

## OPTICS OF SINGLE COLD PLASMA FOR PHOTONIC APPLICATIONS

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**Abstract.** *In the present paper, we have discussed the optical properties of a single cold plasma layer with varying applied static magnetic field; here the electric permittivity of the cold plasma (intrinsic) is dependent on the applied magnetic field. The gyro frequency of such plasma is related to the applied magnetic field. The applied magnetic field affects the refractive index of the plasma for certain incident frequencies and we have obtained negative value of the real part of the permittivity ( $\epsilon'$ ) when the value of imaginary part of the permittivity ( $\epsilon''$ ) becomes maximum for a particular incident frequency. Our observations revealed that a single layer of the cold plasma may be used as a perfect reflector and transmitter for certain values of the applied magnetic field and frequency. It is found that the static magnetic field acts as a controlling factor for optical behaviour of the cold plasma layer and this analysis can be useful for several photonic applications, including optical mirrors, transmitters, and sensors.*

**Keywords:** *Single cold plasma, perfect reflector, perfect tunneling behavior, photonic applications.*

### 1. INTRODUCTION

The pioneer work of Yablonovitch and John in 1987 sparked a new idea in the field of photonic crystals (PCs)[1, 2]. The study of photonic crystals has become popular and continues to be more excited for the optical community till today. The essential physics of photonic crystals has unique property due to a band gap and such structures are called as photonic band structure (PBS). The PBS is analogous to the electronic band structure (EBS) in solids. Photonic crystals have received emerging attention in the field of solid state and optical physics due to exhibition of many unique features [2, 3]. The photonic band gap (PBG) prohibits the propagation of an electromagnetic wave of certain frequency through it or the periodic structures. Different periodic materials have been admitted to study and design tunable PBG materials.

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The development of nanoscience and nanotechnology made the plasma materials, a popular material in fabrication of photonic devices. One-dimensional plasma photonic crystals (1DPPCs) were fabricated by Hojo and Mase in 2004. They studied the dispersion relation of electromagnetic waves in one-dimensional plasma photonic crystals (PPCs) at different plasma frequencies and thicknesses [4]. Such PPCs are known as intrinsic plasma photonic crystals. In 1DPPCs, there are two kinds of materials with distinct refractive indices and they are used like dielectric and plasma in periodic structures. There is another kind of plasma photonic crystal, which is not periodic in nature but it becomes periodic in the presence of external magnetic field and is known as extrinsic plasma photonic crystal [5]. The extrinsic plasma photonic crystal is formed from bulk material, which is influenced by externally applied periodic applied field in space [6]. King et al. have investigated the tunable photonic band structure for extrinsic plasma photonic crystal in which a cold bulk plasma layer is influenced by an externally periodic static magnetic field [5]. The photonic band structure is tuned by the variation of the externally applied magnetic field, electron density as well as the thickness of plasma.

In recent years, plasma PCs have attracted considerable attention due to their tunable characteristics and advantages that conventional PCs composed of dielectrics or metals do not offer. Since plasmas are controlled by changing the applied voltage, gas pressure and temperature, so they enable us to realize the tunable PC devices [4-8]. The interfaces of the materials are responsible for producing the effect of the reflectance and transmittance inside the materials that is why the photonic devices of different materials are possible. Several researchers have investigated the optical properties of the single layer materials to understand the optics behind it [9-12].

In the present work, we have analyzed the optical properties of a single layer of extrinsic plasma photonic crystal using simple translational matrix method (TMM) [4, 8]. The reflectance, transmittance and absorption of extrinsic single plasma layer have been calculated.

## 2. THEORETICAL FORMULATION

The propagation of an electromagnetic wave in one-dimensional periodic structures containing metal/dielectric and negative index material/dielectric complex composite materials is an interesting research field because of its unusual behaviour [13]. Unusual behaviours are studied by solving the amplitudes of electric and magnetic fields of electromagnetic wave within composite materials. The TMM method is the prevalent method to solve the amplitude of electromagnetic waves. This method is a systematic approach to analyze the periodic layers with different materials as well as a single layer of the material [9].

Let us consider a cold plasma with refractive index  $n_2 = \sqrt{\epsilon_2}$  sandwiched between two semi-infinite dielectric media of air having refractive index  $n=1.0$  as shown in figure 1, where the distribution of the refractive indices is given below:

$$n(x) = \begin{cases} n_1, & x < 0 & (\text{air}) \\ n_2, & 0 < x < d & (\text{Cold Plasma}) \\ n_3, & d < x & (\text{air}) \end{cases} \quad (1)$$

where electromagnetic wave is incident on the X-Z plane. For the plane wave solution of the wave equation, the electric field is defined as

$$E(x, t) = E(x) \exp[i(\omega t - \beta z)] \quad (2)$$

where  $\beta$  is the Z component of the propagation wave vector considering the wave incident from the left side and electric field is given as

$$E(x) = \begin{cases} Ae^{-ik_{1x}x} + Be^{ik_{1x}x}, & x < 0 \\ Ce^{-ik_{2x}x} + De^{ik_{2x}x}, & 0 < x < d \\ Fe^{-ik_{3x}(x-d)}, & d < x \end{cases} \quad (3)$$

where A, B, C, D and F are arbitrary constants, and  $k_{1x}$ ,  $k_{2x}$ ,  $k_{3x}$ , are x-components of wave vectors and  $k_{ix}$ , is related as

$$k_{ix} = \left[ \left( \frac{n_i \omega}{c} \right)^2 - \beta^2 \right]^{\frac{1}{2}} = \left( \frac{\omega}{c} \right) n_i \cos \theta_i$$

$i=1, 2, 3$ , and  $\theta_i$  is the ray angle measured from x-axis. Here, B and F are amplitudes of the reflected and transmitted waves, respectively. Further, the magnetic field is defined by following equation:

$$\vec{H} = \frac{i}{\omega \mu} \vec{\nabla} \times \vec{E} = \frac{i}{\omega \mu} \frac{\partial E}{\partial x} \quad (4)$$

If we apply the boundary conditions on the tangential component of the electric field, or TE modes and the tangential component of magnetic field or TM modes, there is no coupling between these components in the whole medium.

Applying boundary conditions at  $x = 0$  and  $x=d$  in equations (3)-(6), we obtain the reflection and transmission coefficients as

$$t = \frac{F}{A} = \frac{4k_{1x}k_{2x}}{\left(k_{1x} + \frac{k_{2x}}{\mu_2}\right)(k_{2x} + \mu_2 k_{3x})e^{ik_{2x}d} + \left(k_{1x} - \frac{k_{2x}}{\mu_2}\right)(k_{2x} - \mu_2 k_{3x})e^{-ik_{2x}d}}$$

$$t = \frac{4k_{1x}k_{2x}e^{-ik_{2x}d}}{\left(k_{1x} + \frac{k_{2x}}{\mu_2}\right)(k_{2x} + \mu_2 k_{3x}) + \left(k_{1x} - \frac{k_{2x}}{\mu_2}\right)(k_{2x} - \mu_2 k_{3x})e^{-2ik_{2x}d}}$$

$$t = \frac{4k_{1x}k_{2x}e^{-ik_{2x}d}}{\mu_2 \left(k_{1x} + \frac{k_{2x}}{\mu_2}\right) \left(\frac{k_{2x}}{\mu_2} + k_{3x}\right) \left[ 1 + \frac{\left(k_{1x} - \frac{k_{2x}}{\mu_2}\right) \left(\frac{k_{2x}}{\mu_2} - k_{3x}\right) e^{-2ik_{2x}d}}{\left(k_{1x} + \frac{k_{2x}}{\mu_2}\right) \left(\frac{k_{2x}}{\mu_2} + k_{3x}\right)} \right]}$$

Let

$$r_{12} = \frac{\left(k_{1x} - \frac{k_{2x}}{\mu_2}\right)}{\left(k_{1x} + \frac{k_{2x}}{\mu_2}\right)}, \quad r_{23} = \frac{\left(\frac{k_{2x}}{\mu_2} - k_{3x}\right)}{\left(\frac{k_{2x}}{\mu_2} + k_{3x}\right)}$$

$$t_{12} = \frac{2k_{1x}}{\left(k_{1x} + \frac{k_{2x}}{\mu_2}\right)}$$

$$t_{23} = \frac{2k_{2x}}{\left(k_{3x} + \frac{k_{2x}}{\mu_2}\right)}$$

Now, the transmittance can be represented by and  $\phi = k_2 x d$

$$t = \frac{t_{12} t_{23} e^{-i\phi}}{\mu_2 [1 + r_{12} r_{23} e^{-2i\phi}]} \quad (5)$$

Similarly, for the reflection coefficient

$$t = \frac{B}{A} = \frac{[(k_{2x} - \mu_2 k_{1x})(k_{2x} + \mu_2 k_{3x}) + (\mu_2 k_{1x} + k_{2x})(\mu_2 k_{3x} - k_{2x}) e^{-2ik_{2x}d}] A}{[(\mu_2 k_{1x} - k_{2x})(\mu_2 k_{3x} - k_{2x}) e^{-2ik_{2x}d} - (\mu_2 k_{1x} + k_{2x})(k_{2x} + \mu_2 k_{3x})]}$$

$$r = \frac{[(k_{2x} - \mu_2 k_{1x})(k_{2x} + \mu_2 k_{3x}) + (\mu_2 k_{1x} + k_{2x})(\mu_2 k_{3x} - k_{2x}) e^{-2ik_{2x}d}]}{[(\mu_2 k_{1x} - k_{2x})(\mu_2 k_{3x} - k_{2x}) e^{-2ik_{2x}d} - (\mu_2 k_{1x} + k_{2x})(k_{2x} + \mu_2 k_{3x})]}$$

$$\text{Hence, } r = \frac{r_{12} + r_{23} e^{-2i\phi}}{1 + r_{12} r_{23} e^{-2i\phi}} \quad (6)$$

Here, the electromagnetic wave through a medium interacts with the optical density of the material. The optical density of the material is related with electric permittivity and magnetic permeability of the interacting medium as follows:

$$n^2 = \mu \epsilon \quad (7)$$

Using the concept of optical density, the periodic structure of two distinct dielectric materials ( $\mu=1$ ) is called intrinsic photonic crystal. Other photonic crystals are called extrinsic photonic crystals for such structures, which are not periodic in structure. The extrinsic photonic crystal is formed from a bulk material, which is influenced by externally applied

periodic field in space. The periodic structure of cold plasma may be considered as extrinsic plasma photonic crystal by applying an external magnetic field. The permittivity function for the cold plasma in the presence of a static magnetic field can be expressed as given below

$$\begin{aligned}\epsilon_{\text{plasma}}(\omega) &= \text{Re}(\epsilon_{\text{plasma}}) - \text{Im}(\epsilon_{\text{plasma}}) \\ &= 1 - \frac{\omega_{\text{pe}}^2}{\omega^2 \left[ \left( 1 - i \frac{\gamma}{\omega} \mp \frac{\omega_{\text{le}}}{\omega} \right) \right]}\end{aligned}\quad (8)$$

where  $\omega_{\text{pe}}$  is the plasma frequency given by

$$\omega_{\text{pe}} = \left( \frac{n_e e^2}{m \epsilon_0} \right)^{1/2} \quad (9)$$

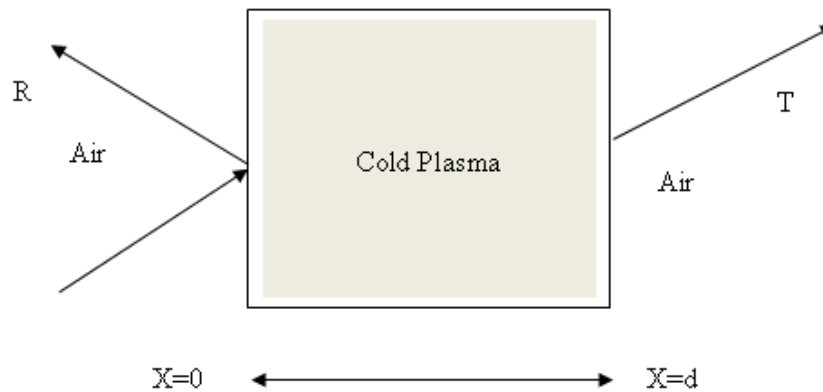
where  $m$  is the electronic mass,  $e$  is the electronic charge and  $n_e$  is the electron density,  $\omega_{\text{le}}$  is the gyro frequency that is expressed as

$$\omega_{\text{le}} = \frac{eB}{m} \quad (10)$$

and  $\gamma$  is the effective collision frequency, respectively. The minus  $\omega_{\text{le}}$  corresponds to a magnetic field in the positive  $x$  direction and is called right hand polarization (RHP).

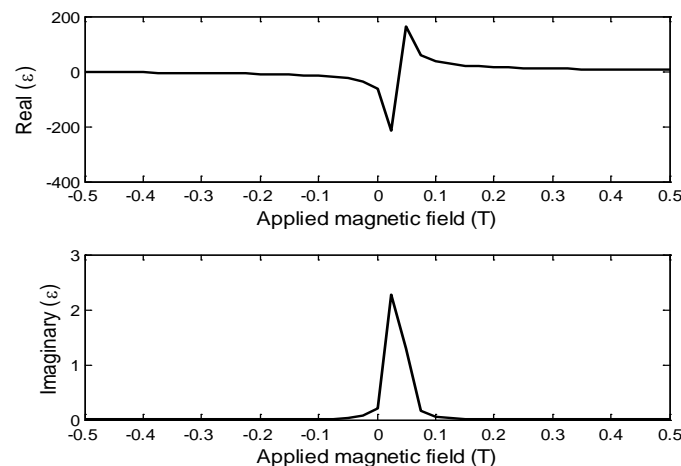
### 3. RESULTS AND DISCUSSION

The optical properties of a single layer of cold plasma influenced by static magnetic field is shown in Fig. 1. The thickness of the single layer is taken 15mm and incident wave is considered in the microwave region. Using equations (2) and (5), we have studied the electric permittivity ( $\epsilon$ ) of cold plasma with varying the static magnetic field for 1 GHz frequency, which is shown in Fig. 2. Fig. 2 has two parts, which have been plotted for real part of the permittivity ( $\epsilon'$ ) and imaginary part of the permittivity ( $\epsilon''$ ) versus applied magnetic field. Figure 2(a) shows that the variation of real part of the permittivity ( $\epsilon'$ ) with applied magnetic field ( $B$ ) for a constant frequency (1GHz). From the Fig. 1, it is clearly seen that the value of real part of the permittivity ( $\epsilon'$ ) is constant for low and high static magnetic fields, however there is a discontinuity at a particular value of the magnetic field. When the magnetic field increases, value of real part of the permittivity ( $\epsilon'$ ) becomes negative as well as positive. So we can say that the negative and positive values of the real part of the permittivity ( $\epsilon'$ ) are obtained due to applied magnetic field. It means plasma material has negative real part of the permittivity ( $\epsilon'$ ) at a particular frequency when the magnetic field strength is varied.



**Figure 1. Single layer of cold plasma with thickness  $d=15\text{mm}$ .**

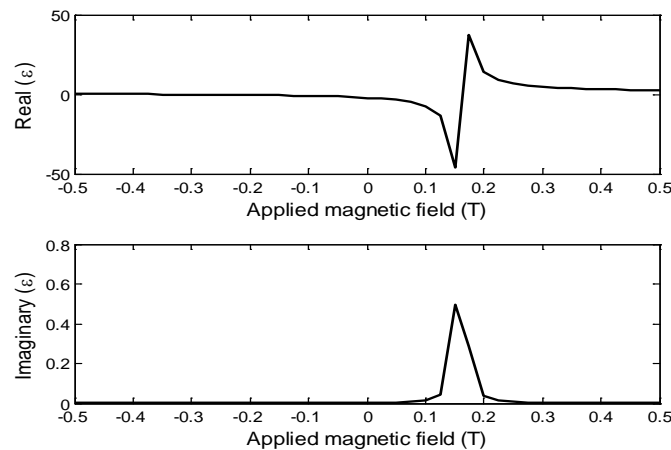
In Fig. 2(b), we plot a graph showing the variation of imaginary part of the permittivity ( $\epsilon''$ ) versus static magnetic field ( $B$ ) at a constant wave frequency. The value of imaginary part of the permittivity ( $\epsilon''$ ) is constant and variable both with the static magnetic field. But the value of imaginary part of the permittivity ( $\epsilon''$ ) increases sharply when the magnetic field is in the range  $0\text{T}$  to  $0.04\text{T}$ , which is shown in figure 2(b). From the figure 2, we observe that the values of real part of the permittivity ( $\epsilon'$ ) and imaginary part of the permittivity ( $\epsilon''$ ) are affected by an applied static magnetic field for certain values of the field. It is clearly seen that the negative value of the real part of the permittivity ( $\epsilon'$ ) is obtained when imaginary part of the permittivity ( $\epsilon''$ ) becomes maximum at a particular value of the magnetic field.



**Figure 2. Refractive index of cold plasma versus applied magnetic field plots with frequency 1GHz for (a) Real ( $\epsilon'$ ) and (b) Imaginary ( $\epsilon''$ ).**

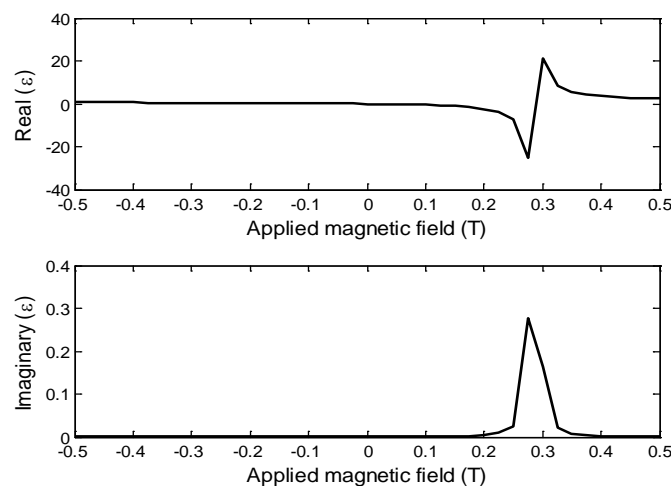
The variation in electric permittivity of the cold plasma with magnetic field is further determined at fixed wave frequencies 4.5 GHz and 8 GHz, respectively, which are shown in the Figs. 3 and 4, respectively. Fig. 3 shows that the negative value of the real part of the permittivity ( $\epsilon'$ ) is obtained when the value of imaginary part of the permittivity ( $\epsilon''$ ) is maximum for the range of the magnetic field from  $0.1\text{T}$  to  $0.2\text{T}$ . The value of real part of the permittivity ( $\epsilon'$ ) becomes negative as well as positive as the magnetic field increases. So, one can say that the negative and positive values of the real part of the permittivity ( $\epsilon'$ ) are also found here due to variation of applied static magnetic field. Thus, the values of real part of the permittivity ( $\epsilon'$ ) and imaginary part of the permittivity ( $\epsilon''$ ) are changing with applied static

magnetic field and the negative real part of the permittivity ( $\epsilon'$ ) is obtained when imaginary part of the permittivity ( $\epsilon''$ ) becomes maximum for the same value of the magnetic field.



**Figure 3. Refractive index of cold plasma versus applied magnetic field plots with frequency 4.5GHz for (a) Real ( $\epsilon$ ) and (b) Imaginary ( $\epsilon$ ).**

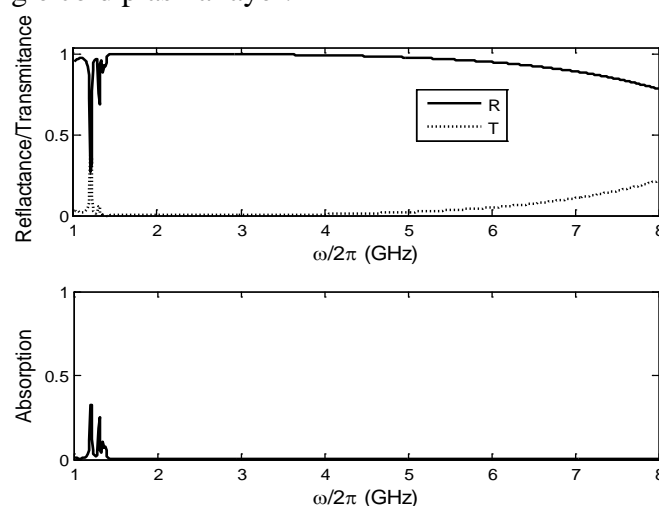
Fig. 4 shows the real part of the permittivity ( $\epsilon'$ ) and imaginary part of the permittivity ( $\epsilon''$ ) versus applied magnetic field curves at 8GHz frequency. Again, the real part of the permittivity ( $\epsilon'$ ) attains negative value when the value of imaginary part of the permittivity ( $\epsilon''$ ) is maximum for range of the magnetic field from 0.25T to 0.35T. We again observe that the values of real part of the permittivity ( $\epsilon'$ ) and imaginary part of the permittivity ( $\epsilon''$ ) are varying with applied static magnetic field and the negative real part of the permittivity ( $\epsilon'$ ) is again obtained when imaginary part of the permittivity ( $\epsilon''$ ) is maximum.



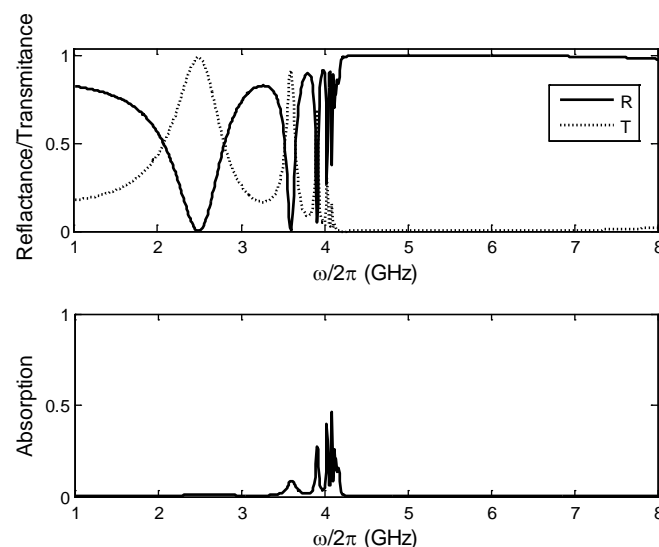
**Figure 4. Refractive index of cold plasma versus applied magnetic field plots with frequency 8GHz for (a) Real ( $\epsilon$ ) and (b) Imaginary ( $\epsilon$ ).**

Thus, we see that the characteristics of both graphs Figs. 3 and 4 are similar to the discussed graph in Fig. 2. From Figs. 2, 3 and 4, we conclude that, if we vary the applied magnetic field, then the electric permittivity of the cold plasma changes for a fixed value of the considered frequency. Moreover, the electric permittivity is shifted towards the higher applied magnetic field for larger frequency. We notice that the real part of the permittivity ( $\epsilon'$ ) has achieved negative value at maximum value of the imaginary part of the permittivity ( $\epsilon''$ )

for all frequencies, but these values are shifted towards the high magnetic field when the frequency is large enough. With reference to the above discussions, we have already analyzed the electric permittivity of the plasma with varying applied magnetic field at high frequency. So in the next calculation, we focus on the optical properties of the plasma depending upon frequency for constant applied field. Therefore, we now plot and study the reflectance, transmittance and absorption curves of the single cold plasma layer with varying frequency at a fixed applied magnetic field. The constant applied magnetic field is taken to include the negative value of the real part of the permittivity ( $\epsilon'$ ) and study the variation of the optical properties of the single layer of the cold plasma, which is embedded with semi-infinite materials like air. Here, the plots have also two parts: (a) reflectance/transmittance and (b) absorption of the single cold plasma layer.



**Figure 5. Reflectance/transmittance and absorption of single layer of cold plasma versus frequency plots with magnetic field  $B=0.05T$ .**



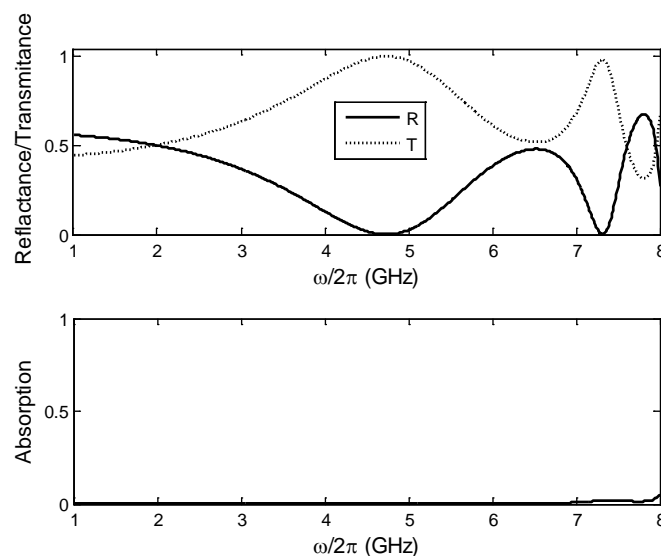
**Figure 6. Reflectance/transmittance and absorption of single layer of cold plasma versus frequency plots with magnetic field  $B=0.15T$ .**

In Fig. 5, we have plotted the transmittance, reflectance and absorption of a single plasma layer versus frequency curves at constant magnetic field 0.05T. In this figure, the solid line shows the reflectance and the dotted line shows the transmittance of the plasma with



varying frequency. The graph shows 100% reflectance and 0% transmittance in the frequency range 1.5GHz - 4.5GHz, but the reflectance decreases slightly and transmittance increases above the 4.5GHz frequency. The reflectance of the material is zero at 1.2GHz frequency. On the other hand, the transmittance is found high and the absorption is maximum at the same frequency. The optical properties of the considered material at 1.2GHz have high transmittance and high absorption but low reflectance. Such specific characteristics are due to negative values of real electric permittivity and the large value of the imaginary electric permittivity.

Fig. 6 shows the plots between optical properties (transmittance, reflectance and absorption) of single plasma layer and frequency at constant magnetic field 0.15T. In this figure, the reflectance and the transmittance of the plasma layer are shown with solid and dotted lines respectively. The reflectance of the material is about 100% above 4.2GHz frequency and the transmittance is 0% above the same frequency range. The optical properties of the plasma in the range of frequency 3.8GHz to 4.2GHz have specific characteristics as discussed earlier for the magnetic field strength 0.05T. Here we are interested in the optical properties below 3.8GHz frequencies, where the obtained transmittance is 100% and the reflectance is found to be 0% at 2.5GHz frequency that indicate the tunnelling behavior of the electromagnetic wave. It means the plasma layer at 0.15T field behaves like metal as well as glass due to the abnormal behavior of the plasma in the presence of the applied magnetic field.



**Figure 7. Reflectance/transmittance and absorption of single layer of cold plasma versus frequency plots with magnetic field  $B=0.31\text{T}$ .**

Similarly, the optical properties of the cold plasma layer versus frequency curves at constant magnetic field 0.31T, are shown in Fig. 7. The absorption is 0% but the reflectance (solid line) and transmittance (dotted line) are varying with the frequency. Maximum transmittances of plasma are obtained at 4.6GHz and 7.2GHz, which show the tunnelling behaviour of the plasma at constant applied magnetic field. It means the plasma layer will show the tunnelling characteristics when the applied field on the plasma is 0.31T due to existence of the negative real part of the permittivity ( $\epsilon'$ ) at higher frequency and positive value of the real part of the permittivity ( $\epsilon'$ ) at a lower frequency.

Thus the cold plasma behaves like a perfect reflector and transmitter in the influence of an applied magnetic field. The electric permittivity of the cold plasma is dependent upon the applied field as well as the frequency. So the electric permittivity of the cold plasma can

be tuned with the applied field and the frequency of the incident wave. It is also revealed that the cold plasma may be used as a perfect reflector or perfect transmitter by controlling the applied field

#### 4. CONCLUSION

We have studied the optical properties of a single cold plasma layer by varying the applied static magnetic field because the electric permittivity of the cold/intrinsic plasma depends on the applied magnetic field strength. The gyro frequency of such plasma is related to the applied magnetic field. The applied magnetic field affects the refractive index of the plasma for certain incident frequencies and shows a negative value of the real part of the permittivity ( $\epsilon'$ ) when the value of imaginary part of the permittivity ( $\epsilon''$ ) is found to be maximum. Here we can also conclude that the external static magnetic field is a controlling factor for interesting optical behaviours of a single layer cold plasma like perfect reflector or transmitter. Such unprecedented behaviors of single layer cold plasma can be useful to design and realize tunable PC devices. This analysis can be a basis for further analysis of different intrinsic as well as extrinsic plasma photonic crystals having several interesting applications in photonic devices such as optical mirrors, sensors and switching applications.

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