ORIGINAL PAPER

ENERGY LEVELS, REDUCED MATRIX ELEMENTS &THE PROBABILITY OF ELECTRIC TRANSITIONS FOR $^{248}_{96}Cm_{152}$ ISOTOPE

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Abstract. *The Cm (A=248) isotope belong to SU(3) dynamical symmetry. Energy levels were calculated by using (IBM-1.For), there is a good agreement with the experimental results. As confirmation one energy level and prediction for energy levels number (6). The computer code (IBMT) was used to study the probability of electric transitions B(E2) for $^{248}_{96}Cm_{152}$ isotope, As it was calculated reduced matrix elements $\langle L_f||\hat{T}^{(E_2)}||L_i\rangle$ for isotope under study. In addition to that it has been found the electric quadrupole moments (Q_L) to clarify the shape of the nucleus.*

Keywords: *IBM-1, IBMT, Energy levels, The electric transitions, B(E2) ratio, Reduce matrix elements, Quadrupole moments.*

1. INTRODUCTION

Throughout history in nuclear physics, there are a lot of attempts to model the characteristic structure of nuclei. The amount of data has been generated, which it is need for more models of nuclear structure to reach it. It has been associated with the calculation ability. Much of work has been done on studying the structure and characteristics of medium and heavy mass nuclei, which suggest some kinds of nuclear model. To understand the structure of medium- and heavy-mass nuclei, it must therefore be resorted to approximations. The selection of a suitable approximation depends on the nature of nuclei in accordance with supposition [1]. In 1974 a new nuclear model was suggested by Arima and Iachello, Interacting Boson Model (IBM) of nuclear structure [2-5].

The interacting boson model (IBM) is suitable for describing the low-lying collective states in even – even nuclei by a system of interacting s-boson and d-boson carrying angular momentum's 0 and 2 respectively [6]. The IBM is built on a closed shell i.e. the total number of bosons (N) depends on the number of active nucleon particle (or hole) pairs outside a closed shell [6]. The IBM-1 does not distinguish between proton and neutron bosons. The total bosons number $(N = N_\pi + N_v)$ is finite and conserved in a given nucleus and is simply given by half the total number of valence nucleus [6].

The interacting boson model offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, based on general algebraic group theoretical techniques which have also recently found application in problems in atomic,

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molecular, and high-energy physics [7, 8]. The IBM Hamiltonian provide exact solutions in three dynamical symmetry limits of theory U(5), SU(3) and O(6) which correspond to vibrational limit, rotational limit and unstable limit, respectively. Another case complex happens where the Hamiltonian considered in terms of invariant operator of these chains of symmetries which is considered as shape phase transition between these dynamical symmetry limits [9, 10].

Alya'a A. have studied of the $^{246}_{96}Cm_{150}$ isotope is belong to the SU(3) – O(6) transition region, it has properties located between the SU(3) and O(6) dynamical symmetries [11]. Saiqa S. *et al*. have studied of even – even curium isotopes, where show comparison between experimental and calculated values of B(E2), as show the quadrupole and hexadecapole deformation parameters to calculations for Cm (A=242-250) isotopes [12].

The aim in this study is to investigate Cm (A=248) isotope belong to the SU(3) dynamical symmetry by calculate the energy ratios, find the energy levels, B(E2) transition probabilities, calculate the reduced matrix elements $\langle L_f||\hat{T}^{(E_2)}||L_i\rangle$, B(E2) ratio, calculate electric quadrupole moment (Q_L) and deformation parameter β with in framework of IBM-1 and comparison all that with available experimental results.

2. THEORETICAL BASICS

2.1. THE HAMILTONIAN OPERATOR OF THE (IBM-1)

It can be expressed by the IBM-1 Hamiltonian as [13]:

$$\hat{H} = \varepsilon_s\,(\hat{S}^{\dagger}.\,\hat{\tilde{S}}) + \varepsilon_d\,\left(\hat{d}^{\dagger}.\hat{\tilde{d}}\right) + \sum_{L=0,2,4}\frac{1}{2}\sqrt{2L+1}\,C_L\,\left[\left[\hat{d}^{\dagger}\times\hat{d}^{\dagger}\right]^{L}\times\left[\hat{d}\times\hat{\tilde{d}}\right]^{L}\right]^{0} + \frac{1}{\sqrt{2}}\,V_2\left[\left[\hat{d}^{\dagger}\times\hat{d}^{\dagger}\right]^{2}\times\left[\hat{\tilde{d}}\times\hat{\tilde{S}}\right]^{2} + \left[\hat{d}^{\dagger}\times\hat{S}^{\dagger}\right]^{2}\times\left[\hat{d}\times\hat{\tilde{d}}\right]^{2}\right]^{0} + \frac{1}{2}\,V_0\left[\left[\hat{d}^{\dagger}\times\hat{d}^{\dagger}\right]^{0}\times\right.$$

S× S 0+ S†× S†0× d× d 0O+ U2d†× Ŝ†2× d× S 20+ 12 U0Ŝ†× Ŝ†0× S×S 00

(1)

Creation and Annihilation operators are $(\hat{S}^{\dagger}, \hat{d}^{\dagger})$ and $(\hat{\tilde{S}},\ \hat{\tilde{d}})$ for s and d- bosons, respectively [9], Two terms of one body interactions (ε_s and ε_d) and seven terms of two-body interactions $[C_L(L=0,2,4), V_L(L=0,2), U_L(L=0,2)]$ in this Hamiltonian, where the single-boson energies are ε_s and ε_d, and the two-boson interactions had been described by C_L, V_L and U_L, and so on, it shows that for a fixed boson number N, only one of the one-body term and five of the two body terms are independent, It can be seen by noting (N) is the total number of bosons. However, it is more common to write the Hamiltonian of the IBM-1 as a multipole expansion, grouped into different boson-boson interactions Equation (2) [13]:

$$\hat{H} = \varepsilon\,\hat{n}_d + a_0(\hat{p}.\hat{p}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad (2)$$

where:

$\hat{n}_d = \left(\hat{d}^{\dagger}.\hat{\tilde{d}}\right)$ is the total number of the d_{boson} operator (3)

$\hat{p} = 1/2\left[\left(\hat{\tilde{d}}.\hat{\tilde{d}}\right) - (\hat{\tilde{S}}.\hat{\tilde{S}})\right]$ is the pairing operator (4)

$\hat{L} = \sqrt{10}\left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{1}$ is the angular momentum operator (5)

$\hat{Q} = \left[\hat{d}^{\dagger} \times \hat{\tilde{S}} + \hat{S}^{\dagger} \times \hat{\tilde{d}}\right]^{(2)} + \chi\left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(2)}$ is the quadrupole operator (6)

where χ is the quadrupole structure parameter and take the values 0 and $\pm\frac{\sqrt{7}}{2}$ [13, 14].

$\hat{T}_r = \left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(r)}$ is the octupole ($r = 3$) and hexadecapole ($r = 4$) operator (7)

and $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy. (8)

The parameters $a_0,\ a_1, a_2, a_3$ and a_4 designated the strength of the pairing, angular momentum, quadrupole, octupole and hexadecapole interaction between the bosons.

2.2. ELECTRIC QUADRUPOLE TRANSITION OPERATOR ($T^{(E2)}$)

The electric quadrupole transition operator $T^{(E2)}$ can be taken from equation (9) as [13, 15]:

$$\hat{T}^{(E2)} = \alpha_2\left[\hat{d}^{\dagger} \times \hat{\tilde{S}} + \hat{S}^{\dagger} \times \hat{\tilde{d}}\right]^{(2)}_{\mu} + \beta_2\left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(2)}_{\mu} = \alpha_2\left(\left[\hat{d}^{\dagger} \times \hat{\tilde{S}} + \hat{S}^{\dagger} \times \hat{\tilde{d}}\right]^{(2)}_{\mu} + \chi\left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(2)}_{\mu}\right) = e_B\hat{Q} \quad (9)$$

where
α_2 is the effective charge of s and d boson.
β_2 is the effective charge of d boson.
The $\beta_2 = \chi\alpha_2, \alpha_2 = e_B$.

The selection rules of the electric quadrupole transition operator of the three dynamical symmetries U(5), SU(3) and O(6) are as follows [16]:

$$U(5)\text{: } \Delta n_d = 0, \pm 1\ ;\ \Delta\nu = \pm 1;\ \ \Delta n_{\Delta} = 0\ ;\ |\Delta L| \leq 2$$
$$SU(3)\text{: } \Delta\lambda = 0;\ \Delta\mu = 0$$
$$O(6)\text{: } \Delta\sigma = 0;\ \Delta\tau = \pm 1$$

The electric quadrupole transition rates are governed by B(E2) values, these are defined as [15]:

$$B\left(E2; L_i \rightarrow L_f\right) = \frac{1}{2L_i+1}\left|\left\langle L_f\right|\left|\hat{T}^{(E_2)}\right|\left|L_i\right\rangle\right|^2 \quad (10)$$

where
L_i is the initial angular momentum.
L_f is the final angular momentum.

The electric quadrupole moments of the nuclei can be derived from the $B\left(E2; L_i \rightarrow Lf\right.$ values according to equation (16) [8]:

$$Q_L = \left[\frac{16\pi}{5}\right]^{1/2} \left[\frac{L(2L-1)}{(2L+1)(L+1)(2L+3)}\right]^{1/2} \left[B(E2; L_i \rightarrow L_f\right]^{1/2} \quad (11)$$

The L is the angular momentum.

2.3. B(E2) RATIO

The (Branching ratios) of the reduced transition probabilities for electric quadrupole transition of the dynamical symmetry U(5), SU(3) and O(6) obeys the following relations [15]:

$$R = \frac{B(E2;4_1^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}, R' = \frac{B(E2;2_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}, R'' = \frac{B(E2;0_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)} \quad (12)$$

In addition, they can be written in terms of the total boson number [15]:

$$R = R' = R'' = \frac{2(N-1)}{N} \rightarrow 2_{N\rightarrow\infty} \quad \text{In U(5) limit} \quad (13)$$

$$R = \frac{10}{7}\frac{(N-1)(2N+5)}{N(2N+3)} \rightarrow_{N\rightarrow\infty} \frac{10}{7};\ R' = R'' = 0 \quad \text{In SU(3) limit} \quad (14)$$

$$R = R' = \frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)} \rightarrow_{N\rightarrow\infty} \frac{10}{7};\ R'' = 0 \quad \text{In O(6) limit} \quad (15)$$

where N is the total boson number.

3. RESULTS AND DISCUSSION

3.1. ENERGY RATIOS AND DYNAMICAL SYMMETRY

To find the dynamical symmetry for any isotope, we compare the experimental energy ratios for the isotope under study with identical energy ratios of corresponding limits as shown in Table 1.

Table 1. The energy ratios of corresponding limits [6, 13, 14].

Limit	$R4 = E(4_1^+)/E(2_1^+)$	$R6 = E(6_1^+)/E(2_1^+)$	$R8 = E(8_1^+)/E(2_1^+)$
U(5)	2	3	4
SU(3)	3.33	7	12
O(6)	2.5	4.5	7

Table 2. The comparison between the experimental data and theoretical energy ratios according to the (IBM-1) for the Cm (A=248) isotope.

Isotope	$R4 = E(4_1^+)/E(2_1^+)$		$R6 = E(6_1^+)/E(2_1^+)$		$R8 = E(8_1^+)/E(2_1^+)$	
$^{248}_{96}Cm_{152}$	EXP. [17]	IBM-1	EXP. [17]	IBM-1	EXP. [17]	IBM-1
	3.3134	3.3336	6.8871	7.0005	11.6682	12.0007

From Table 2 shows that the energy ratios for $^{248}_{96}Cm_{152}$ isotope quite close to the energy ratios for the dynamical symmetry SU(3).This isotope has properties located in the SU(3) dynamical symmetry region.

The equation of the Hamiltonian operator function for dynamical symmetry SU(3) shows in equation (16).

$$\hat{H} = a_1\hat{L}^2 + a_2\hat{Q}^2 \quad (16)$$

a_1 and a_2 represent the angular momentum and quadrupole respectively.

3.2. ENERGY LEVELS

The energy levels for the Cm (A=248) isotope classified according to the energy bands $(g, \beta, \gamma - bands)$.

Table 3 shows the parameters was used in the (IBM-1.For) program to obtain with the best fitting from the Hamiltonian operator with the experimental results and depending on the convenient dynamical symmetry.

Table 3. The parameters values of Hamiltonian operator for the Cm (A=248) isotope by using (IBM-1. For) program.

Isotope	N_π	N_ν	N	EPS (Mev)	$\hat{P}^\dagger.\hat{P}$ (Mev)	$\hat{L}.\hat{L}$ (Mev)	$\hat{Q}.\hat{Q}$ (Mev)	$\hat{T}_3.\hat{T}_3$ (Mev)	$\hat{T}_4.\hat{T}_4$ (Mev)	CHI (Mev)	$SO6$ (Mev)
$^{248}_{96}Cm_{152}$	7	13	20	0.0000	0.0000	0.0040	-0.0086	0.0000	0.0000	-1.3200	0.0000

Table 4 shows the energy band spectrum $(g, \beta, \gamma - bands)$ for $^{248}_{96}Cm_{152}$ isotope, It was compared the resulting data by using (IBM-1.For) program with the available experimental data, the results shows that, there are good agreement of the energy levels between the present work and the experimental results, where it was confirmed the energy level of the spin (2_2^+) from (1.0500) in the experimental results to (1.04761) in the present work, as predicted to energy levels for spin and parity $(3_1^+, 5_1^+, 6_3^+, 7_1^+, 9_1^+, 11_1^+)$ to be (1.09096, 1.22099, 1.30864, 1.40882, 1.65444, 1.95785) respectively.

Table 4. The comparison between the theoretical (pw) and experimental [17] energy bands (g, β, γ-bands) for the $^{248}_{96}Cm_{152}$ isotope by using IBM-1.

Isotopes	Spin / Band	0^+ 2^+	2^+ 3^+	4^+ 4^+	6^+ 5^+	8^+ 6^+	10^+ 7^+	12^+ 8^+	14^+ 9^+	16^+ 10^+	18^+ 11^+
$^{248}_{96}Cm_{152}$ SU(3)	$g-exp.$	0.00000	0.04340	0.14380	0.29890	0.50640	0.76280	1.06410	1.40620	1.78400	2.19270
	$g-pw$	0.00000	0.04334	0.14448	0.30340	0.52011	0.79462	1.12691	1.51700	1.96487	2.47053
	$\gamma_1 - exp.$	(1.0500)	–	1.14400	–	1.28440	–	1.45230	–	1.66930	–
	$\gamma_1 - pw$	1.04761	1.09096	1.14875	1.22099	1.30768	1.40882	1.52440	1.65444	1.79892	1.95785
	$\beta_1 - exp.$	1.08400	1.13100	1.22200	–	1.46610	1.65180	1.88020	–	–	–
	$\beta_1 - pw$	1.00524	1.04858	1.14972	1.30864	1.52536	1.79987	2.13123	–	–	–

Fig. 1 represent the comparison between the experimental and theoretical energy levels for $^{248}_{96}Cm_{152}$ isotope, noticed that, there are good agreement of the level sequences of each band with the typical sequence of $(g-band)(0^+, 2^+, 4^+ \ldots.), (\gamma-band)(2^+, 3^+, 4^+ \ldots.)$ and $(\beta-band)(0^+, 2^+, 4^+ \ldots.)$.

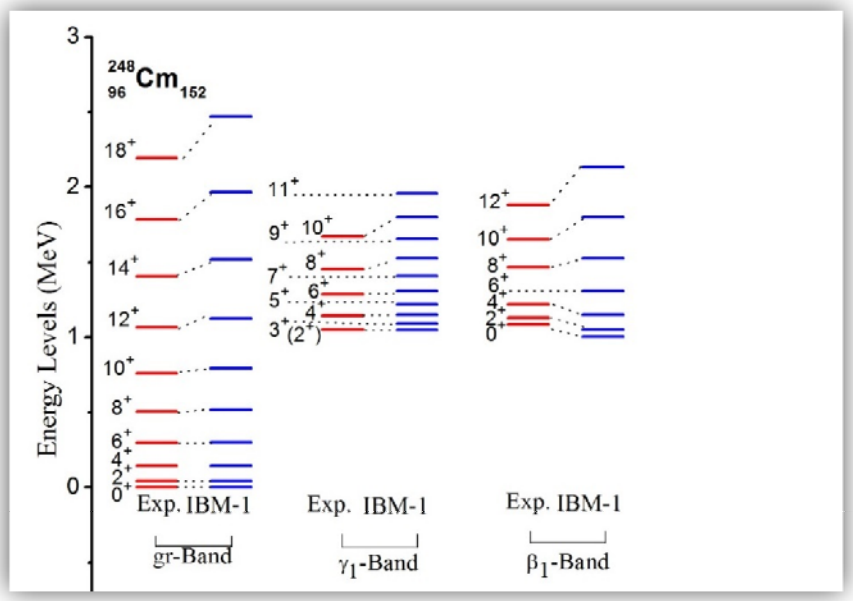

Figure 1. Comparison between calculated IBM (pw); and experimental [17] energy bands states (g, γ, β - bands) in $^{248}_{96}Cm_{152}$ isotope of the dynamical symmetry SU(3).

3.3. REDUCED MATRIX ELEMENTS & THE PROBABILITY OF ELECTRIC TRANSITIONS

It is the probability of electric transitions B(E2) properties of the important to study the nuclear structure for any isotope, (IBMT.FOR) program was used to calculate B(E2) theoretical values, and it must specify values of effective charge e_B. The values of effective charge e_B (α_2) were estimated to reproduce the experimental B(E2), as can through it find the value (β_2) and as shown in the Table 5, where: $\beta_2 = -\frac{\sqrt{7}}{2}\alpha_2$.

The values (E2SD, E2DD) represents the parameters was used in the program to obtain the theoretical results of probability of electric transitions B(E2) and reduced matrix elements $\langle L_f||\hat{T}^{(E_2)}||L_i\rangle$. Where: $E2SD = \alpha_2, E2DD = \sqrt{5}\beta_2$

Table 5. The values of the parameters (α_2, β_2) and (E2SD, E2DD) of the B(E2) and $\langle L_f||\hat{T}^{(E_2)}||L_i\rangle$ for Cm (A=248) isotope by using (IBMT.FOR) program.

Isotope	N_π	N_ν	N	$\alpha_2\ (eb)$	$\beta_2\ (eb)$	$E2SD(eb)$	$E2DD(eb)$
$^{248}_{96}Cm_{152}$	7	13	20	0.12556	-0.16610	0.12556	-0.37141

The comparison of the experimental data [17] with calculations of B(E2) values are given in table 6 for isotope under study. There is no available experimental transition data to many transitions in table 6. Therefore, it has been predicted by using IBM-1. This table shows that in general, there are a good agreement between the experimentally B(E2) values and the (IBM-1) calculated ones.

Table 6 show that, probabilities of electric transitions (up and down) B(E2)↓, B(E2)↑ Respectively, in $(eb)^2$ units. Where: (eb) Electron barn.

The reduced matrix elements $\langle L_f||\hat{T}^{(E_2)}||L_i\rangle$ the amount associated with the probability electric transitions B(E2), It can be found them by using equation (10), and for each electric transition for this isotope. It is measured in (eb) units.

Table 6. Theoretical values of reduced matrix elements $\langle L_f||\hat{T}^{(E_2)}||L_i\rangle$ and probabilities of electric transitions B(E2) compared with the available experimental data for the Cm (A=248) isotope.

Isotope	Spin sequences $L_f^+ - L_i^+$	B(E2)↓ $(eb)^2$		B(E2)↑ $(eb)^2$		$\langle L_f\|\|\hat{T}^{(E_2)}\|\|L_i\rangle$ eb
		IBM-1	EXP.[17]	IBM-1	EXP.[17]	
$^{248}_{96}Cm_{152}$ SU(3)	$2_1^+ - 0_1^+$	2.9936	2.9987	14.9680	14.9935	3.8688
	$0_2^+ - 2_1^+$	0.0022	----	0.0110	----	0.2345
	$0_2^+ - 2_2^+$	0.1010	----	0.5050	----	1.5890
	$2_2^+ - 0_1^+$	0.2154	----	1.0770	----	1.0378
	$2_3^+ - 0_1^+$	0.0004	----	0.0020	----	0.0447
	$2_3^+ - 0_2^+$	2.7077	----	13.5385	----	3.6795
	$2_2^+ - 2_1^+$	0.3306	----	1.6530	----	2.8749
	$2_3^+ - 2_1^+$	0.0006	----	0.0030	----	0.1225
	$2_3^+ - 2_2^+$	0.0232	----	0.1160	----	0.7616
	$2_2^+ - 4_1^+$	0.0196	----	0.0980	----	0.9391
	$2_3^+ - 4_1^+$	0.0012	----	0.0060	----	0.2324
	$3_1^+ - 2_1^+$	0.3823	----	1.9115	----	3.0915
	$3_1^+ - 2_2^+$	4.3312	----	21.6560	----	10.4058
	$3_1^+ - 2_3^+$	0 .0112	----	0.0560	----	0.5291
	$3_1^+ - 4_1^+$	0.1811	----	0.9055	----	2.8547
	$4_1^+ - 2_1^+$	4.1144	4.1648	20.5720	20.8240	10.1420
	$4_2^+ - 2_1^+$	0.1150	----	0.5750	----	1.6956
	$4_2^+ - 2_2^+$	1.3674	----	6.8370	----	5.8468
	$4_2^+ - 2_3^+$	0.0068	----	0.0340	----	0.4123
	$4_3^+ - 2_1^+$	0.0004	----	0.0020	----	0.1000
	$4_3^+ - 2_2^+$	0.0004	----	0.0020	----	0.1000
	$4_3^+ - 2_3^+$	3.6995	----	18.4975	----	9.6170
	$4_2^+ - 3_1^+$	3.2320	----	16.1600	----	10.6358
	$4_3^+ - 3_1^+$	0.0066	----	0.0330	----	0.4806
	$4_2^+ - 4_1^+$	0.4001	----	2.0005	----	4.2432
	$4_3^+ - 4_1^+$	0.0008	----	0.0040	----	0.1897
	$4_3^+ - 4_2^+$	0.0285	----	0.1425	----	1.1325
	$4_2^+ - 6_1^+$	0.0454	----	0.2270	----	1.7178
	$4_3^+ - 6_1^+$	0.0011	----	0.0055	----	0.2674
	$5_1^+ - 3_1^+$	2.1382	----	10.6910	----	8.6508
	$5_1^+ - 4_1^+$	0.3189	----	1.5945	----	3.7882
	$5_1^+ - 4_2^+$	2.3117	----	11.5585	----	10.1993
	$5_1^+ - 6_1^+$	0.2380	----	1.1900	----	3.9332
	$7_1^+ - 5_1^+$	2.7980	----	13.9900	----	12.4052
	$7_1^+ - 6_1^+$	0.2792	----	1.3960	----	4.2600
	$7_1^+ - 6_2^+$	1.3054	----	6.5270	----	9.2115
	$7_1^+ - 8_1^+$	0.2683	----	1.3415	----	4.7755
	$6_1^+ - 4_1^+$	4.3366	4.1649	21.6830	20.8245	13.9695
	$6_2^+ - 4_1^+$	0.0842	----	0.4210	----	1.9465
	$6_2^+ - 4_2^+$	2.5670	----	12.8350	----	10.7478
	$6_3^+ - 4_1^+$	0.0003	----	0.0015	----	0.1162
	$6_3^+ - 4_2^+$	0.0006	----	0.0030	----	0.1643
	$6_2^+ - 5_1^+$	1.7146	----	8.5730	----	9.7110

	$6_2^+-6_1^+$	0.4057	----	2.0285	----	5.1352
	$6_3^+-6_1^+$	0.0011	----	0.0055	----	0.2674
	$6_3^+-6_2^+$	0.0296	----	0.1480	----	1.3871
	$6_2^+-8_1^+$	0.0625	----	0.3125	----	2.3049
	$6_3^+-8_1^+$	0.0010	----	0.0050	----	0.2915
	$8_1^+-6_1^+$	4.3207	4.1649	21.6035	20.8245	16.7584
	$8_2^+-6_1^+$	0.0670	----	0.3350	----	2.0869
	$8_2^+-6_2^+$	2.9300	----	14.6500	----	13.8004
	$8_2^+-6_3^+$	0.0119	----	0.0595	----	0.8795
	$8_2^+-7_1^+$	1.0413	----	5.2065	----	8.8373
	$8_2^+-8_1^+$	0.3985	----	1.9925	----	5.8200
	$8_2^+-10_1^+$	0.0761	----	0.3805	----	2.8267
	$9_1^+-7_1^+$	2.9746	----	14.8730	----	14.9364
	$9_1^+-8_1^+$	0.2478	----	1.2390	----	4.5894
	$9_1^+-8_2^+$	0.8336	----	4.1680	----	8.4176
	$10_1^+-8_1^+$	4.1997	4.4425	20.9985	22.2125	18.8938
	$10_2^+-8_1^+$	0.0550	----	0.2750	----	2.1622
	$10_2^+-8_2^+$	2.9982	----	14.9910	----	15.9639
	$12_1^+-10_1^+$	4.0204	4.7202	20.1020	23.6010	20.5461
	$12_2^+-10_1^+$	0.0455	----	0.2275	----	2.1857
	$12_2^+-10_2^+$	2.9361	----	14.6805	----	17.5582

3.4. B(E2) RATIO

By using other important quantities, the B(E2) ratio shows that the $^{248}_{96}Cm_{152}$ isotope is belong to the dynamical symmetry SU(3). The formulas for calculating the B(E2) ratio are showing in equation (12).Since Curium isotope ($^{248}_{96}Cm_{152}$) belongs to the SU(3) dynamical symmetry in IBM-1, therefore, Table 7 show the B(E2) ratio by using IBM-1 and the B(E2) ratio to available experimental values for isotope under study. Also from this table, we can see that the theoretical value of B(E2) ratio (R) for this isotope is approximately equal with the experimental value, (R′) represent small value, (R′′) equal to a very small value close to zero and these ratios in good agreement a fairly with B(E2) ratio to the dynamical symmetry SU(3).

Table 7. The comparison between IBM-1 and experimental [17] values of B(E2) ratio for the Cm (A=248) isotope.

Isotope		$^{248}_{96}Cm_{152}$
N		20
IBM-1	R	1.3744
	R′	0.1104
	R′′	0.0007
EXP.	R	1.3889
	R′	---
	R′′	---

3.5. QUADRUPOLE MOMENTS

3.5.1. INTRINSIC QUADRUPOLE MOMENT AND DEFORMATION PARAMETER

The intrinsic quadrupole moment Q_0 is an important property for nuclei and can be derived from the transition rate $B(E2\uparrow)$values according to equation (17) [18].

$$Q_0 = [(16\pi/5)B(E2)\uparrow/e^2]^{1/2} barn \quad (17)$$

where:

$$B(E2\uparrow; L-2 \rightarrow L) = \frac{2L+1}{2(L-2)+1} B(E2\downarrow; L \rightarrow L-2) \quad (18)$$

In addition to, there are other quantities dependent on the value of B(E2) called the deformation parameter β , and it can be calculated by the following equation:

$$\beta = \left(4\pi/\left(3ZR_0{}^2\right)\right)[B(E2)\uparrow/e^2]^{1/2} \quad (19)$$

where:

$$R_0{}^2 = 0.144A^{2/3} barn \quad (20)$$

Note that $Q_0, R_0{}^2$ measured with barn unit.

Table 8 represents the calculations of intrinsic quadrupole moment Q_0 and deformation parameter β with in framework of IBM-1 for the even – even ${}^{248}_{96}Cm_{152}$ isotope. The presented result for Q_0 is consistent with the expectations and from phenomenological systematic, and is compared with experimental result [17].

Table 8. The IBM-1 values and the experimental data [17] of Q_0 and β for Cm (A=248) isotope.

Isotope		${}^{248}_{96}Cm_{152}$
$Q_0(b)$	IBM-1	12.2637
	EXP.	12.2741
β	IBM-1	0.0297
	EXP.	0.0297

3.5.2. ELECTRIC QUADRUPOLE MOMENTS

The values of electric quadrupole moments (Q_L) are one of the most important properties for measuring the nuclear deformation. The deformed nuclei of prolate shape have a positive electric quadrupole moment $(Q_L > 0)$, the nuclei of oblate shape have a negative electric quadrupole moment $(Q_L < 0)$, and $(Q_L = 0)$ for the isotopes belong to O(6) limit.

Table 9 shows the IBM-1 electric quadrupole moment measuring with (eb) unit of the ground state $(Q_{2_1^+})$ according to the dynamical symmetry.

Table 9. The IBM-1 value of electric quadrupole moment $Q_{2_1^+}$ (pw) for Cm (A=248) isotope.

Isotope	Electric Quadrupole Moment $Q_{2_1^+}(eb)$
${}^{248}_{96}Cm_{152}$	-3.3370

4. CONCLUSIONS

From this work, we can determine the dynamical symmetry to the isotope under study, show that, the $^{248}_{96}Cm_{152}$ isotope is belonging to the dynamical symmetry SU(3) as it calculate the energy ratio to this isotope by using (IBM-1) and comparison with the experimental results, as it also study the energy levels for $(g, \beta, \gamma - bands)$ by using (IBM-1) program and comparison with the experimental results where confirmation one energy level and prediction for energy levels number (6). In addition to that it calculate the probability of electric transitions B(E2) values and compared with available experimental results and find the reduce matrix elements $\langle L_f || \hat{T}^{(E_2)} || L_i \rangle$ the associated with the probability electric transitions B(E2). As it calculate the B(E2) ratio (R, R', R'') for this isotope and comparing with the available experimental B(E2) ratio. As it also calculate the value of intrinsic quadrupole moment Q_0 and electric quadrupole moment Q_L depending B(E2)↑ for determine the shape of the nucleus and the amount of deformation happening in it.

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