV)^2" \quad (17)$$

2.5. THE POTENTIAL ENERGY SURFACE (P.E.S)

The potential energy surface $E(N,\beta,\gamma)$ gives a final shape to the nucleus that corresponds to the function of Hamiltonian; as the equation [7- 9, 17]:

$$"E(N,\beta,\gamma) = \langle N,\beta,\gamma|H|N,\beta,\gamma\rangle/\langle N,\beta,\gamma|N,\beta,\gamma\rangle" \quad (18)$$

"To create the IBM, energy surface had used the expectation value of the IBM-1 Hamiltonian with the coherent state $(|N,\beta,\gamma\rangle)$" [9, 18].

"The state is a product of boson creation operators $\left(b_c^{\dagger}\right)$"; with

$$"|N,\beta,\gamma\rangle = 1/\sqrt{N!}\left(b_c^{\dagger}\right)^N|0\rangle" \quad (19)$$

$$"b_c^{\dagger} = (1+\beta^2)^{-1/2}\left\{s^{\dagger} + \beta\left[\cos\gamma\left(d_0^{\dagger}\right) + \sqrt{1/2}\sin\gamma\left(d_2^{\dagger} + d_{-2}^{\dagger}\right)\right]\right\}" \quad (20)$$

"The energy surface, as a function of β and γ, has been given by" [9]:

$$"E(N,\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2}(\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4)" \quad (21)$$

The "α_i's" "are related to the coefficients C_L, v_2, v_0, u_2 and u_0 of Eq. (2-1). β is a measure of the total deformation of nucleus. The shape is spherical when $\beta = 0$, and when $\beta \neq 0$ be distorted, γ correlates with the nucleus and it's the amount of deviation from the focus symmetry, the shape is prolate when $\gamma = 0$, and the shape becomes oblate when $\gamma = 60$" [19].

The potential energy surface for three dynamical symmetries represents by the following equations [4]:

$$"E(N,\beta,\gamma) = \varepsilon_d\, N\frac{\beta^2}{1+\beta^2}" \quad \text{"in U(5) limit"} \quad (22)$$

$$"E(N,\beta,\gamma) = KN(N-1)\frac{1+\frac{3}{4}\beta^4-\sqrt{2}\beta^2\cos 3\gamma}{(1+\beta^2)}" \quad \text{"in SU(3) limit"} \quad (23)$$

$$"E(N,\beta,\gamma) = K'N(N-1)\left[\frac{1-\beta^2}{1+\beta^2}\right]" \quad \text{"in O(6) limit"} \quad (24)$$

where K, K' are the same as in equation (2).

3. RESULTS AND DISCUSSION

3.1. ENERGY RATIOS AND DYNAMICAL SYMMETRY

Energy ratios considered as the first and most important step to determine the type of nuclei and its dynamical symmetry, it is compared to the experimental energy ratios for the isotope under study with identical energy ratios of corresponding limits as shown in table 1.

Table 1. The energy ratios of corresponding limits [4, 7, 9].

Limit	$R4 = E(4_1^+)/E(2_1^+)$	$R6 = E(6_1^+)/E(2_1^+)$	$R8 = E(8_1^+)/E(2_1^+)$
U(5)	2	3	4
SU(3)	3.33	7	12
O(6)	2.5	4.5	7

Table 2 shows the energy ratios for Hg (A=200) isotope, which showing that the isotope under study belong to the O(6) γ- Unstable limit, as elucidated the comparison between the theoretical and experimental energy ratios for this isotope.

Table 2. Theoretical energy ratios values according to the (IBM-1) and their comparison with the experimental data for Hg (A=200) isotope.

Isotope	$R4 = E(4_1^+)/E(2_1^+)$		$R6 = E(6_1^+)/E(2_1^+)$		$R8 = E(8_1^+)/E(2_1^+)$	
$^{200}_{80}Hg_{120}$	EXP. [8, 20]	IBM-1	EXP. [8, 20]	IBM-1	EXP. [8, 20]	IBM-1
	2.5744	2.4235	4.6387	4.2704	7.2837	6.5408

The equation of the Hamiltonian operator function for dynamical symmetry O(6) is shown in equation (25) [7, 9].

$$\hat{H} = a_0 \hat{P}^{\dagger}.\hat{P} + a_1 \hat{L}^2 + a_3 \hat{T}_3^2 \quad \ldots. (25)$$

a_0, a_1 and a_3 are phenomenological parameters, $CHI = 0$ and $SO6 = 1$ for O(6) dynamical symmetry.

3.2. ENERGY LEVELS AND ENERGY TRANSITIONS

To understand the nuclear structure clearly for any isotope, we should at start study the energy levels for this isotope, where for each dynamical symmetry there is an equivalent equation of the Hamiltonian operator function, which is used to calculate the energy levels, their ratios, their transitions and the (g, β, γ)– energy band.

Table 3 shows the parameter values used of Hamiltonian operator to obtain the best fitting with the convenient dynamical symmetry.

Table 3. The parameter values of Hamiltonian operator for the Hg (A=200) isotope by using (IBM-1. For) program.

Isotope	N_π	N_ν	N	EPS (Mev)	$\hat{P}^{\dagger}.\hat{P}$ (Mev)	$\hat{L}.\hat{L}$ (Mev)	$\hat{Q}.\hat{Q}$ (Mev)	$\hat{T}_3.\hat{T}_3$ (Mev)	$\hat{T}_4.\hat{T}_4$ (Mev)	CHI (Mev)	$SO6$ (Mev)
$^{200}_{80}Hg_{120}$	1	3	4	0.0000	0.2058	0.0154	0.0000	0.2140	0.0000	0.0000	1.0000

Table 4 elucidate the energy levels and their transitions were getting from (IBM-1.For) program for Hg (A=200) isotope. There is good agreement between the theoretical results by using aforementioned program and experimental data for isotope under study.

Table 4. Theoretical energy levels for (g, β, γ-bands) and energy transitions compared with the experimental data for the $^{200}_{80}Hg_{120}$ isotope by using (IBM-1. For) program.

Isotope	I_i^+	Energy level (MeV)		Spin sequences $I_f^+ - I_i^+$	Transition Energy (MeV)	
		Exp.[8, 20]	IBM-1		Exp.[20]	IBM-1
$^{200}_{80}Hg_{120}$ O(6)	$0_1^+(g)$	0.0000	0.0000	–	–	–
	$2_1^+(g)$	0.3679	0.3920	$2_1^+ - 0_1^+$	0.3679	0.3920
	$4_1^+(g)$	0.9472	0.9500	$4_1^+ - 2_1^+$	0.5793	0.5580
	$0_2^+(\beta_1)$	1.0293	1.0290	$0_2^+ - 2_1^+$ $0_2^+ - 0_1^+$	0.6614 1.0293	0.6370 1.0290
	$2_2^+(\gamma_1)$	1.2541	1.0340	$2_2^+ - 0_2^+$ $2_2^+ - 4_1^+$ $2_2^+ - 2_1^+$ $2_2^+ - 0_1^+$	0.2247 0.3069 0.8862 1.2541	0.0050 0.0840 0.6420 1.0340
	$2_3^+(\beta_1)$	(1.5737)	1.4210	$2_3^+ - 2_2^+$ $2_3^+ - 0_2^+$ $2_3^+ - 4_1^+$ $2_3^+ - 2_1^+$ $2_3^+ - 0_1^+$	0.3196 0.5442 0.6265 1.2057 1.5736	0.3870 0.3920 0.4710 1.0290 1.4210
	$0_3^+(\beta_2)$	1.5152	1.6464	$0_3^+ - 0_2^+$ $0_3^+ - 2_1^+$ $0_3^+ - 0_1^+$	0.4856 1.1472 1.5150	0.6174 1.2544 1.6464
	$6_1^+(g)$	1.7067	1.6740	$6_1^+ - 4_1^+$	0.7595	0.7240
	$4_2^+(\beta_1)$	–	1.8060	–	–	–
	$3_1^+(\gamma_1)$	1.6590	1.8540	$3_1^+ - 2_2^+$ $3_1^+ - 4_1^+$ $3_1^+ - 2_1^+$	0.4049 0.7117 1.2911	0.8200 0.9040 1.4620
	$0_4^+(\beta_3)$	1.8568	1.9260	$0_4^+ - 0_3^+$ $0_4^+ - 2_2^+$ $0_4^+ - 0_2^+$ $0_4^+ - 2_1^+$ $0_4^+ - 0_1^+$	0.3418 0.6027 0.8275 1.4885 1.8568	0.2796 0.8920 0.8970 1.5340 1.9260
	$4_3^+(\gamma_1)$	–	1.9790	–	–	–
	$2_4^+(\beta_2)$	(2.2889)	2.0630	$2_4^+ - 2_2^+$ $2_4^+ - 4_1^+$ $2_4^+ - 2_1^+$ $2_4^+ - 0_1^+$	1.0349 1.3417 1.9211 2.2896	1.0290 1.1130 1.6710 2.0630
	$8_1^+(g)$	2.6796	2.5640	$8_1^+ - 6_1^+$	0.9729	0.8900
	$6_2^+(\beta_1)$	–	2.7440	–	–	–
	$5_1^+(\gamma_1)$	–	2.8160	–	–	–
	$4_4^+(\beta_2)$	–	2.8760	–	–	–
	$2_5^+(\beta_3)$	–	2.9600	–	–	–

3.3. (g, β, γ) – ENERGY BANDS SPECTRUM

Been classified the energy levels of Hg (A=200) isotope, according to the energy bands (g, β, γ)– bands.

Table 5 elucidate the energy band values for Hg (A=200) isotope and their comparison with the available experimental data, the results shows that, there is good agreement for the

energy levels resulting from (IBM-1.For) program was used in the present work comparison with the experimental results, where it was assured some the energy levels of the spin and parity $(2_3^+, 2_4^+)$ were not exactly determined in experimental data to be (1.5737, 2.2889) respectively, by using the mentioned program. In the experimental results there are many the energy levels, but the spin and parity have been confirmed unknown, it was chosen as the most suitable for the existing energy bands. As it predicted some energy levels for spins $(4_2^+, 4_3^+, 6_2^+, 5_1^+, 4_4^+, 2_5^+)$ to be (1.8060, 1.9790, 2.7440, 2.8160, 2.8760, 2.9600) respectively.

Table 5. The comparison between the theoretical (PW) and experimental [8, 20] energy bands (g, β, γ-bands) for the $^{200}_{80}Hg_{120}$ isotope by using IBM-1.

Isotopes	Spin / Band	0^+ 2^+	2^+ 3^+	4^+ 4^+	6^+ 5^+	8^+ 6^+
$^{200}_{80}Hg_{120}$ O(6)	$g-exp.$	0.00000	0.3679	0.9472	1.7067	2.6796
	$g-pw$	0.00000	0.3920	0.9500	1.6740	2.5640
	$\beta_1-exp.$	1.0293	(1.5737)	–	–	–
	β_1-pw	1.0290	1.4210	1.8060	2.7440	–
	$\gamma_1-exp.$	1.2541	1.6590	–	–	–
	γ_1-pw	1.0340	1.8540	1.9790	2.8160	–
	$\beta_2-exp.$	1.5152	(2.2889)	–	–	–
	β_2-pw	1.6464	2.0630	2.8760	–	–
	$\beta_3-exp.$	1.8568	–	–	–	–
	β_3-pw	1.9260	2.9600	–	–	–

Fig. 1 shows the energy levels for the Hg (A=200) isotope, where that obey the typical energy bands spectrum. Noticed that, there is good agreement of the level sequences of each band with the typical sequence of g −band $(0^+, 2^+, 4^+ \ldots\ldots)$, β −band $(0^+, 2^+, 4^+ \ldots\ldots)$ and γ −band $(2^+, 3^+, 4^+ \ldots\ldots)$. There are good agreement between the present results and the experimental values of the energy bands.

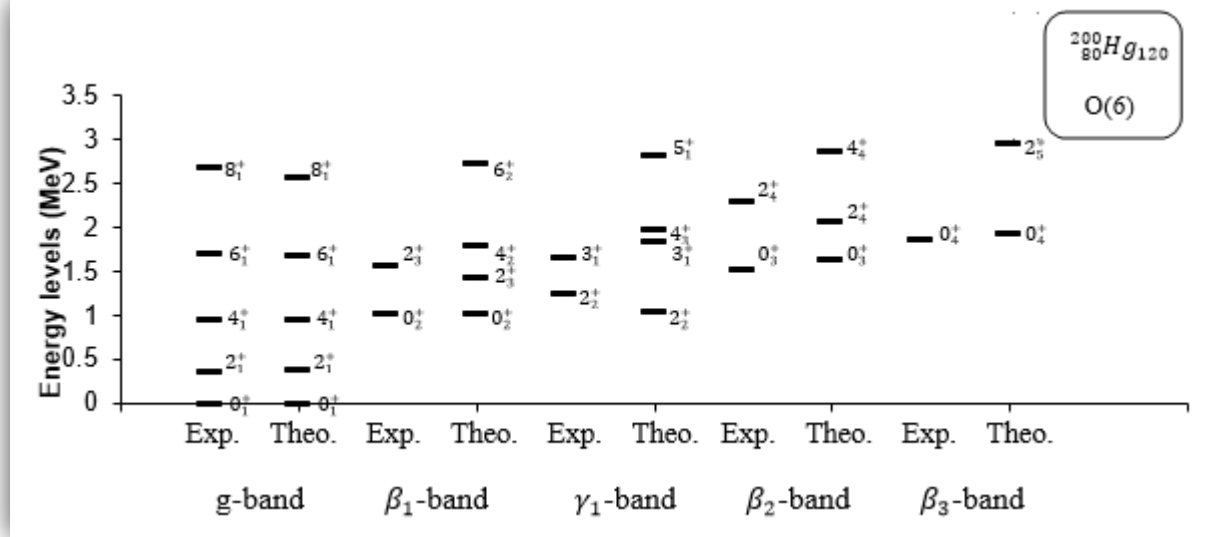

Figure 1. Comparison between calculated IBM (PW); and experimental [8, 20] energy band states (g, β, γ)– bands in Hg (A=200) isotope of the dynamical symmetry O(6).

3.4. REDUCED MATRIX ELEMENTS & THE PROBABILITY OF ELECTRIC TRANSITIONS

Much information can be obtained by studying the reduced matrix elements and the probability of electric transitions B(E2). The computer code IBMT was used to calculate the B(E2) values, and it must specify values of effective charge (e_B). The values of effective charge $e_B(\alpha_2)$ were estimated to reproduce the experimental B(E2) and it is tabulated in table 6.

The other parameters were used in the program $(\beta_2, SO6)$ equal (0 – 1) respectively, for O(6) dynamical symmetry.

Table 6. The values of the parameters $(\alpha_2, \beta_2, SO6)$ of the B(E2) and $\langle L_f || \hat{T}^{(E_2)} || L_i \rangle$ for Hg (A=200) isotope by using (IBST.FOR) program.

Isotope	N_π	N_ν	N	$\alpha_2\ (eb)$	$\beta_2\ (eb)$	SO6
$^{200}_{80}Hg_{120}$	1	3	4	0.1633	0.0000	1.0000

The comparison of the experimental data [8] with calculations of B(E2) values are given in table 7 for isotope under study. There are no available experimental transition data to many transitions in table 7. Therefore, it has been predicted by using IBMT. This table shows that in general, there are a good agreement between the experimentally B(E2) values and the (IBMT) calculated ones.

Table 7 show that, probabilities of electric transitions (up and down) B(E2)↓, B(E2)↑ Respectively, in $(eb)^2$ units, where: (eb) Electron barn.

The reduced matrix elements $\langle \boldsymbol{L_f} || \hat{\boldsymbol{T}}^{(\boldsymbol{E_2})} || \boldsymbol{L_i} \rangle$ the amount associated with the probability electric transitions B(E2), It can be found them by using equation (11), and for each electric transition for this isotope. It is measured in $(\boldsymbol{eb})$ units.

Table 7. Theoretical values of reduced matrix elements $\langle L_f||\hat{T}^{(E_2)}||L_i\rangle$ and Probabilities of electric transitions B(E2) compared with the available experimental data for the Hg (A=200) isotope.

Isotope	Spin sequences $L_f^+ - L_i^+$	B(E2)↓ $(eb)^2$		B(E2)↑ $(eb)^2$		$\langle L_f\|\|\hat{T}^{(E_2)}\|\|L_i\rangle$ eb
		IBM-1	EXP.[8]	IBM-1	EXP.[8]	
$^{200}_{80}Hg_{120}$	$2_1^+ - 0_1^+$	0.1707	0.1707	0.8585	0.8585	0.9265
	$2_2^+ - 0_1^+$	0.0000	0.0012	0.0000	0.0060	0.0774
	$2_3^+ - 0_2^+$	0.0640	----	0.3200	----	0.5657
	$0_2^+ - 2_1^+$	0.0000	----	0.0000	----	0.0000
	$0_2^+ - 2_2^+$	0.0000	----	0.0000	----	0.0000
	$2_2^+ - 2_1^+$	0.2057	----	1.0285	0.1040	2.2677
	$2_4^+ - 2_3^+$	0.0533	----	0.2665	----	1.1543
	$2_1^+ - 4_1^+$	0.3703	----	1.8515	----	4.0821
	$2_2^+ - 4_3^+$	0.1676	----	0.8380	----	2.7463
	$2_3^+ - 4_2^+$	0.0960	----	0.4800	----	2.0785
	$3_1^+ - 4_1^+$	0.0508	----	0.2540	----	1.5119
	$4_1^+ - 2_1^+$	0.2057	0.2626	1.0285	1.3130	2.2677
	$4_2^+ - 2_3^+$	0.0533	----	0.2665	----	1.1543
	$4_3^+ - 2_2^+$	0.0931	----	0.4655	----	1.5256
	$4_1^+ - 3_1^+$	0.0395	----	0.1975	----	1.1758
	$4_4^+ - 3_1^+$	0.0565	----	0.2825	----	1.4062
	$4_3^+ - 4_1^+$	0.0847	----	0.4235	----	1.9523
	$4_3^+ - 5_1^+$	0.0311	----	0.1555	----	1.3079
	$6_1^+ - 4_1^+$	0.2568	0.3196	1.2840	1.5980	3.3994
	$4_3^+ - 6_2^+$	0.1051	----	0.5255	----	2.6137
	$5_1^+ - 3_1^+$	0.0560	----	0.2800	----	1.4000
	$5_1^+ - 6_1^+$	0.0252	----	0.1260	----	1.2798
	$8_1^+ - 6_1^+$	----	0.2848	----	1.4240	4.3025

3.5. B(E2) RATIO

The B(E2) ratio considered from the important quantities to find out the dynamical symmetry and therefore study the nuclear structure for any isotope. The B(E2) ratio shows that the $^{200}_{80}Hg_{120}$ isotope is deformed nucleus and it belongs to the dynamical symmetry O(6). The formulas for calculating the B(E2) ratio are shown in equation (15).

Table 8 shows the comparison between the B(E2) ratio for Mercury isotope ($^{200}_{80}Hg_{120}$) by using IBMT program with the B(E2) ratio for O(6) dynamical symmetry.

Table 8. The IBMT and O(6) dynamical symmetry [21] values of B(E2) ratio of the Hg (A=200) isotope.

Isotope		$^{200}_{80}Hg_{120}$
N		4
IBMT	R	1.2050
	R′	1.2050
	R″	0.0000
O(6)	R	$< \frac{10}{7}$
	R′	$< \frac{10}{7}$
	R″	0

3.6. INTRINSIC QUADRUPOLE MOMENT AND DEFORMATION PARAMETER

By using equation (26), we can find the intrinsic quadrupole moment (Q_0), which can be derived from the transition rate B(E2)↑ values.

$$Q_0 = [(16\pi/5)B(E2)\uparrow/e^2]^{1/2} barn \tag{26}$$

where:

$$B(E2\uparrow; L-2 \rightarrow L) = \frac{2L+1}{2(L-2)+1} B(E2\downarrow; L \rightarrow L-2) \tag{27}$$

From other quantities depends on the value of B(E2), that the deformation parameter β, which can be calculated according from the equation (28).

$$\beta = \left(4\pi/\left(3ZR_0{}^2\right)\right)[B(E2)\uparrow/e^2]^{1/2} \tag{28}$$

where:

$$R_0{}^2 = 0.144A^{2/3} barn \tag{29}$$

Note that $Q_0, R_0{}^2$ measured with barn unit. The intrinsic quadrupole moment (Q_0) and the deformation parameter β consider from the important quantities to determine the shape of the nuclei. Table 9 presents the calculations of intrinsic quadrupole moment Q_0 and deformation parameter β within the framework of IBMT for the even – even $^{200}_{80}Hg_{120}$ isotope. The presented result for Q_0 is consistent with the expectations and from phenomenological systematic, and is compared with experimental result [8].

Table 9. The IBM-1 values and the experimental data [8] of Q_0 and β for Hg (A=200) isotope.

Isotope		$^{200}_{80}Hg_{120}$
$Q_0(b)$	IBM-1	2.9370
	EXP.	2.9370
β	IBM-1	0.0098
	EXP.	0.0098

All other quadrupole moments $(Q_L = 0)$ in O(6) limit, because of the selection rule $(\Delta\tau = \pm 1)$.

3.7. THE SQUARE OF ROTATIONAL ENERGY AND THE MOMENT OF INERTIA

"In some nuclei occurs a sudden change in the value of the moment of inertia at the high angular momentum that relatively leads to landing in the rotational energy of the nuclei. These sudden changes lead to occur" "curvatures" in the field of energy with angular momentum. When we draw the relationship between moment of inertia and the square of rotational energy, the change clearly shows the shape of the letter Z inverses. After finding the energy levels using IBM-1 and angular moment to the yrast energy levels, the square of rotational energy and the moment of inertia can be calculated from equations (16) and (17).

Fig. 4 shows the relation of the moment of inertia as a function of the square of the energy (backbending curve) of the emitted photon for all Hg isotopes under study. When the nuclei transform from the (L) state of the (L-2) state, the even-even Hg isotopes do not suffer from any curvature which indicates the absence of change in their properties. The comparison between the theoretical and the experimental values [8, 21-25] for all square rotational energy and the moment of inertia are shown in Fig. 4.

Table 1. Comparison between the IBM-1 and experimental [8, 20] values for square of rotational energy & moment of inertia for Hg (A=200) isotope.

L	$\hbar^2\omega^2$		$\frac{2\vartheta}{\hbar^2}$	
	IBM-1	EXP.	IBM-1	EXP.
2	0.025610666	0.022558401	15.30612245	16.30691794
4	0.07610769	0.08202811	25.08960573	24.16709822
6	0.129923635	0.142965838	28.96642528	28.96642528
8	0.197128885	0.235557931	33.70786517	30.83564601

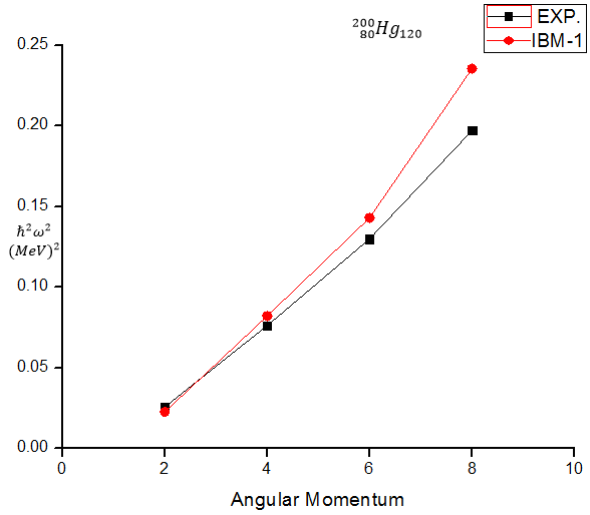

Figure 2. The relation between $\hbar^2\omega^2$ as a function of angular momentum for Hg (A=200) isotope. The experimental data are taken from [8, 20].

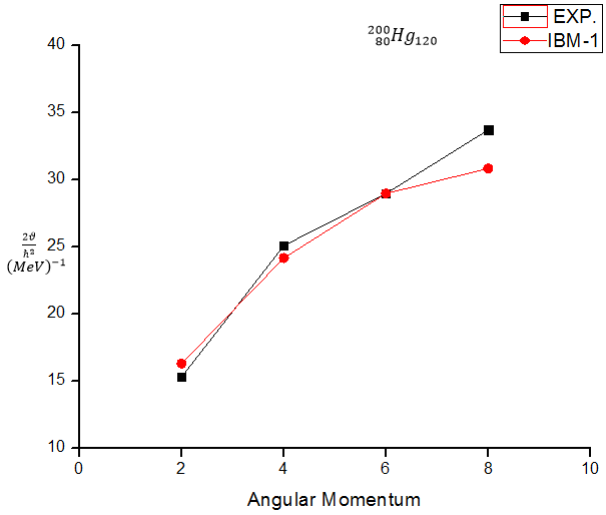

Figure 3. The relation between $\frac{2\vartheta}{\hbar^2}$ as a function of angular momentum for Hg (A=200) isotope. The experimental data are taken from [8, 20].

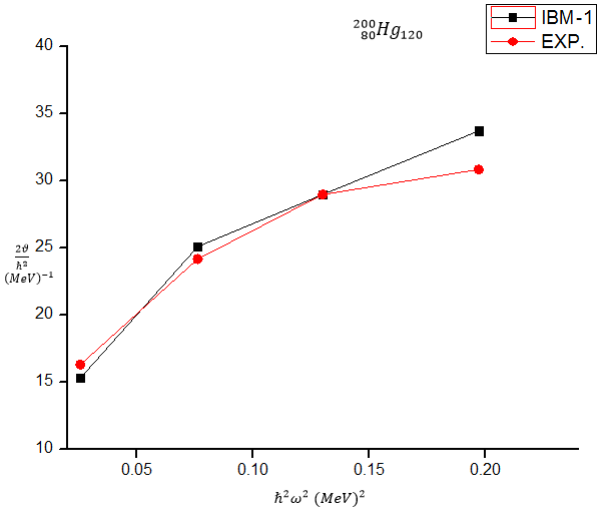

Figure 4. The calculated and observed moment of inertia $\frac{2\vartheta}{\hbar^2}$ vs. $\hbar^2\omega^2$ for yrast levels of Hg (A=200) isotope. The experimental data are taken from [8, 20].

3.8. THE POTENTIAL ENERGY SURFACE (P.E.S)

The potential energy surface is one of the nuclei properties, and it gives a final shape of nuclei. The PES.FOR program is used to calculate the potential energy surface $E(N,\beta,\gamma)$. In this work, we calculate the potential energy surface from equations (21) and (24). In the figure 5, the contour plots in the γ-β plane, resulting from $E(N,\beta,\gamma)$ are shown for 190-198Hg isotopes. For most of the considered Hg nuclei the mapped IBM energy surfaces are triaxial shape. Triaxial shape is associated with intermediate values $0 < \gamma < \pi/3$. The triaxial deformation helps to understand the prolate-to-oblate shape transition that occurs in the considered Hg isotopes. The Hg nuclei considered here do not display any rapid structural change, but remain γ-soft. This evolution reflects the triaxial deformed as one approaches the neutron shell closure N = 126.

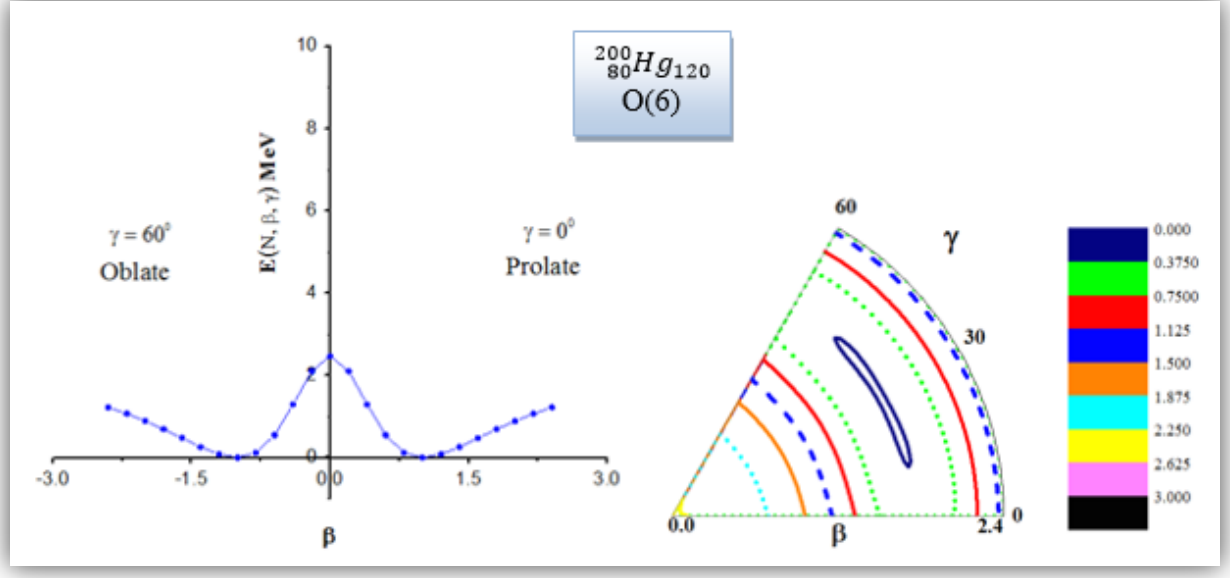

Figure 5. The Contour plots and the axial symmetric for the Hg (A=200) isotope of the dynamical symmetry O(6).

4. CONCLUSIONS

From the present work, we can conclude the following:

- Determine the dynamical symmetry to the isotope under study, Show that, the $^{200}_{80}Hg_{120}$ isotope belongs to the dynamical symmetry O(6).
- Calculate the energy levels, their ratios and transition energies using IBM-1 for $^{200}_{80}Hg_{120}$ isotope is in a good agreement with the experimental values.
- Determining spin and parity for some energy levels, which not exactly determined experimentally and prediction to some other energy levels is not identified experimentally.
- Calculate the probability of electric transitions (up and down) (B(E2)↓ and B(E2)↑) values, respectively and compared with available experimental results for isotope under study.
- Find the reduce matrix elements $\langle L_f || \hat{T}^{(E_2)} || L_i \rangle$ the associated with the probability electric transitions B(E2).

- Calculate the B(E2) ratios (R, R', R'') for $^{200}_{80}Hg_{120}$ isotope and comparing with B(E2) ratio O(6) dynamical symmetry.
- Calculate the value of intrinsic quadrupole moment Q_0 and the deformation parameter β depending B(E2)↑ for determining the shape of the nucleus.

REFERENCES

[1] Cohen, B.L., *Concepts of Nuclear physics*, Mc Grow-Hill Book Company, New York, 1971.
[2] Abrahams, K., Allaart, K., Dieperink A.E.L., *Nuclear Structure*, Plenum press, New York and London, 1981.
[3] Bijker, R., Scholten, O., *Phys. Rev.* C**32**, 591, 1985.
[4] Bonatsos, D., *Interacting Boson Models of Nuclear Structure*, Pub. Oxford University Press, New York, 1988.
[5] Yates, S.W., Baum, E.M., Henry, E.A., Mann, L.G., Roy, N., Aprahamian, A., Meyer, R.A., Estep, R., *Phys. Rev.* C**37**, 5, 1988.
[6] Casten, R.F., Cizewski, J.A., *Nucl. Phys.* A**309**, 477, 1978.
[7] Iachello, F., Arima, A., *Interacting boson model*, Pub. Cambridge University Press, Cambridge, England, 1987.
[8] http://www.nndc.bnl.gov/chart/getENSDFD.
[9] Casten, R.F., Warner, D.D, *Rev. Mod. Phys.* **60**, 389, 1988.
[10] Sethi, A., Hintz, N.M., Mihailidis, D.N., Mack, A.M., Gazzaly, M., *Phys. Rev.* C**44**, 700, 1991.
[11] Arima, A., Iachello, F., *Ann. Phys.* (NY) **111**, 201, 1978.
[12] Krane, K.S., Shobaki, J.M., *Phys. Rev.* C**16**, 4, 1977.
[13] Krane, K.S., Steffen, R.M., *Phys. Rev.* C**2**, 724, 1970.
[14] Johnson, A., Ryde, H., Sztarkier, J., *Phys. Lett. B***43**, 605, 1971.
[15] Sorenson, R.A., *Rev. Mod. Phys.*, **45**, 353, 1973.
[16] Wong, S.M., *Introductory Nuclear Physics*, Prentice-Hall International, Inc., 1990.
[17] Hamilton, W.D., *The Electromagnetic Interaction in Nuclear Spectroscopy*, American Elsevier Publishing Company, New York, 1975.
[18] Sharrad, F.I., Abdullah, H.Y., AL-Dahan, N., Umran, N.M., Okhunov, A.A., Abu Kassim, H., *Chinese Physics C*, **37**, 034101, 2013.
[19] Hollas, C.L., Aniol, K.A., Gebbie, D.W., *Nucl. Phys. A*, **276**, 1, 1977.
[20] Sakai, M., *Atomic Data and Nuclear Data Tables*, **31**(3), 399, 1984.
[21] Xiaolong, H., *Nucl. Data Sheets*, **110**, 2533, 2009.
[22] Singh, B.J., *Nucl. Data Sheets*, **99**, 275, 2003.
[23] Baglin, C.M., *Nucl. Data Sheets*, **84**,717, 1998.
[24] Singh, B.J., *Nucl. Data Sheets*, **107**, 1531, 2006.
[25] Xiaolong, H., *Nucl. Data Sheets*, **108**, 1093, 2007.