STUDY ON INTUITIONISTIC FUZZY BI-IDEALS IN GAMMA NEAR RINGS

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Abstract. *In this paper, we introduce the intuitionistic fuzzy bi ideals in Γ-near-rings and investigate some of their related properties.*

Keywords: *Γ-near-rings, Intuitionistic Fuzzy ideals, Intuitionistic Fuzzy bi-ideals.*

1. INTRODUCTION

Following the introduction of fuzzy sets by L. A. Zadeh [18], the fuzzy set theory has been used for many applications in the domain of mathematics and elsewhere. The idea of "Intuitionistic Fuzzy Set" was first published by K.T.Atanassov [1] as a generalization of the notion of fuzzy set. The concept of Γ-near-ring, a generalization of both the concepts nearing and Γ-ring was introduced by Satyanarayana [16]. Later, several authors such as Booth [4] and Satyanarayana [15] studied the ideal theory of Γ-near-rings. Later Jun et.al [8] considered the fuzzification of left (resp. right) ideals of Γ-near-rings. In this paper, we introduce the notion an intuitionistic fuzzy bi-ideal in a Γ-near-ring and some properties of such bi-ideals are investigated. The homomorphic property of intuitionistic fuzzy bi-ideals is established.

2. PRELIMINARIES

In this section we include some elementary aspects that are necessary for this paper.

Definition 2.1[15] A non–empty set R with two binary operations "+" (addition) and "." (multiplication) is called a near-ring if it satisfies the following axioms:

(i) (R, +) is a group,

(ii) (R, .) is a semigroup,

(iii) $(x + y) . z = x . z + y . z$, for all $x, y, z \in R$. It is a right near-ring because it satisfies the right distributive law.

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Definition 2.2[16] A Γ-near-ring is a triple $(M, +, \Gamma)$ where
(i) $(M, +)$ is a group,
(ii) Γ is a nonempty set of binary operators on M such that for each $\alpha \in \Gamma$, $(M, +, \alpha)$ is a near-ring,
(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.3[16] A subset A of a Γ-near-ring M is called a left (resp. right) ideal of M if
(i) $(A, +)$ is a normal divisor of $(M, +)$,
(ii) $u\alpha(x + v) - u\alpha v \in A$ (resp . $x\alpha u \in A$) for all $x \in A$, $\alpha \in \Gamma$ and $u, v \in M$.

Definition 2.4[17] Let M be Γ-near-ring. A subgroup A of M is called a bi-ideal of M if $(A\Gamma M\Gamma A) \cap (A\Gamma M)\Gamma^* A \subseteq A$. where the operation '* 'is defined by

$$A\Gamma^*B = \{a\gamma(a' + b) - a\gamma a') / a, a' \in A, \gamma \in \Gamma, b \in B \}$$

Definition 2.5[17] Let M be Γ-near-ring. A subgroup Q of M is called a quasi-ideal of M if $(Q\Gamma M) \cap (M\Gamma Q) \cap (M\Gamma)^*Q \subseteq Q$.

Definition 2.6[8] Let M and N be Γ-near-rings. A mapping $f: M \to N$ is said to be a homomorphism if $f(a\alpha b) = f(a)\alpha f(b)$ for all $a, b \in M$ and $\alpha \in \Gamma$.

Definition 2.7[8] A fuzzy set μ in a Γ-near-ring M is called a fuzzy left (resp.right) ideal of M if
(i) $\mu(x-y) \geq \min\{ \mu(x), \mu(y) \}$,
(ii) $\mu(y + x-y) \geq \mu(x)$, for all $x, y \in M$,
(iii) $\mu(u\alpha(x + v) -u\alpha v) \geq \mu(x)$ (resp. $\mu(x\alpha u) \geq \mu(x)$) for all $x, u, v \in M$ and $\alpha \in \Gamma$.

Definition 2.9[1] Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{ < x, \mu_A(x) , \nu_A(x) >/ x \in X \}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A: X \to [0, 1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Notation. For the sake of simplicity, we shall use the symbol $A= <\mu_A, \nu_A>$ for the IFS

$$A = \{< x, \mu_A(x), \nu_A(x) > /x \in X\}.$$

Definition 2.10[1] Let X be a non-empty set and let $A = <\mu_A, \nu_A>$ and $B = <\mu_B, \nu_B>$ be IFSs in X. Then
(1) $A \subset B$ iff $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.
(2) $A = B$ iff $A \subset B$ and $B \subset A$.
(3) $A^c = < \nu_A, \mu_A >$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$.
(6) $\Box A = (\mu_A, 1-\mu_A)$, $\Diamond A = (1-\nu_A, \nu_A)$.

Definition 2.11 Let A be an IFS in a Γ-near-ring M. For each pair $< t, s > \in [0, 1]$ with $t + s \leq 1$, the set $A_{<t,s>} = \{ x \in X / \mu_A(x) \geq t \text{ and } \nu_A(x) \leq s \}$ is called a $<t, s>$ level subset of A.

Definition 2.12 Let $A = < \mu_{A,} \nu_A >$ be an intuitionistic fuzzy set in M and let $t \in [0, 1]$. Then the sets $U(\mu_A ; t) = \{x \in M : \mu_A(x) \geq t\}$ and $L(\nu_A ; t) = \{x \in M : \nu_A(x) \leq t\}$ are called upper level set and lower level set of A respectively.

3. INTUITIONISTIC FUZZY BI-IDEALS OF Γ-NEAR-RINGS

In what follows, M will denote a Γ-near-ring unless otherwise specified.

Definition 3.1. An intuitionistic fuzzy ideal $A = \langle \mu_A, \nu_A \rangle$ of M is called an intuitionistic fuzzy bi-ideal of M if

(i) $\mu_A(x - y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$,
(ii) $\mu_A(y + x - y) \geq \mu_A(x)$,
(iii)$\mu_A((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z)) \geq \{\mu_A(x) \wedge \mu_A(z)\}$for all x, y, z $\in$ M, $\alpha, \beta \in \Gamma$.
(iv)$\nu_A(x - y) \leq \{\nu_A(x) \vee \nu_A(y)\}$,
(v) $\nu_A(y + x - y) \leq \nu_A(x)$,
(vi)$\nu_A((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z)) \leq \{\nu_A(x) \vee \nu_A(z)\}$for all x, y, z $\in$ M, $\alpha, \beta \in \Gamma$.

Example 3.2. Let R be the set of all integers then R is a ring.
Take $M = \Gamma = R$. Let a, b $\in$ M, $\alpha \in \Gamma$, suppose $a\alpha b$ is the product of a, α, b $\in$ R.
Then M is a Γ –near-ring.
Define an IFS $A = < \mu_A, \nu_A >$ in R as follows.

$$\mu_A(0) = 1 \text{ and } \mu_A(\pm 1) = \mu_A(\pm 2) = \mu_A(\pm 3) = ... = t \text{ and}$$
$$\nu_A(0) = 0 \text{ and } \nu_A(\pm 1) = \quad \nu_A(\pm 2) = \nu_A(\pm 3) = ... = s,$$

where $t \in [0, 1)$, $s \in (0, 1]$ and $t + s \leq 1$.
By routine calculations,

Clearly A is an intuitionistic fuzzy bi-ideal of a Γ-near-ring M.

Lemma 3.3. If B is a bi-ideal of M then for any $0 < t, s < 1$, there exists an intuitionistic fuzzy bi-ideal $C = < \mu_{C,} \nu_C >$ of M such that $C_{<t, s>} = B$.

Proof: Let $C \to [0, 1]$ be a function defined by

$$\mu_B(x) = \begin{cases} t & \text{if } x \in B, \\ 0 & \text{if } x \notin B, \end{cases} \qquad \nu_B(x) = \begin{cases} s & \text{if } y \in B, \\ 1 & \text{if } y \notin B. \end{cases}$$

for all $x \in M$ and the pair s, t $\in$[0, 1]. Then $C_{<t, s>} = B$ is an intuitionistic fuzzy bi-ideal of M with $t + s \leq 1$.

Now suppose that B is a bi-ideal of M. For all x, y $\in$ B, such that x - y $\in$ B, we have

$\mu_c(x - y) \geq t = \{\mu_c(x) \wedge \mu_c(y)\}$,
$\nu_c(x - y) \leq s = \{\nu_c(x) \vee \nu_c(y)\}$,
$\mu_c(y + x - y) \geq t = \mu_c(x)$,
$\nu_c(y + x - y) \leq s = \nu_c(x)$,

Also, for all x, y, z $\in$ B and α, $\beta \in \Gamma$ such that $x\alpha y\beta z \in B$, we have

$$\mu_c((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z))) \geq t = \{\mu_c(x) \wedge \mu_c(z)\},$$
$$\nu_c((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z))) \leq s = \{\nu_c(x) \vee \nu_c(z)\}$$

Thus $C_{<t, s>}$ is an intuitionistic fuzzy bi-ideal of M.

Lemma 3.4. Let B be a non-empty subset of M. Then B is a bi-ideal of M if and only if the IFS $\bar{B} = < \chi_B , \bar{\chi}_B >$ is an intuitionistic fuzzy ideal of M.

Proof: Let x, y $\in$ B. From the hypothesis, x - y $\in$ B.

(i) If x, y $\in$ B, then $\chi_B(x) = 1$, $\bar{\chi}_B(x) = 0$, $\chi_B(y) = 1$ and $\bar{\chi}_B(y) = 0$. In this case
$\chi_B(x - y) = 1 \geq \{\chi_B(x) \wedge \chi_B(y)\}$.
$\bar{\chi}_B(x - y) = 0 \leq \{ \bar{\chi}_B(x) \vee \bar{\chi}_B(y)\}$.

(ii) If x $\in$ B, y $\notin$ B, then $\chi_B(x) = 1$, $\bar{\chi}_B(x) = 0$, $\chi_B(y) = 0$, and $\bar{\chi}_B(y) = 1$. Thus,
$\chi_B(x - y) = 0 \geq \{\chi_B(x) \wedge \chi_B(y)\}$.
$\bar{\chi}_B(x - y) = 1 \leq \{ \bar{\chi}_B(x) \vee \bar{\chi}_B(y)\}$.

(iii)If x $\notin$ B, y $\in$ B, then $\chi_B(x) = 0$, $\bar{\chi}_B(x) = 1$ and $\chi_B(y) = 1$ and $\bar{\chi}_B(y) = 0$. Thus,
$\chi_B(x - y) = 0 \geq \{\chi_B(x) \wedge \chi_B(y)\}$.
$\bar{\chi}_B(x - y) = 1 \leq \{ \bar{\chi}_B(x) \vee \bar{\chi}_B(y)\}$.

(iv) If x $\notin$ B, y $\notin$ B, then $\chi_B(x) = 0$, $\bar{\chi}_B(x) = 1$, $\chi_B(y) = 0$ and $\bar{\chi}_B(y) = 1$. Thus,
$\chi_B(x - y) \geq 0 = \{\chi_B(x) \wedge \chi_B(y)\}$.
$\bar{\chi}_B(x - y) \leq 1 = \{ \bar{\chi}_B(x) \vee \bar{\chi}_B((y)\}$.

Thus (i) of Definition 3.1 holds good.

Let x, y $\in$ B. From the hypothesis, y + x - y $\in$ B.

(i) If x, y $\in$ B, then $\chi_B(x) = 1$, $\bar{\chi}_B(x) = 0$, $\chi_B(y) = 1$ $\bar{\chi}_B(y) = 0$. In this case
$\chi_B(y + x - y) = 1 \geq \chi_B(x)$.
$\bar{\chi}_B(y + x - y) = 0 \leq \bar{\chi}_B(x)$.

(ii) If x $\in$ B, y $\notin$ B, then $\chi_B(x) = 1$, $\bar{\chi}_B(x) = 0$, $\chi_B(y) = 0$ and $\bar{\chi}_B(y) = 1$. Thus,
$\chi_B(y + x - y) = 0 \geq \chi_B(x)$.
$\bar{\chi}_B(y + x - y) = 1 \leq \bar{\chi}_B(x)$.

(iii) If x $\notin$B, y $\in$B, then $\chi_B(x) = 0$, $\bar{\chi}_B(x) =1$, $\chi_B(y) =1$ and $\bar{\chi}_B (y)= 0$. Thus,
$\chi_B(y + x - y) = 0 \geq \chi_B(x)$.
$\bar{\chi}_B(y + x - y) = 1 \leq \bar{\chi}_B(x)$.

(iv) If x $\notin$ B, y $\notin$ B, then $\chi_B(x) = 0$, $\bar{\chi}_B(x) = 1$, $\chi_B(y) = 0$ and $\bar{\chi}_B(y)= 1$. Thus,
$\chi_B(y + x - y) \geq 0 = \chi_B(x)$.
$\bar{\chi}_B(y + x - y) \leq 1 = \bar{\chi}_B(x)$.

Thus (ii) of Definition 3.1 holds good.

Let x, y, z $\in$ B and α, $\beta \in \Gamma$. From the hypothesis, $x\alpha y\beta z$, $x\alpha(y + z) - x\alpha z \in B$.

(i) If x, z $\in$ B, then $\chi_B(x) = 1$, $\bar{\chi}_B(x) = 0$, $\chi_B(z) = 1$ and $\bar{\chi}_B(z) = 0$. Thus,
$\chi_B(\mu((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z))) = 1 \geq \{\chi_B(x) \wedge \chi_B(z)\}$.
$\bar{\chi}_B(\mu((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z))) = 0 \leq \{ \bar{\chi}_B(x) \vee \bar{\chi}_B(z)\}$.

(ii) If x $\in$ B, z $\notin$ B, then $\chi_B(x) = 1$, $\bar{\chi}_B(x) = 0$, $\chi_B(z) = 0$ and $\bar{\chi}_B(z) = 1$. Thus,
$\chi_B(\mu((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z))) = 0 \geq \{\chi_B(x) \wedge \chi_B(z)\}$.
$\bar{\chi}_B(\mu((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z))) = 1 \leq \{ \bar{\chi}_B(x) \vee \bar{\chi}_B(z)\}$.

(iii) If x $\notin$ B, z $\in$ B, then $\chi_B(x) = 0$, $\bar{\chi}_B(x) = 1$, $\chi_B(z) = 1$ and $\bar{\chi}_B(z) = 0$. Thus,
$\chi_B(\mu((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z))) = 0 \geq \{\chi_B(x) \wedge \chi_B(z)\}$.
$\bar{\chi}_B(\mu((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z))) = 1 \leq \{ \bar{\chi}_B(x) \vee \bar{\chi}_B(z)\}$.

(iv) If x $\notin$ B, z $\notin$ B, then $\chi_B(x) = 0$, $\bar{\chi}_B(x) = 1$, $\chi_B(z) = 0$ and $\bar{\chi}_B(z) = 1$. Thus,
$\chi_B(\mu((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z))) \geq 0 = \{\chi_B(x) \wedge \chi_B(z)\}$.
$\bar{\chi}_B(\mu((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z))) \leq 1 = \{ \bar{\chi}_B(x) \vee \bar{\chi}_B(z)\}$.

Thus (iii) of Definition 3.1 holds good.

Conversely, suppose that IFS $\bar{B} = < \chi_B , \bar{\chi}_B >$ is an intuitionistic fuzzy ideal of M. Then by Lemma 3.3, χ_B is two-valued, Hence B is a bi-ideal of M.
This completes the proof.

Theorem 3.5. If $\{A_i\}_{i\in\wedge}$ is a family of intuitionistic fuzzy bi-ideals of M then $\cap A_i$ is an intuitionistic fuzzy bi-ideals of M, where $\cap A_i = \{\Lambda\mu_{Ai}, V\nu_{Ai}\}$,
$\Lambda\mu_{Ai}(x) = \inf \{\mu_{Ai}(x) \mid i \in \Lambda, x \in M\}$ and $V\nu_{Ai}(x) = \sup \{\nu_{Ai}(x) \mid i \in V, x \in M\}$.

Proof: Let x, y $\in$ M. Then we have

$$\begin{aligned}\Lambda\mu_{Ai}(x - y) &= \inf \{\{\mu_{Ai}(x) \wedge \mu_{Ai}(y)\} \mid i \in \Lambda, x, y \in M\}\\ &= \{\{\inf (\mu_{Ai}(x)) \wedge \inf (\mu_{Ai}(y))\} \mid i \in \Lambda, x, y \in M\}\\ &= \{\{\inf (\mu_{Ai}(x) \mid i \in \Lambda, x\in M)\} \wedge \{\inf (\mu_{Ai}(y) \mid i \in \Lambda, y \in M)\}\}\\ &= \{\Lambda\mu_{Ai}(x) \wedge \Lambda\mu_{Ai}(y)\}.\end{aligned}$$

Let x, y $\in$ M. Then we have

$$\begin{aligned}\Lambda\mu_{Ai}(y + x - y) &= \inf \{\mu_{Ai}(x) \mid i \in \Lambda, x, y \in M\}\\ &= \Lambda\mu_{Ai}(x).\end{aligned}$$

Let x, y, z $\in$ M and α, $\beta \in \Gamma$.

$$\begin{aligned}\Lambda\mu_{Ai}((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z)) &= \inf\{\{\mu_{Ai}(x) \wedge \mu_{Ai}(z)\} \mid i \in \Lambda, x, z \in M\}\\ &= \{\{\inf (\mu_{Ai}(x)) \wedge \inf (\mu_{Ai}(z))\} \mid i \in \Lambda, x, z \in M\}\end{aligned}$$

$$= \{\{\inf (\mu_{Ai}(x) \mid i \in \Lambda, x \in M)\} \wedge \{\inf (\mu_{Ai} \mid i \in \Lambda, z \in M)\}\}$$
$$= \{\Lambda\mu_{Ai}(x) \wedge \Lambda\mu_{Ai}(z)\}.$$

Let x, y $\in$ M. Then we have

$$V\nu_{Ai}(x - y) = \sup \{\{\nu_{Ai}(x) \vee \nu_{Ai}(y)\} \mid i \in V, x, y \in M\}$$
$$= \{\{\sup (\nu_{Ai}(x)) \vee \sup (\nu_{Ai}(y))\} \mid i \in V, x, y \in M\}$$
$$=\{\{\sup (\nu_{Ai}(x) \mid i \in V, x \in M)\} \vee\{\sup (\nu_{Ai}(y) \mid i \in V, y \in M)\}\}$$
$$=\{V\nu_{Ai}(x) \vee V\nu_{Ai}(y)\}.$$

Let x, y $\in$ M. Then we have

$$V\nu_{Ai}(y + x - y) = \sup\{\nu_{Ai}(x) \mid i \in V, x, y \in M\}$$
$$= V\nu_{Ai}(x).$$

= Let x, y, z $\in$ M and $\alpha, \beta \in \Gamma$.

$$V\nu_{Ai}((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z)) = \sup\{\{\nu_{Ai}(x) \vee \nu_{Ai}(z)\} \mid i \in V, x, z \in M\}$$
$$= \{\{\sup(\nu_{Ai}(x)) \vee \sup(\nu_{Ai}(z))\} \mid i \in V, x, z \in M\}$$
$$= \{\{\sup(\nu_{Ai}(x) \mid i \in V, x \in M)\} \vee \{\sup(\nu_{Ai}(z) \mid i \in \Lambda, z \in M)\}\}$$
$$= \{V\nu_{Ai}(x) \vee V\nu_{Ai}(z)\}.$$

Hence, $\cap A_i = \{\Lambda\mu_{Ai}, V\nu_{Ai}\}$ is an intuitionistic fuzzy bi-ideal of M.

Theorem 3.6. If A is an intuitionistic fuzzy bi-ideal of M then A' is also an intuitionistic fuzzy bi-ideal of M.

Proof: Let x, y $\in$ M. We have

$$\mu_A'(x - y) = 1 - \mu_A(x - y)$$
$$= 1 - \{\mu_A(x) \wedge \mu_A(y)\},$$
$$\nu_A'(x - y) = 1 - \nu_A(x - y)$$
$$= 1 - \{\nu_A(x) \vee \nu_A(y)\}.$$

Let x, y $\in$ M. We have

$$\mu_A'(y + x - y) = 1 - \mu_A(y + x - y)$$
$$= 1 - \mu_A(x)$$
$$= \mu_A'(x),$$

$$\nu_A'(y + x - y) = 1 - \nu_A(y + x - y)$$
$$= 1 - \nu_A(x)$$
$$= \nu_A'(x).$$

Let x, y, z $\in$ M and $\alpha, \beta \in \Gamma$. We have

$$\begin{aligned}\mu_A{}'((x\alpha y\beta z)\wedge(x\alpha(y+z)-x\alpha z)) &= 1-\mu_A((x\alpha y\beta z)\wedge(x\alpha(y+z)-x\alpha z))\\ &= 1-\{\mu_A(x)\wedge\mu_A(z)\}\\ &= \{1-\mu_A(x)\wedge 1-\mu_A(z)\}\\ &= \{\mu_A{}'(x)\wedge\mu_A{}'(z)\},\end{aligned}$$

$$\begin{aligned}\nu_A{}'((x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z)) &= 1-\nu_A((x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z))\\ &= 1-\{\nu_A(x)\vee\nu_A(z)\}\\ &= \{1-\nu_A(x)\vee 1-\nu_A(z)\}\\ &= \{\nu_A{}'(x)\vee\nu_A{}'(z)\}.\end{aligned}$$

Therefore, A' is also an intuitionistic fuzzy bi-ideal of M.

Theorem 3.7. An intuitionistic fuzzy set A of M is an intuitionistic fuzzy bi-ideal of M if and only if the level sets $U(\mu_A; t) = \{x \in M \mid \mu(x) \geq t\}$ and $L(\nu_A ; t) = \{x \in M : \nu_A(x) \leq t\}$ are a bi-ideal of M when it is non-empty.

Proof: Let A be an intuitionistic fuzzy bi-ideal of M.
Then $\mu_A(x - y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$.

$$\begin{aligned}x, y \in U(\mu_A; t) &\Rightarrow \mu_A(x) \geq t,\ \mu_A(y) \geq t\\ &\quad \mu_A(x-y) \geq \{\mu_A(x)\wedge\mu_A(y)\} \geq t\\ &\quad \mu_A(x-y) \geq t\\ &\Rightarrow x-y \in U(\mu_A; t).\end{aligned}$$

$\mu_A(x - y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$.

$$\begin{aligned}x, y \in L(\nu_A ; t) &\Rightarrow \nu_A(x) \leq t,\ \nu_A(y) \leq t\\ &\quad \nu_A(x-y) \leq \{\nu_A(x)\vee\nu_A(y)\} \leq t\\ &\quad \nu_A(x-y) \leq t\\ &\Rightarrow x-y \in L(\nu_A ; t).\end{aligned}$$

Let $\mu_A(y+ x - y) \geq \mu_A(x)$.

$$\begin{aligned}x, y \in U(\mu_A; t) &\Rightarrow \mu_A(x) \geq t,\ \mu_A(y) \geq t\\ &\quad \mu_A(y+x-y) \geq \mu_A(x) \geq t\\ &\quad \mu_A(y+x-y) \geq t\\ &\Rightarrow y+x-y \in U(\mu_A; t).\end{aligned}$$

Let $\nu_A(y+ x - y) \leq \nu_A(x)$.

$$\begin{aligned}x, y \in L(\nu_A ; t) &\Rightarrow \nu_A(x) \leq t,\ \nu_A(y) \leq t\\ &\quad \nu_A(y+x-y) \leq \nu_A(x) \leq t\\ &\quad \nu_A(y+x-y) \leq t\\ &\Rightarrow y+x-y \in L(\nu_A ; t).\end{aligned}$$

Also, let

$$\mu_A((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z)) \geq \{\mu_A(x) \wedge \mu_A(z)\}.$$

$x, y, z \in U(\mu_A; t)$, $\alpha, \beta \in \Gamma \Rightarrow \mu_A(x) \geq t, \mu_A(y) \geq t, \mu_A(z) \geq t$

$\mu_A((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z)) \geq \{\mu_A(x) \wedge \mu_A(z)\} \geq t$

$\mu_A((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z)) \geq t$

$\Rightarrow (x\alpha y\beta z), (x\alpha(y + z) - x\alpha z) \in U(\mu_A; t)$.

Thus, $U(\mu_A; t)$ is a bi-ideal of M.

$$\nu_A((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z)) \leq \{\nu_A(x) \vee \nu_A(z)\}.$$

$x, y, z \in L(\nu_A; t)$, $\alpha, \beta \in \Gamma \Rightarrow \nu_A(x) \leq t, \nu_A(y) \leq t, \nu_A(z) \leq t$

$\nu_A((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z)) \leq \{\nu_A(x) \vee \nu_A(z)\} \leq t$

$\nu_A((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z)) \leq t$

$\Rightarrow (x\alpha y\beta z), (x\alpha(y + z) - x\alpha z) \in L(\nu_A; t)$.

Thus, $L(\nu_A; t)$ is a bi-ideal of M.
Conversely, if $U(\mu_A; t)$ is a bi-ideal of M let $t = \{\mu_A(x) \wedge \mu_A(y)\}$. Then

$x, y \in U(\mu_A; t), \Rightarrow x - y \in U(\mu_A; t)$

$\Rightarrow \mu_A(x - y) \geq t$

$\Rightarrow \mu_A(x - y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$.

Also, $x, y \in U(\mu_A; t), \Rightarrow y + x - y \in U(\mu_A; t)$

$\Rightarrow \mu_A(y + x - y) \geq \mu_A(x)$.

if $L(\nu_A; t)$ is a bi-ideal of M let $t = \{\nu_A(x) \vee \nu_A(y)\}$. Then

$x, y \in L(\nu_A; t), \Rightarrow x - y \in L(\nu_A; t)$

$\Rightarrow \nu_A(x - y) \leq t$

$\Rightarrow \nu_A(x - y) \leq \{\nu_A(x) \vee \nu_A(y)\}$.

Also, $x, y \in L(\nu_A; t), \Rightarrow y + x - y \in L(\nu_A; t)$

$\Rightarrow \nu_A(y + x - y) \leq \nu_A(x)$.

Next, define $t = \{\mu_A(x) \wedge \mu_A(z)\}$. Then

$x, y, z \in U(\mu; t)$, $\alpha, \beta \in \Gamma \Rightarrow (x\alpha y\beta z), (x\alpha(y + z) - x\alpha z) \in U(\mu_A; t)$

$\Rightarrow \mu_A((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z)) \geq t$

$\Rightarrow \mu_A((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z)) \geq \{\mu(x) \wedge \mu(z)\}$.

Next, define $t = \{\nu_A(x) \vee \nu_A(z)\}$. Then

$$x, y, z \in U(\mu; t),\ \alpha, \beta \in \Gamma \Rightarrow (x\alpha y\beta z), (x\alpha(y + z) - x\alpha z) \in L(\nu_A ; t)$$
$$\Rightarrow \nu_A((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z)) \leq t$$
$$\Rightarrow \nu_A((x\alpha y\beta z) \vee (x\alpha(y + z) - x\alpha z)) \leq \{\nu_A(x) \vee \nu_A(z)\}.$$

Consequently, A is an intuitionistic fuzzy bi-ideal of M.

Theorem 3.8. Let A be an intuitionistic fuzzy bi-ideal of M. If M is completely regular, then $\mu_A(a) = \mu_A(a\alpha a)$, $\nu_A(a) = \nu_A(a\alpha a)$ for all $a \in M$ and $\alpha \in \Gamma$.

Proof: Straightforward.

Let f be mappings from a set X to Y, and A be intuitionistic fuzzy set on Y. Then the preimage of μ under f, denoted by $f^{-1}(A)$, is defined by

$$f^{-1}(\mu_A(x)) = \mu_A(f(x)),\ f^{-1}(\nu_A(x)) = \nu_A(f(x)) \text{ for all } x \in X.$$

Theorem 3.9. Let the pair of mappings $f : M \to N$ be a homomorphism of Γ-near-rings. If μ is an intuitionistic fuzzy bi-ideal of N, then the preimage $f^{-1}(A)$ of A under f is an intuitionistic fuzzy bi-ideal of M.

Proof: Let x, y ∈ M. Then we have

$$\begin{aligned} f^{-1}(\mu_A)(x - y) &= \mu_A(f(x - y)) \\ &= \mu_A(f(x) - f(y)) \\ &\geq \{\mu_A(f(x)) \wedge \mu_A(f(y))\} \\ &= \{f^{-1}(\mu_A(x)) \wedge f^{-1}(\mu_A(y))\}. \end{aligned}$$

$$\begin{aligned} f^{-1}(\nu_A)(x - y) &= \nu_A(f(x - y)) \\ &= \nu_A(f(x) - f(y)) \\ &\leq \{\nu_A(f(x)) \vee \nu_A(f(y))\} \\ &= \{f^{-1}(\nu_A(x)) \vee f^{-1}(\nu_A(y))\}. \end{aligned}$$

Let x, y ∈ M. Then we have

$$\begin{aligned} f^{-1}(\mu_A)(y + x - y) &= \mu_A(f(y + x - y)) \\ &\geq \mu_A(f(x)) \\ &= f^{-1}(\mu_A(x)). \end{aligned}$$

$$\begin{aligned} f^{-1}(\nu_A)(y + x - y) &= \nu_A(f(y + x - y)) \\ &\leq \nu_A(f(x)) \\ &= f^{-1}(\nu_A(x)). \end{aligned}$$

Let x, y, z ∈ M and α, β ∈ Γ. Then:

$$\begin{aligned} f^{-1}(\mu_A)((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z)) &= \mu_A(f((x\alpha y\beta z) \wedge (x\alpha(y + z) - x\alpha z))) \\ &= \mu_A((f(x\alpha y\beta z)) \wedge (f(x\alpha(y + z) - x\alpha z))) \\ &\geq \mu_A(f(x)) \wedge \mu_A(f(z))\} \\ &= \{f^{-1}(\mu_A(x)) \wedge f^{-1}(\mu_A(z))\}. \end{aligned}$$

Therefore $f^{-1}(\mu_A)$ is an intuitionistic fuzzy bi-ideal of M .

$$\begin{aligned} f^{-1}(\nu_A)((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) &= \nu_A(f((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z))) \\ &= \nu_A((f(x\alpha y\beta z)) \vee (f(x\alpha(y+z) - x\alpha z))) \\ &\leq \{\nu_A(f(x)) \vee \nu_A(f(z))\} \\ &= \{f^{-1}(\nu_A(x)) \vee f^{-1}(\nu_A(z))\}. \end{aligned}$$

Therefore $f^{-1}(\nu_A)$ is an intuitionistic fuzzy bi-ideal of M.
Therefore $f^{-1}(A)$ is an intuitionistic fuzzy bi-ideal of M.

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