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SURFACES FAMILY WITH COMMON SMARANDACHE GEODESIC CURVE

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Abstract. *In this paper, we analyzed the problem of finding a surfaces family through a given some special Smarandache curves in Euclidean 3-space. Using the Frenet frame of the curve in Euclidean 3-space, we constructed a differential equation system such that a given Smarandache curves satisfy both the geodesic and isoparametric requirements. The extension to ruled surfaces is also outlined. Finally, we demonstrated some interesting surfaces about subject.*

Keywords: *Geodesic Curve, Smarandache Curves, Frenet Frame, Ruled Surface.*
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1. INTRODUCTION

One of most significant curve on a surface is geodesic curve. Most people have heard the phrase; a straight line is the shortest distance between two points. But in differential geometry, they say this same thing in a different language. They say instead geodesics for the Euclidean metric are straight lines. A geodesic is a curve that represents the extremal value of a distance function in some space. In the Euclidean space, extremal means 'minimal',so geodesics are paths of minimal arc length. Geodesics are important in the relativistic description of gravity. In general relativity, geodesics generalize the notion of "straight lines" to curved space time. This concept is based on the mathematical concept of a geodesic. Importantly, the world line of a particle free from all external force is a particular type of geodesic. In other words, a freely moving particle always moves along a geodesic. Einstein’s principle of equivalence tells us that geodesics represent the paths of freely falling particles in a given space. (Freely falling in this context means moving only under the influence of gravity, with no other forces involved). The geodesics principle states that the free trajectories are the geodesics of space.

The problem of finding surfaces with a given common curve as a special curve was firstly handled by Wang et.al. [1]. They constructed surfaces with a common geodesic. Kasap et.al. [2] generalized the work of Wang by introducing new types of marching-scale functions, coefficients of the Frenet frame appearing in the parametric representation of surfaces. With the inspiration of work of Wang, Li et.al.[3] changed the characteristic curve from geodesic to line of curvature and defined the surface pencil with a common line of curvature. In [4] Bayram et.al. defined the surface pencil with a common asymptotic curve. They introduced three types of marching-scale functions and derived the necessary and sufficient conditions on them to satisfy both parametric and asymptotic requirements.

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In addition, differential geometry has an important role in the study of special curves. Smarandache curves are one of these curves. Special Smarandache curves have been investigated by some differential geometers [5-11].

This curve is defined as, a regular curve in Minkowski space-time, whose position vector is composed by Frenet frame vectors on another regular curve, is Smarandache curve [5]. A.T. Ali has introduced some special Smarandache curves in the Euclidean space [6]. Special Smarandache curves according to Bishop Frame in Euclidean 3-space have been investigated by Çetin et al [7]. In addition, Special Smarandache curves according to Darboux Frame in Euclidean 3-space has introduced in [8]. They found some properties of these special curves and calculated normal curvature, geodesic curvature and geodesic torsion of these curves. Also, they investigate special Smarandache curves in Minkowski 3-space, [9]. Furthermore, they find some properties of these special curves and they calculate curvature and torsion of these curves. Special Smarandache curves such as $\gamma t, td, \gamma td$ -Smarandache curves according to Sabban frame in Euclidean unit sphere S^2 has introduced in [10]. Also, they give some characterization of Smarandache curves and illustrate examples of their results. On the Quaternionic Smarandache Curves in Euclidean 3-Space have been investigated in [11]. Atalay and Kasap [12], studied the problem: given a curve (with Bishop frame), how to characterize those surfaces that posess this curve as a common isogeodesic and Smarandache curve in Euclidean 3-space. Atalay and Kasap [13-14] analyzed the problem of consructing a family of surfaces from a given some special Smarandache curves using the Bishop frame and Frenet frame of the curve in Euclidean 3-space. So, they are given to show the family of surfaces with common Smarandache asymptotic curve. Bayram et al. [15], found a surface family possessing the natural lift of a given curve as a common asymptotic curve. In [16], Bayram and Bilici consructed a surface family possessing an involute of a given curve as an asymptotic curve. Ergün et.al [17] constructed a surface pencil from a given spacelike (timelike) line of curvature in Minkowski 3-space. However, they solved the problem using Frenet frame of the given curve.

In this paper, we study the problem: given a curve (with Frenet frame), how to characterize those surfaces that posess this curve as a common isogeodesic and Smarandache curve in Euclidean 3-space. In section 2, we give some preliminary information about Smarandache curves in Euclidean 3-space and define isogeodesic curve. We express surfaces as a linear combination of the Frenet frame of the given curve and derive necessary and sufficient conditions on marching-scale functions to satisfy both isogeodesic and Smarandache requirements in Section 3. In section 4, the part of the study related to the ruled surfaces is given. Finally, we illustrate the method by giving some examples.

2. PRELIMINARIES

Let E^3 be a 3-dimensional Euclidean space provided with the metric given by

$$\langle\, ,\, \rangle = dx_1^{\,2} + dx_2^{\,2} + dx_3^{\,2}$$

where (x_1, x_2, x_3) is a rectangular coordinate system of E^3. Recall that, the norm of a arbitrary vector $X \in E^3$ is given by $\|X\| = \sqrt{\langle X, X\rangle}$. Let $\alpha = \alpha(s): I \subset IR \to E^3$ is an arbitrary curve of arc-length parameter s. The curve α is called a unit speed curve if velocity vector α' of a satisfies $\|\alpha'\|=1$. Let $\{T(s),N(s),B(s)\}$ be the moving Frenet frame along α, Frenet formulas is given by

$$\frac{d}{ds}\begin{pmatrix}T(s)\\N(s)\\B(s)\end{pmatrix}=\begin{pmatrix}0&\kappa(s)&0\\-\kappa(s)&0&\tau(s)\\0&-\tau(s)&0\end{pmatrix}\begin{pmatrix}T(s)\\N(s)\\B(s)\end{pmatrix},$$

where the function $\kappa(s)$ and $\tau(s)$ are called the curvature and torsion of the curve $\alpha(s)$, respectively.

A curve on a surface is geodesic if and only if the normal vector to the curve is everywhere parallel to the local normal vector of the surface. Another criterion for a curve in a surface M to be geodesic is that its geodesic curvature vanishes.

An isoparametric curve α(s) is a curve on a surface Ψ=Ψ(s,t) is that has a constant s or t-parameter value. In other words, there exist a parameter s_0 or t_0 such that α(s)= Ψ(s,t_0) or α(t)= Ψ(s_0 ,t).

Given a parametric curve α(s), we call α(s) an isogeodesic of a surface Ψ if it is both a geodesic and an isoparametric curve on Ψ.

Let $\alpha=\alpha(s)$ be a unit speed regular curve in E^3 and $\{T(s),N(s),B(s)\}$ be its moving Frenet-Serret frame . Smarandache TN curves are defined by

$$\beta=\beta(s^*)=\frac{1}{\sqrt{2}}(T(s)+N(s)).$$

Smarandache NB curves are defined by

$$\beta=\beta(s^*)=\frac{1}{\sqrt{2}}(N(s)+B(s)).$$

Smarandache TNB curves are defined by

$$\beta=\beta(s^*)=\frac{1}{\sqrt{2}}(T(s)+N(s)+B(s)), [6].$$

3. SURFACES WITH COMMON SMARANDACHE GEODESIC CURVE

Let $\varphi=\varphi(s,v)$ be a parametric surface. The surface is defined by a given curve $\alpha=\alpha(s)$ as follows:

$$\varphi(s,v)=\alpha(s)+[x(s,v)T(s)+y(s,v)N(s)+z(s,v)B(s)],\ L_1\le s\le L_2,\ T_1\le v\le T_2, \quad (3.1)$$

where $x(s,v), y(s,v)$ and $z(s,v)$ are C^1 functions. The values of the marching-scale functions $x(s,v), y(s,v)$ and $z(s,v)$ indicate, respectively; the extension-like, flexion-like and retortionlike effects, by the point unit through the time v, starting from $\alpha(s)$ and $\{\mathrm{T(s)}, \mathrm{N(s)}, \mathrm{B(s)}\}$ is the Frenet frame associated with the curve $\alpha(s)$.

Our goal is to find the necessary and sufficient conditions for which the some special Smarandache curves of the unit space curve $\alpha(s)$ is an parametric curve and an geodesic curve on the surface $\varphi(s,v)$.

Firstly, since Smarandache curve of $\alpha(s)$ is an parametric curve on the surface $\varphi(s,v)$, there exists a parameter $v_0 \in [T_1, T_2]$ such that

$$x(s,v_0) = y(s,v_0) = z(s,v_0) = 0,\ L_1 \le s \le L_2,\ T_1 \le v_0 \le T_2. \tag{3.2}$$

Secondly, since Smarandache curve of $\alpha(s)$ is an geodesic curve on the surface $\varphi(s,v)$, there exist a parameter $v_0 \in [T_1, T_2]$ such that

$$n(s,v_0) \,//\, \mathrm{N(s)}. \tag{3.3}$$

where n is a normal vector of $\varphi = \varphi(s,v)$ and N is a normal vector of $\alpha(s)$.

Theorem 3.1. Smarandache NB curve of the curve $\alpha(s)$ is isogeodesic on a surface $\varphi(s,v)$ if and only if the following conditions are satisfied:

$$\begin{cases} \mathrm{x(s,v_0)} = \mathrm{y(s,v_0)} = \mathrm{z(s,v_0)} = 0, \\ \dfrac{\partial \mathrm{y(s,v_0)}}{\partial \mathrm{v}} = 0 \\ \dfrac{\partial \mathrm{z(s,v_0)}}{\partial \mathrm{v}} = -\dfrac{\tau(\mathrm{s})}{\kappa(\mathrm{s})} \dfrac{\partial \mathrm{x(s,v_0)}}{\partial \mathrm{v}}. \end{cases}$$

Proof: Let $\alpha(s)$ be a Smarandache NB curve . From (3.1) , $\varphi(s,v)$ parametric surface is defined by a given Smarandache NB curve of curve $\alpha = \alpha(s)$ as follows:

$$\varphi(\mathrm{s,v}) = \frac{1}{\sqrt{2}}((\mathrm{N(s)} + \mathrm{B(s)})) + [\mathrm{x(s,v)T(s)} + \mathrm{y(s,v)N(s)} + \mathrm{z(s,v)B(s)}].$$

Let $\alpha(s)$ be a Smarandache NB curve on surface $\varphi(s,v)$. If Smarandache NB curve is an parametric curve on this surface , then there exist a parameter $v = v_0$ such that $\frac{1}{\sqrt{2}}(\mathrm{N(s)} + \mathrm{B(s)}) = \varphi(\mathrm{s,v_0})$, that is,

$$x(s,v_0) = y(s,v_0) = z(s,v_0) = 0 \tag{3.4}$$

The normal vector can be expressed as

$$n(s,v)=\left[\frac{\partial z(s,v)}{\partial v}\left(x(s,v)\kappa(s)+\frac{\partial y(s,v)}{\partial s}-\tau(s)z(s,v)\right)-\frac{\partial y(s,v)}{\partial v}\left(\frac{\tau(s)}{\sqrt{2}}+\frac{\partial z(s,v)}{\partial s}+\tau(s)y(s,v)\right)\right]T(s)$$
$$+\left[\frac{\partial x(s,v)}{\partial v}\left(\frac{\tau(s)}{\sqrt{2}}+\frac{\partial z(s,v)}{\partial s}+\tau(s)y(s,v)\right)-\frac{\partial z(s,v)}{\partial v}\left(-\frac{\kappa(s)}{\sqrt{2}}+\frac{\partial x(s,v)}{\partial s}-\kappa(s)y(s,v)\right)\right]N(s)$$
$$+\left[\frac{\partial y(s,v)}{\partial v}\left(-\frac{\kappa(s)}{\sqrt{2}}+\frac{\partial x(s,v)}{\partial s}-\kappa(s)y(s,v)\right)-\frac{\partial x(s,v)}{\partial v}\left(\frac{\partial y(s,v)}{\partial s}+\kappa(s)x(s,v)-\tau(s)z(s,v)\right)\right]B(s) \tag{3.5}$$

Thus, if we let

$$\begin{cases}\phi_1(s,v_0)=-\dfrac{\partial y(s,v_0)}{\partial v}\dfrac{\tau(s)}{\sqrt{2}},\\ \phi_2(s,v_0)=\dfrac{\partial x(s,v_0)}{\partial v}\dfrac{\tau(s)}{\sqrt{2}}+\dfrac{\partial z(s,v_0)}{\partial v}\dfrac{\kappa(s)}{\sqrt{2}},\\ \phi_3(s,v_0)=-\dfrac{\partial y(s,v_0)}{\partial v}\dfrac{\kappa(s)}{\sqrt{2}}.\end{cases} \tag{3.6}$$

We obtain

$$n(s,v_0)=\phi_1(s,v_0)T(s)+\phi_2(s,v_0)N(s)+\phi_3(s,v_0)B(s).$$

We know that $\alpha(s)$ is a geodesic curve if and only if

$$\phi_1(s,v_0)=\phi_3(s,v_0)=0,\ \phi_2(s,v_0)\neq 0 \tag{3.7}$$

From (3.7),

$$\phi_1(s,v_0)=0,\ \text{we have}\ \frac{\partial y(s,v_0)}{\partial v}=0,\quad \tau(s)\neq 0 \tag{3.8}$$

$$\phi_3(s,v_0)=0,\ \text{we have}\ \frac{\partial y(s,v_0)}{\partial v}=0,\ \kappa(s)\neq 0 \tag{3.9}$$

$$\phi_2(s,v_0)\neq 0,\ \text{we have}\ \frac{\partial z(s,v_0)}{\partial v}=-\frac{\tau(s)}{\kappa(s)}\frac{\partial x(s,v_0)}{\partial v} \tag{3.10}$$

Combining the conditions (3.4), (3.9) and (3.10), we have found the necessary and sufficient conditions for the $\varphi(s,v)$ to have the Smarandache NB curve of the curve $\alpha(s)$ is an isogeodesic. □

Now let us consider other types of the marching-scale functions. In the Eqn. (3.1) marching-scale functions $x(u,v)$, $y(u,v)$ and $z(u,v)$ can be choosen in two different forms:

1) If we choose

$$\begin{cases} x(s,v)=\sum_{k=1}^{p} a_{1k} l(s)^k x(v)^k , \\ y(s,v)=\sum_{k=1}^{p} a_{2k} m(s)^k y(v)^k , \\ z(s,v)=\sum_{k=1}^{p} a_{3k} n(s)^k z(v)^k , \end{cases}$$

then we can simply express the sufficient condition for which the curve $\alpha(s)$ is an geodesic curve on the surface $\varphi(s,v)$ as

$$\begin{cases} x(v_0)=y(v_0)=z(v_0)=0, \\ a_{21}=0 \ or \ m(s)=0 \ \text{ or } \ \frac{dy(v_0)}{dv}=0 \\ a_{31} n(s)\frac{dz(v_0)}{dv}=-\frac{\tau(s)}{\kappa(s)} a_{11} l(s)\frac{dx(v_0)}{dv}, \end{cases} \tag{3.11}$$

where $l(s), m(s), n(s), x(v), y(v)$ and $z(v)$ are C^1 functions, $a_{ij}\in IR$, $i=1,2,3,\ \ j=1,2,...,p.$

2) If we choose

$$\begin{cases} x(u,v)=f\left(\sum_{k=1}^{p} a_{1k} l(u)^k x(v)^k\right), \\ y(u,v)=g\left(\sum_{k=1}^{p} a_{2k} m(u)^k y(v)^k\right), \\ z(u,v)=h\left(\sum_{k=1}^{p} a_{3k} n(u)^k z(v)^k\right), \end{cases}$$

then we can write the sufficient condition for which the curve $\alpha(s)$ is an Smarandache NB geodesic curve on the surface $\varphi(s,v)$ as

$$\begin{cases} x(v_0)=y(v_0)=z(v_0)=f(0)=g(0)=h(0)=0, \\ a_{21}=0 \ or \ m(s)=0 \ \text{ or } \ \frac{dy(v_0)}{dv}=0 \ \ or \ \ g'(0)=0 \\ a_{31} n(s)\frac{dz(v_0)}{dv}h'(0)=-\frac{\tau(s)}{\kappa(s)} a_{11} l(s)\frac{dx(v_0)}{dv} f'(0) \end{cases} \tag{3.12}$$

where $l(s),\ m(s),\ n(s),\ x(v),\ y(v),\ z(v),\ f,\ g$ and h are C^1 functions.

Also conditions for different types of marching-scale functions can be obtained by using the Eqn. (3.7) and (3.10).

Let $\alpha(s)$ be a Smarandache TN curve .Thus, from (3.1) , $\varphi(s,v)$ parametric surface is defined by a given Smarandache TN curve of curve $\alpha=\alpha(s)$ as follows:

$$\varphi(s,v)=\frac{1}{\sqrt{2}}(T(s)+N(s))+[x(s,v)T(s)+y(s,v)N(s)+z(s,v)B(s)].$$

Let $\alpha(s)$ be a Smarandache TN curve on surface $\varphi(s,v)$. If Smarandache TN curve is an parametric curve on this surface , then there exist a parameter $v=v_0$ such that $\frac{1}{\sqrt{2}}(T(s)+N(s))=\varphi(s,v_0)$, that is,

$$x(s,v_0)=y(s,v_0)=z(s,v_0)=0$$

The normal vector can be expressed as

$$\begin{aligned} n(s,v)=&\left[\frac{\partial z(s,v)}{\partial v}\left(\frac{\kappa(s)}{\sqrt{2}}+\frac{\partial y(s,v)}{\partial s}+\kappa(s)x(s,v)-\tau(s)z(s,v)\right)-\frac{\partial y(s,v)}{\partial v}\left(\frac{\tau(s)}{\sqrt{2}}+\frac{\partial z(s,v)}{\partial s}+\tau(s)y(s,v)\right)\right]T(s)\\ &+\left[\frac{\partial x(s,v)}{\partial v}\left(\frac{\tau(s)}{\sqrt{2}}+\frac{\partial z(s,v)}{\partial s}+\tau(s)y(s,v)\right)-\frac{\partial z(s,v)}{\partial v}\left(-\frac{\kappa(s)}{\sqrt{2}}+\frac{\partial x(s,v)}{\partial s}-\kappa(s)y(s,v)\right)\right]N(s)\\ &+\left[\frac{\partial y(s,v)}{\partial v}\left(-\frac{\kappa(s)}{\sqrt{2}}+\frac{\partial x(s,v)}{\partial s}-\kappa(s)y(s,v)\right)-\frac{\partial x(s,v)}{\partial v}\left(\frac{\kappa(s)}{\sqrt{2}}+\frac{\partial y(s,v)}{\partial s}+\kappa(s)x(s,v)-\tau(s)z(s,v)\right)\right]B(s) \end{aligned} \tag{3.13}$$

Thus, if we let

$$\begin{cases} \phi_1(s,v_0)=\dfrac{\partial z(s,v_0)}{\partial v}\dfrac{\kappa(s)}{\sqrt{2}}-\dfrac{\partial y(s,v_0)}{\partial v}\dfrac{\tau(s)}{\sqrt{2}}, \\ \phi_2(s,v_0)=\dfrac{\partial x(s,v_0)}{\partial v}\dfrac{\tau(s)}{\sqrt{2}}+\dfrac{\partial z(s,v_0)}{\partial v}\dfrac{\kappa(s)}{\sqrt{2}}, \\ \phi_3(s,v_0)=-\dfrac{\partial y(s,v_0)}{\partial v}\dfrac{\kappa(s)}{\sqrt{2}}-\dfrac{\partial x(s,v_0)}{\partial v}\dfrac{\kappa(s)}{\sqrt{2}}. \end{cases} \tag{3.14}$$

We obtain

$$n(s,v_0)=\phi_1(s,v_0)T(s)+\phi_2(s,v_0)N(s)+\phi_3(s,v_0)B(s).$$

From (3.7),

$\phi_1(s,v_0)=0$, we have $\dfrac{\partial z(s,v_0)}{\partial v}\dfrac{\kappa(s)}{\sqrt{2}}=\dfrac{\partial y(s,v_0)}{\partial v}\dfrac{\tau(s)}{\sqrt{2}}$ (3.15)

$\phi_3(s,v_0)=0$, we have $\dfrac{\partial y(s,v_0)}{\partial v}=-\dfrac{\partial x(s,v_0)}{\partial v}$ (3.16)

(3.15) and (3.16) , we obtain $\frac{\partial z(s,v_0)}{\partial v}\frac{\kappa(s)}{\sqrt{2}}=-\frac{\partial x(s,v_0)}{\partial v}\frac{\tau(s)}{\sqrt{2}}$ (3.17)

$\phi_2(s,v_0)\neq 0$, we have $\frac{\partial x(s,v_0)}{\partial v}\frac{\tau(s)}{\sqrt{2}}+\frac{\partial z(s,v_0)}{\partial v}\frac{\kappa(s)}{\sqrt{2}}\neq 0$ (3.18)

In equations (3.17) and (3.18) , the contradiction is obtained.

Let $\alpha(s)$ be Smarandache TNB curve. Thus, from (3.1) , $\varphi(s,v)$ parametric surface is defined by a given Smarandache TNB curve of curve $\alpha=\alpha(s)$ as follows:

$$\varphi(s,v)=\frac{1}{\sqrt{3}}(T(s)+N(s)+B(s))+[x(s,v)T(s)+y(s,v)N(s)+z(s,v)B(s)].$$

Let $\alpha(s)$ be a Smarandache TNB curve on surface $\varphi(s,v)$. If Smarandache TNB curve is an parametric curve on this surface , then there exist a parameter $v=v_0$ such that $\frac{1}{\sqrt{3}}(T(s)+N(s)+B(s))=\varphi(s,v_0)$, that is,

$$x(s,v_0)=y(s,v_0)=z(s,v_0)=0.$$

The normal vector can be expressed as

$$\begin{aligned} n(s,v)=&\left[\frac{\partial z(s,v)}{\partial v}\left(\frac{\kappa(s)-\tau(s)}{\sqrt{3}}+\frac{\partial y(s,v)}{\partial s}+\kappa(s)x(s,v)-\tau(s)z(s,v)\right)-\frac{\partial y(s,v)}{\partial v}\left(\frac{\tau(s)}{\sqrt{3}}+\frac{\partial z(s,v)}{\partial s}+\tau(s)y(s,v)\right)\right]T(s)\\ &+\left[\frac{\partial x(s,v)}{\partial v}\left(\frac{\tau(s)}{\sqrt{3}}+\frac{\partial z(s,v)}{\partial s}+\tau(s)y(s,v)\right)-\frac{\partial z(s,v)}{\partial v}\left(-\frac{\kappa(s)}{\sqrt{3}}+\frac{\partial x(s,v)}{\partial s}-\kappa(s)y(s,v)\right)\right]N(s)\\ &+\left[\frac{\partial y(s,v)}{\partial v}\left(-\frac{\kappa(s)}{\sqrt{3}}+\frac{\partial x(s,v)}{\partial s}-\kappa(s)y(s,v)\right)-\frac{\partial x(s,v)}{\partial v}\left(\frac{\kappa(s)-\tau(s)}{\sqrt{3}}+\frac{\partial y(s,v)}{\partial s}+\kappa(s)x(s,v)-\tau(s)z(s,v)\right)\right]B(s)\end{aligned}$$

Thus, if we let

$$\begin{cases}\phi_1(s,v_0)=\frac{\partial z(s,v_0)}{\partial v}\left(\frac{\kappa(s)-\tau(s)}{\sqrt{3}}\right)-\frac{\partial y(s,v_0)}{\partial v}\frac{\tau(s)}{\sqrt{3}},\\ \phi_2(s,v_0)=\frac{\partial x(s,v_0)}{\partial v}\frac{\tau(s)}{\sqrt{3}}+\frac{\partial z(s,v_0)}{\partial v}\frac{\kappa(s)}{\sqrt{3}},\\ \phi_3(s,v_0)=-\frac{\partial y(s,v_0)}{\partial v}\frac{\kappa(s)}{\sqrt{3}}-\frac{\partial x(s,v_0)}{\partial v}\left(\frac{\kappa(s)-\tau(s)}{\sqrt{3}}\right).\end{cases}$$

We obtain

$$n(s,v_0)=\phi_1(s,v_0)T(s)+\phi_2(s,v_0)N(s)+\phi_3(s,v_0)B(s).$$

From (3.7),

$\phi_1(s,v_0)=0$, we have $\frac{\partial z(s,v_0)}{\partial v}(\kappa(s)-\tau(s))=\frac{\partial y(s,v_0)}{\partial v}\tau(s)$ (3.19)

$\phi_3(s,v_0)=0$, we have $\frac{\partial y(s,v_0)}{\partial v}=-\frac{(\kappa(s)-\tau(s))}{\kappa(s)}\frac{\partial x(s,v_0)}{\partial v}, \kappa(s)\neq 0$ (3.20)

(3.19) and (3.20) , we obtain $\frac{\partial z(s,v_0)}{\partial v}=-\frac{\partial x(s,v_0)}{\partial v}\frac{\tau(s)}{\kappa(s)}$ (3.21)

$\phi_2(s,v_0)\neq 0$, we have $\frac{\partial x(s,v_0)}{\partial v}\tau(s)\neq -\frac{\partial z(s,v_0)}{\partial v}\kappa(s)$ (3.22)

In equations (3.21) and (3.22) ,the contradiction is obtained. Thus, we can give the following results:

Conclusion 3.1: Smarandache TN and TNB curves of unit speed curve $\alpha=\alpha(s)$, is not geodesic on the surface $\varphi(s,v)$.

4. RULED SURFACES WITH COMMON SMARANDACHE NB GEODESIC

Ruled surfaces are one of the simplest objects in geometric modelling as they are generated basically by moving a line in space. A surface φ is a called a ruled surface in Euclidean space, if it is a surface swept out by a straight line l moving alone a curve α. The generating line l and the curve α are called the rulings and the base curve of the surface, respectively.

We show how to derive the formulations of a ruled surfaces family such that the common Smarandache NB geodesic is also the base curve of ruled surfaces.

Let $\varphi=\varphi(s,v)$ be a ruled surface with the Smarandache NB isogeodesic base curve. From the definition of ruled surface, there is a vector $R=R(s)$ such that;

$$\varphi(s,v)-\varphi(s,v_0)=(v-v_0)R(s)$$

From (3.1), we get

$$(v-v_0)R(s)=x(s,v)T(s)+y(s,v)N(s)+z(s,v)B(s)$$

For the solutions of three unknows $x(s,v)$, $y(s,v)$ and $z(s,v)$ we have,

$$\begin{aligned} x(s,v)&=(v-v_0)\langle T(s),R(s)\rangle \\ y(s,v)&=(v-v_0)\langle N(s),R(s)\rangle \\ z(s,v)&=(v-v_0)\langle B(s),R(s)\rangle . \end{aligned} \tag{4.1}$$

From (3.9) and (4.1), we have

$$\langle N(s),R(s)\rangle=0 \tag{4.2}$$

Including, $R(s)=x(s)T(s)+y(s)N(s)+z(s)B(s)$ using (4.2) we obtain,

$$y(s)=0 \tag{4.3}$$

From (3.10) and (4.1), we have

$$\langle B(s),R(s)\rangle=-\frac{\tau}{\kappa}\langle T(s),R(s)\rangle \tag{4.4}$$

Including, $R(s)=x(s)T(s)+y(s)N(s)+z(s)B(s)$ using (4.4) we obtain,

$$z(s)=-\frac{\tau}{\kappa}x(s) \tag{4.5}$$

So, the ruled surfaces family with common Smarandache NB isogeodesic given by;

$$\varphi(s,v)=\frac{1}{\sqrt{2}}(N(s)+B(s))+v[x(s)T(s)-\frac{\tau}{\kappa}x(s)B(u)] \tag{4.6}$$

5. EXAMPLES OF GENERATING SIMPLE SURFACES RULED SURFACE WITH COMMON SMARANDACHE NB GEODESIC CURVE

Example 5.1. Let $\alpha(s)=\left(\frac{2}{9}\cos(3s),\frac{2}{9}\sin(3s),\frac{\sqrt{5}}{3}s\right)$ be a unit speed curve. Then it is easy to show that

$$\begin{cases} T(s)=\left(-\frac{2}{3}\sin(3s),\frac{2}{3}\cos(3s),\frac{\sqrt{5}}{3}\right), \\ N(s)=\left(-2\cos(3s),-2\sin(3s),0\right), \qquad \kappa=\|T'\|=2 \qquad , \qquad \tau=\langle N',B\rangle=4\sqrt{5} \\ B(s)=\left(\frac{2\sqrt{5}}{3}\sin(3s),\frac{2\sqrt{5}}{3}\cos(3s),\frac{4}{3}\right). \end{cases}$$

If we take

$x(s,v)=\sin(sv)$, $y(s,v)=0$, $z(s,v)=-2\sqrt{5}\sin(sv)$ we obtain a member of the surface with curve $\alpha(s)$ as

$$\varphi(s,v)=\left(\frac{2}{9}\cos(3s)-\frac{22}{3}\sin(3s)\sin(sv),\ \frac{2}{9}\sin(3s)-6\cos(3s)\sin(sv),\ \frac{\sqrt{5}}{3}(s-7\sin(sv))\right)$$

where $0\le s\le\pi$, $-1\le v\le 1$ (Fig. 5.1).

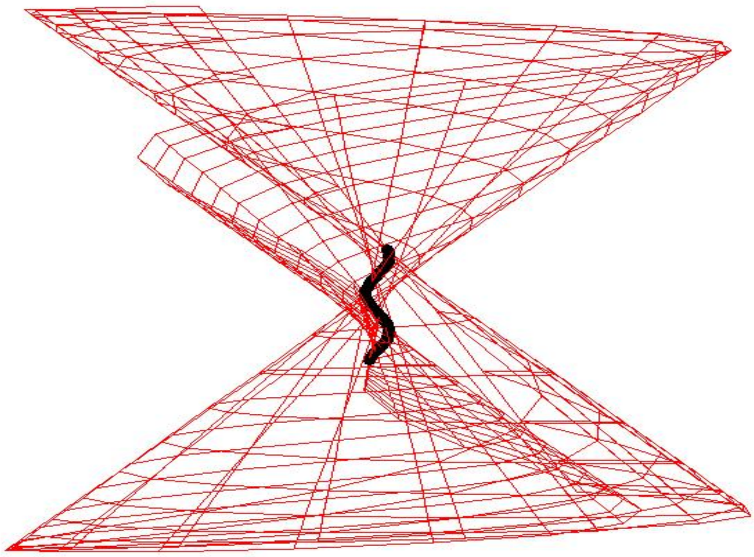

Figure 5.1. $\varphi(s,v)$ **surface with curve** $\alpha(s)$.

If we take

$x(s,v)=\sin(sv),\ y(s,v)=0,\ z(s,v)=-2\sqrt{5}\sin(sv)$ and $v_0=0$ then the Eqn. (3.4), (3.9) and (3.10) are satisfied. Thus, we obtain a member of the surface with common Smarandache NB geodesic curve as

$$\varphi(s,v)=\begin{pmatrix}\frac{2}{\sqrt{2}}(-\cos(3s)+\frac{\sqrt{5}}{3}\sin(3s))-\frac{22}{3}\sin(3s)\sin(sv),\frac{2}{\sqrt{2}}(-\sin(3s)+\frac{\sqrt{5}}{3}\cos(3s))-6\cos(3s)\sin(sv),\\ \frac{4}{3\sqrt{2}}-\frac{7\sqrt{5}}{3}\sin(sv)\end{pmatrix}$$

where $0\le s\le\pi$, $-1\le v\le 1$ (Fig. 5.2).

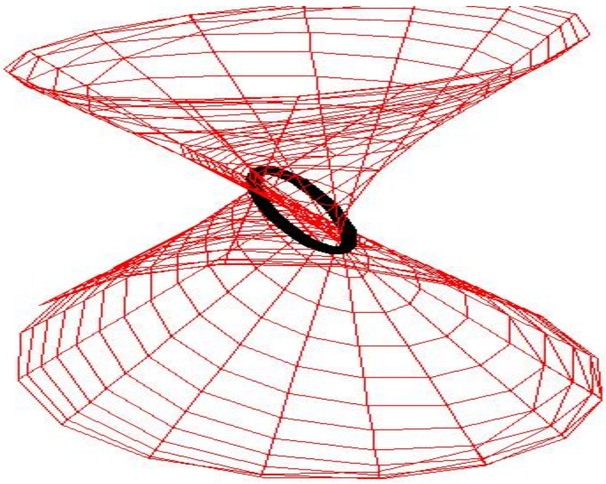

Figure 5.2. $\varphi(s,v)$ **as a member of the surface and its Smarandache NB geodesic curve of** $\alpha(s)$.

If we take

$$x(s,v)=\sum_{k=1}^{4}(\cos s)^k \sin(v)^k,\ y(s,v)=0,\ z(s,v)=\sum_{k=1}^{4}(-2\sqrt{5})(\cos s)^k \sin(v)^k \text{ and } v_0=0 \text{ then}$$

the Eqn. (3.11) is satisfied. Thus, we obtain a member of the surface with common Smarandache NB geodesic curve as

$$\varphi(s,v)=\begin{pmatrix} \frac{2}{\sqrt{2}}(-\cos(3s)+\frac{\sqrt{5}}{3}\sin(3s))-\frac{22}{3}\sin(3s)(\sum_{k=1}^{4}(\cos s)^k \sin(v)^k), \\ \frac{2}{\sqrt{2}}(-\sin(3s)+\frac{\sqrt{5}}{3}\cos(3s))-6\cos(3s)(\sum_{k=1}^{4}(\cos s)^k \sin(v)^k), \frac{4}{3\sqrt{2}}-\frac{7\sqrt{5}}{3}(\sum_{k=1}^{4}(\cos s)^k \sin(v)^k) \end{pmatrix}$$

where $0 \le s \le \pi$, $-1 \le v \le 1$ (Fig. 5.3).

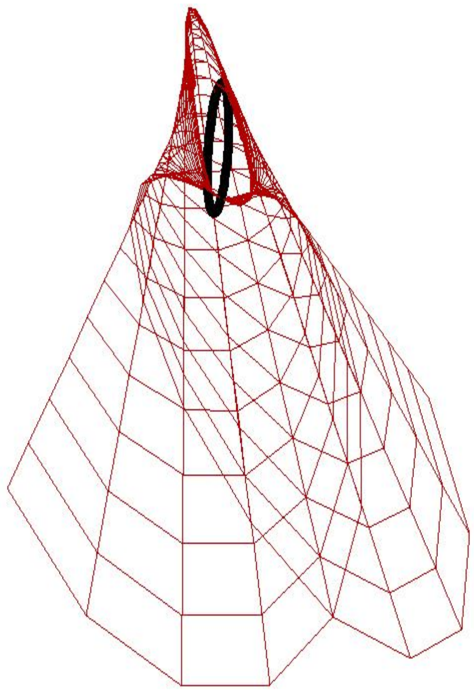

Figure 5.3. $\varphi(s,v)$ **as a member of the surface and its Smarandache NB geodesic curve of** $\alpha(s)$.

If we take

$x(u)=\sin s\cos s,\ y(s)=0,\ z(s)=-2\sqrt{5}\sin s\cos s$ and $v_0=0$ then the Eqn. (4.6) is satisfied.

Thus, we obtain a member of the ruled surface with common Smarandache NB geodesic curve as

$$\varphi(s,v)=\begin{pmatrix}\frac{2}{\sqrt{2}}(-\cos(3s)+\frac{\sqrt{5}}{3}\sin(3s))-\frac{22}{3}v\sin(3s)\cos(3s),\frac{2}{\sqrt{2}}(-\sin(3s)+\frac{\sqrt{5}}{3}\cos(3s))-6v\cos(3s)\sin(3s),\\ \frac{4}{3\sqrt{2}}-\frac{7\sqrt{5}}{3}v\cos(3s)\sin(3s)\end{pmatrix}$$

where $0\le s\le \pi/2,\ 0\le v\le 1$ (Fig. 5.4).

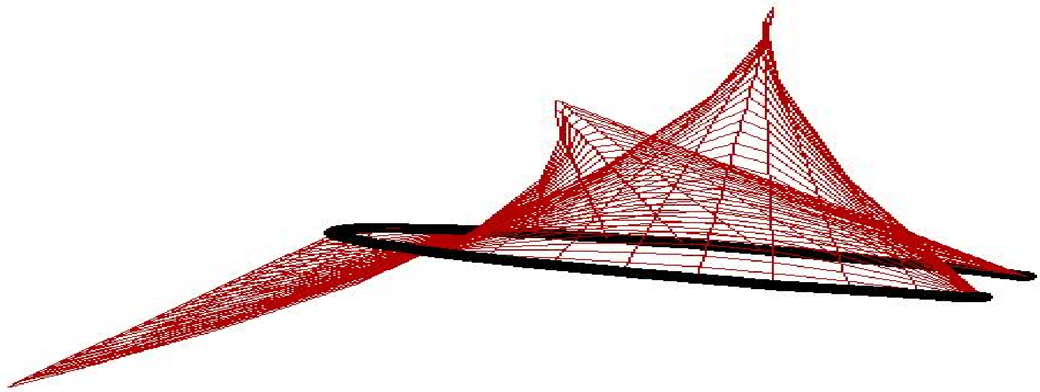

Figure 5.4. $\varphi(s,v)$ **as a member of the ruled surface and its Smarandache NB geodesic curve.**

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