ORIGINAL PAPER

THE EFFECT OF THIN FLAME VELOCITY AND OTHER SENSITIVE FACTORS ON THE MAXIMUM TEMPERATURE OF THERMAL EXPLOSION IN THE EARTH'S INTERIOR

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Abstract. *The paper examines the effects of thin flame velocity and other sensitive factors on the maximum temperature of the reaction during gravitational differentiation (GD) and radioactive decay (RD) processes in the interior of the earth.The partial differential equation governing the model together with the boundary conditions is transformed into ordinary differential equation by thin flame technique. The resulting equation is investigated for the effects of some sensitive factors such as activation energies ratio, flame velocity, gravitational differentiation and radioactive decay parameters, on maximum temperature of the reaction. We establish the criteria for the existence of unique solution of the resulting equations. Numerical results were obtained by shooting method.The results show that flame velocity, and other sensitive factors have appreciable effects on maximum temperature of the reaction. In particular, regulating the flame velocity has appreciable impact on the reactions in terms of heat release.*

Keywords: *thermal explosion, activation energy, Gravitational Differentiation (GD), , radioactive decay.*

1. INTRODUCTION

Our previous contributions in combustion theory and modelling emphasized on heat release during the chemical reaction [1-3]. Those works are basically helpful in terms of safety and industrial purposes.

Literatures have shown that in the interior of the earth, there are two major sources of heat that constitute exothermic regimes which are Gravitational Differentiation (GD) and decay of radioactive elements [4]. Previous works further established that in the early stages of planetary build up, the earth was much less compact than what it is today. This build-up process led to more gravitational attractions which force the earth to contract into smaller volume. During the GD, the potential energy generated becomes heat energy due to viscous dissipation. Also, radioactive elements are inherently unstable. The unstable Uranium isotope (Uranium-238) slowly decays to Lead - 206 and the radioactive decay processes continue. They break down over time to more stable forms and release intense heat (which may include flame) as by-product of the chain reaction.

This heat is continually radiated outward through several concentric shells that form the solid portion of the planet. The work done by [4] explained the above processes in the earth interior and gave further insight that thermal processes that occur in the earth interior

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differ from characteristic thermal explosion but they are analogous. Other contributions by [5-7] also established that although multiple steps are involved in reaction but two major steps (chemical decomposition and combustion process) are basically involved.

Our previous works on thermal explosion showed that explosion generally results from two exothermic reactions; one step follows the other in rapid succession depending on the activation energies of the reactions [9]. [10] presented some remarks on thermal explosion in the early evolution of the earth. The paper considered the unsteady and steady state energy equation associated with the earth evolution, and establishes the criteria for the occurrence of thermal runaway.

[11] investigated the effect of radioactive heat source and gravitational differentiation on unsteady state thermal explosion in the evolution of the earth. The resulting energy equation was solved by shooting method. The authors showed that critical temperature which signifies the onset of thermal instability due to gravitational differentiation depends linearly on the intensity of radioactive heat source. We were able to establish that even when thermal conductivity due to gravitational differentiation and ordinary thermal conductivity are comparable, a steady thermal solution exists under specified conditions.[12] revisited the theory of evolution of the earth. The effect of gravitational differentiation in the separation of heavier material forming the earth's core from Silicates in the extended and heated area was studied. Previous literatures focused on small and large thermal conductivities while we focused on all orders of thermal conductivity. The numerical solution of the energy equation was provided by shooting method. The previous results in the literatures were special cases of the new results in that paper.

Our previous works on heat transfer were not restricted to [13-14] alone. Article [15] investigated the effect of activation energies on thermal explosion that occurs in the interior of the earth during gravitational differentiation and decay of radioactive substances. The unsteady, steady and homogeneous reactions were investigated. Theorems on the existence of unique solution were formulated and proved. Results on blow-up were obtained, and the criteria for a blow up to occur in the chain reaction, were established. The analytical and numerical results showed that activation energies have different implications in terms of heat release.

While our contributions on safety and modelling [16-18] continue, [19] also examined thermal explosion arising from time-dependent gravitational differentiation and radioactive decay in the Earth's Interior. He considered the work done by [15] when the gravitational differentiation and radioactive decay parameters were time dependent. The paper was able to establish that the problem has a unique solution and the activation energies ratio has appreciable effects on the reactions in terms of heat release.

This paper therefore considers [15, 18] and uses thin flame technique to reduce the partial differential equation into ordinary differential equation and to also introduce flame velocity parameter into the model in order to study its effect and other sensitive factors, on the maximum temperature of the reaction.

2. MATHEMATICAL EQUATIONS

According to [15, 18], the dimensionless thermal conductivity equation governing the generation of heat by two major sources; GD and the decay of radioactive elements, is given by

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial \xi}\left(1 + P_{eo} e^{\frac{(1+\alpha)\theta}{n}}\right)\frac{\partial \theta}{\partial \xi} + \Gamma_d \, e^{(1+\alpha)\theta} + \Gamma_r \tag{1}$$

satisfying the initial and boundary conditions

$$\theta(\xi,0) = 0, \qquad \theta(-1,\tau) = \theta\,(1,\tau) = 0 \tag{2}$$

where the parameters are defined as;

θ	Non-dimensionaltemperature
τ	Non-dimensional time variable
ξ	Non dimensionalspace variable.
P_e	Pécletnumber
α	Ratio of activation energies
Γ_d	Non dimensional term for Gravitational Differentiation
Γ_r	Non dimensional term for radioactive source

The equation (1) subject to conditions (2), becomes

$$\frac{\partial\theta}{\partial\tau} = \left(1 + P_{eo}e^{\frac{(1+\alpha)\theta}{n}}\right)\frac{\partial^2\theta}{\partial\xi^2} + \frac{(1+\alpha)}{n}P_{eo}e^{\frac{(1+\alpha)\theta}{n}}\left(\frac{\partial\theta}{\partial\xi}\right)^2 + \Gamma_d\, e^{(1+\alpha)\theta} + \Gamma_r \tag{3}$$

Here we use thin flame technique

$$\text{Let} \quad \theta(\xi,\tau) = g(\eta) \quad such \quad that \quad \eta = \xi - v\tau. \tag{4}$$

where v is the flame velocity.

The equation (3) gives

$$\left(1 + P_{eo}e^{\frac{(1+\alpha)g}{n}}\right)\frac{d^2g}{d\eta^2} + v\frac{dg}{d\eta} + \frac{(1+\alpha)}{n}P_{eo}e^{\frac{(1+\alpha)g}{n}}\left(\frac{dg}{d\eta}\right)^2 + \Gamma_d\, e^{(1+\alpha)g} + \Gamma_r = 0 \tag{5}$$

and the above equation is now subject to conditions

$$g(-1 - v\tau) = 0, \qquad g(1 - v\tau) = 0 \tag{6}$$

Case 1:

$$P_{eo}e^{\frac{(1+\alpha)g}{n}} >> 1, .$$

The equation (5) remains as

$$\left(1 + P_{eo}e^{\frac{(1+\alpha)g}{n}}\right)\frac{d^2g}{d\eta^2} + v\frac{dg}{d\eta} + \frac{(1+\alpha)}{n}P_{eo}e^{\frac{(1+\alpha)g}{n}}\left(\frac{dg}{d\eta}\right)^2 + \Gamma_d\, e^{(1+\alpha)g} + \Gamma_r = 0 \tag{7}$$

Subject to

$$g(-1 - v\tau) = g(1 - v\tau) = 0 \tag{8}$$

Theorem 1: Suppose D denotes the region for which $0 \le \alpha \le N$, $-1 \le y_1 \le 1$, $v, N, \Gamma_r, \Gamma_d, n, P_{eo} > 0$, then problem (7) which satisfies conditions (8) and for which $g'(-1-v\tau)$ is fixed, has a unique solution.

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \eta \\ g \\ g' \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} 1 \\ y_3 \\ -\dfrac{\left[v y_3 + \dfrac{(1+\alpha)p_{e_0}}{n} \exp\left(\dfrac{(1+\alpha)y_2}{n}\right)(y_3)^2 + \Gamma_d \exp\left((1+\alpha)\ y_2\right) + \Gamma_r \right]}{\left(1 + p_{e_0} \exp\left(\dfrac{(1+\alpha)y_2}{n}\right)\right)} \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} g_1(y_1, y_2, y_3) \\ g_2(y_1, y_2, y_3) \\ g_3(y_1, y_2, y_3) \end{pmatrix}$$

Subject to the initial conditions

$$\begin{pmatrix} y_1(-1-v\tau) \\ y_2(-1-v\tau) \\ y_3(-1-v\tau) \end{pmatrix} = \begin{pmatrix} -1-v\tau \\ 0 \\ -\lambda_g \end{pmatrix} \tag{11}$$

Remark: λ_g is guessed such that the boundary condition $y_2(1-v\tau) = 0$.

Theorem 2: Assume that D denotes the region for which $0 \le \alpha \le N$**,** $-1 \le y_1 \le 1$**,** $0 \le y_2 \le M$, $\lambda_g \le y_3 \le -\lambda_g^*$**,** $v, M, N, \Gamma_r, \Gamma_d, n, P_{eo} > 0$**,** then the functions $g(i = 1, 2, 3)$ are Lipschitz continuous in D.

Proof: Obviously, $\left|\dfrac{\partial g_{i1}}{\partial y_j}\right|$, $i, j = 1,2,3$, are continuous in D and bounded on D. So, there exists a constant K such that $K = \left|\dfrac{\partial g_{i1}}{\partial y_j}\right|$, $i, j = 1,2,3$. Hence, $g_i(y_1, y_2, y_3)$, $i = 1, 2, 3$ are Lipschitz continuous and so problem (10) satisfying (11) is Lipschitz continuous.

Proof of theorem 1: The existence of Lipschitz constant in the proof of theorem 2 implies the existence of unique solution of problem (10) which satisfies (11). And this implies the existence of unique solution of problem (6) satisfying the conditions (7).

Case 2:

$$P_{eo} e^{\frac{(1+\alpha)g}{n}} << 1,$$

The equation (4) becomes

$$\frac{d^2 g}{d\eta^2} + v\frac{dg}{d\eta} + \Gamma_d \, e^{(1+\alpha)g} + \Gamma_r = 0 \tag{12}$$

Which is subject to condition (8).

The equation (12) is resolved into a system of equations as follows. Recall that

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \eta \\ g \\ g' \end{pmatrix} \tag{13}$$

Then

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} 1 \\ y_3 \\ -\left[-\frac{y_1 y_3}{2} + \Gamma_d \exp((1+\alpha)\, y_2) + \Gamma_r\right] \end{pmatrix} \tag{14}$$

Subject to the initial conditions

$$\begin{pmatrix} y_1(-1-v\tau) \\ y_2(-1-v\tau) \\ y_3(-1-v\tau) \end{pmatrix} = \begin{pmatrix} -1-v\tau \\ 0 \\ -\lambda_z \end{pmatrix} \tag{15}$$

NUMERICAL COMPUTATION

In this section, numerical solutions of problems (7) and (12) satisfying conditions (8) are provided by using Runge-Kutta Shooting method.

Some computer programmes written in Pascal language were used to solve problems (10) and (14) together with conditions (11) and (15) respectively, and for which λ_g and λ_z are guessed such that the boundary condition $y_2(1-v\tau)=0$. The numerical results obtained are presented in the figures below.

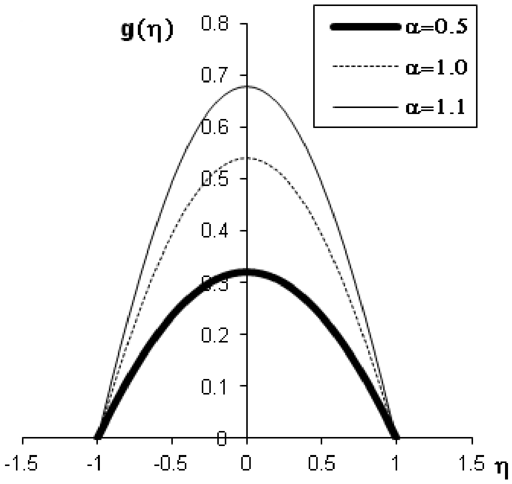

Figure 1. Temperature profile for fixed values of G_r=0.4, G_d=0.2, v=0.01 and for various values of a

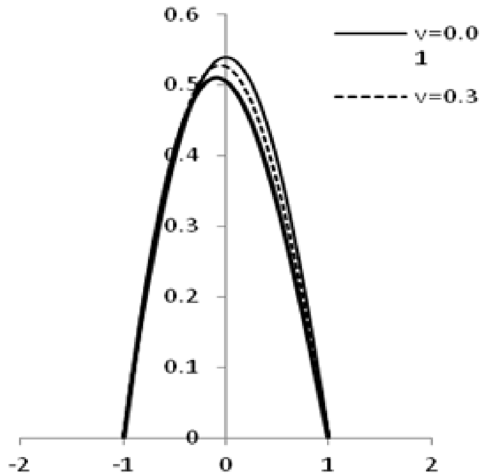

Figure 2. Temperature profile for fixed values of α=1, Γ_d= 0.2,Γ_r= 0.4, and for various values of v

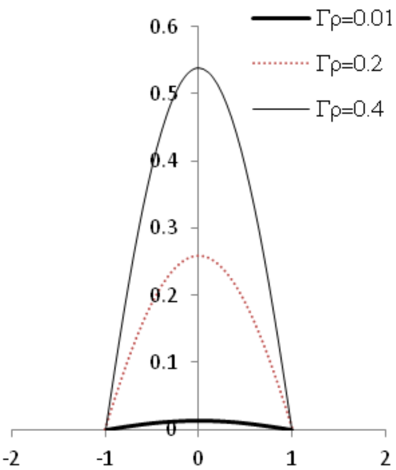

Figure 3. Temperature profile for fixed values of α=1, Γ_d=0.2, v= 0.01 and for various values of Γr

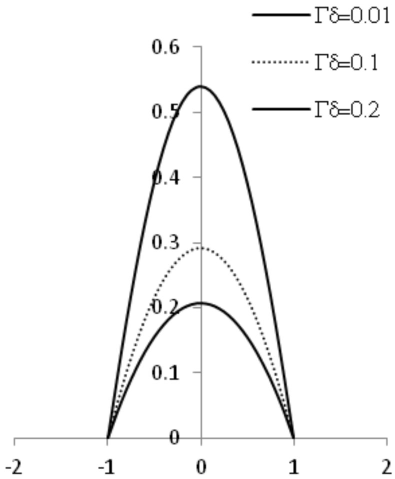

Figure 4. Temperature profile for fixed values of α=1, Γ_ρ=0.2, v= 0.01 and for various values of Γd

Case 2: $P_{eo}e^{\frac{(1+\alpha)g}{n}} << 1$

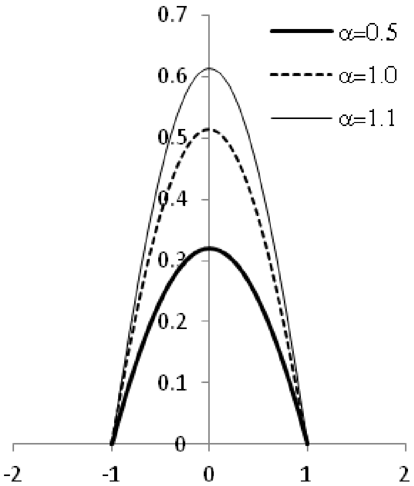

Figure 5. Temperature profile for fixed values of Γ_r= 0.04, Γ_d=0.2, v= 0.01 and for various values of α

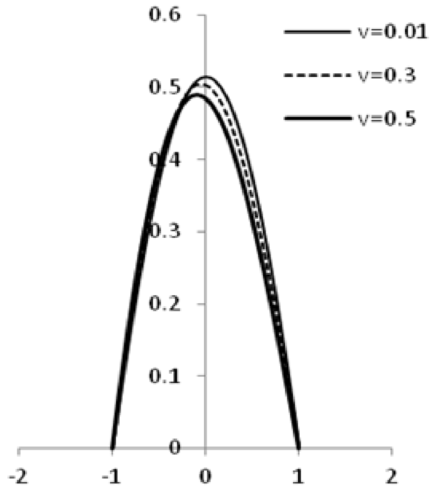

Figure 6. Temperature profile for fixed values of α=1, Γ_d=0.2, Gr= 0.4, and for various values of v

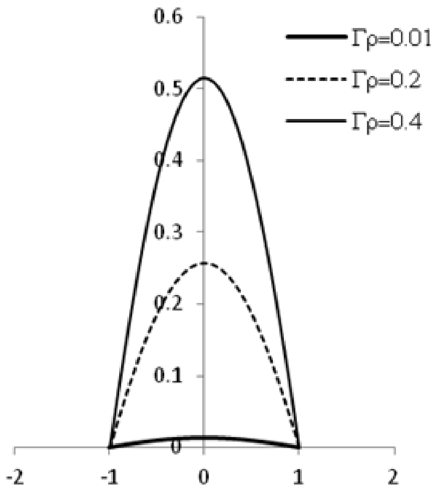

Figure 7. Temperature profile for fixed values of α=1, Γ_r=0.2, v=0.01and for various values of Γ_r

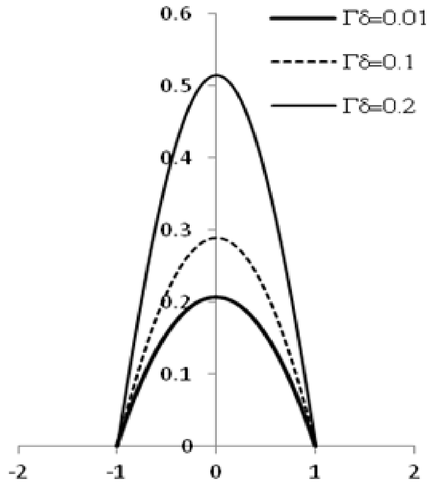

Figure 8. Temperature profile for fixed values of α=1, Γ_ρ=0.4, v= 0.01 and for various values of Γ_d

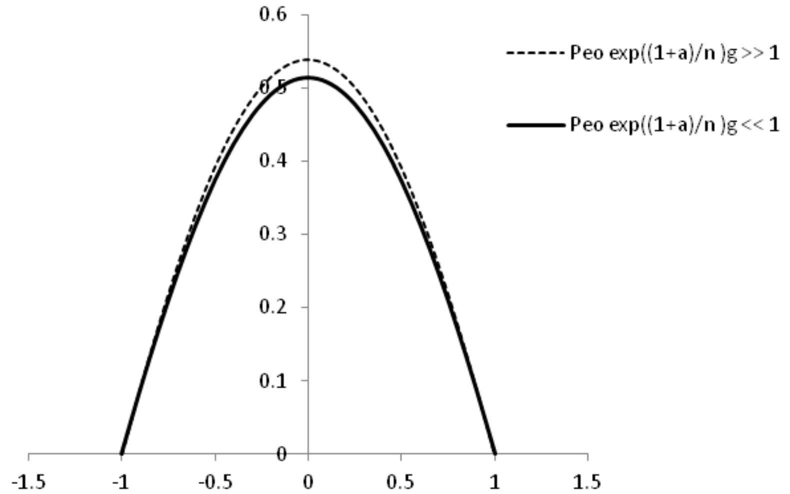

Figure 9. Temperature profile for fixed values of Γ_r=0.4, Γ_d= 0.2, v= 0.01, α=1.0, showing cases 1 and 2

3. RESULTS AND DISCUSSION

The paper examined the effects of thin flame velocity and other sensitive factors such as activation energy ratio, gravitational differentiation and radioactive decay, on maximum temperature of the reaction during thermal explosion which occur in the earth's interior.

We considered two special cases of the model:

(i). Case 1: $P_{eo}e^{\frac{(1+\alpha)g}{n}} >> 1$ (heat transfer due to GD or thermal convection far exceeds the conductive heat transfer, [4])

(ii).Case 2: $P_{eo}e^{\frac{(1+\alpha)g}{n}} << 1$ the contrast of (i).

Two Theorems which established the criteria for the existence of unique solution of the resulting equations were formulated and proved. The proofs of theorems showed that the problems have a unique solution. The interpretation being that the model represents a physical problem. The resulting systems of equations were solved numerically by shooting method.

Figs. 1 and 5 showed that the activation energy ratio (α) had significant effects on maximum temperature of the reaction. As α increases, the maximum temperature also rises. Figs. 2 and 6 showed that the flame velocity (v) had significant effects on maximum temperature of the reaction. A reasonable adjustment of flame velocity v produced the maximum temperature of the reaction. Figures 3 and 7 showed that the heat released from the radioactive decay process (Γ_r) had appreciable effects on maximum temperature of the reaction. A rise in Γ_r increases the maximum temperature of the reaction. Figs. 4 and 8 showed that the heat released during Gravitational Differentiation (Γ_d) had significant effects on maximum temperature of the reaction. A rise in Γ_d increased the maximum temperature of the reaction. Fig. 9 showed the difference between the two cases. The maximum temperature increased in case 1 when heat transfer due to GD or thermal convection exceeded the conductive heat transfer.

4. CONCLUSION

Conclusively, thin flame velocity and other sensitive factors have significant effects on maximum temperature of the reactions during thermal explosion that occured in the interior of the earth. Specifically, a slight regulation of thin flame velocity has notable impact in terms of heat release.

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