ORIGINAL PAPER

TWO NEW APPLICATIONS OF GENERALIZED $\left(\frac{G'}{G}\right)$- EXPANSION METHOD

IBRAHIM ENAM INAN[1], UNAL IC[1]

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Abstract. *In this paper, we implemented an Generalized* $\left(\frac{G'}{G}\right)$ *– expansion method for some soliton wave solution sixth-order Ramani equation and new Konno-Oono equation system. These solutions are hyperbolic function solutions, trigonometric function solutions, exponential function solutions and rational function solutions. We also saw that the solutions provided the equation using Mathematica 11.2. In addition, graphs of some solutions and numerical explanations of these graphs were given. Recently, this method has been studied for obtaining exact travelling wave solutions of nonlinear partial differential equations.*

Keywords: *Sixth-order Ramani equation,* *new Konno-Oono equation system, Generalized* $\left(\frac{G'}{G}\right)$*- Expansion Method.*

1. INTRODUCTION

Nonlinear partial differential equations have an important place in many areas of the scientific World. Some of these areas are applied mathematics, optical fibers, physics, cosmology and fluid dynamics. Many analytical methods have been developed to solve nonlinear partial differential equations [1-10]. Apart from these methods, there are many methods in which these equations are solved using an auxiliary equation. Using these methods, partial differential equations are transformed into ordinary differential equations. These nonlinear partial differential equations are solved with the help of ordinary differential equations. Some of these methods are given in [11-24]. Many authors have applied these and similar methods to various equations [25-34].

We used the generalized $\left(\frac{G'}{G}\right)$- Expansion Method for finding the some soliton wave solution sixth-order Ramani equation and new Konno-Oono equation system. This method is presented by Manafian et al. [24].

2. ANALYSIS OF METHOD

Let's introduce the method briefly. Consider a general partial differential equation of two variables,

$$Q(u, u_t, u_x, u_{xx}, \dots) = 0, \tag{1}$$

[1] Firat University, Faculty of Education, Elazig, Turkey. E-mail: ieinan@yahoo.com ; unalic@firat.edu.tr.

Using the wave variable $u(x,t)=u(\xi),\ \ \xi=x-\mu t$ the equation (1) turns into an ordinary differential equation,

$$Q'(u,u',u'',u''',\dots)=0 \tag{2}$$

here μ is constant. With this conversion, we obtain a nonlinear ordinary differential equation for $u(\xi)$. We can express the solution of equation (2) as below,

$$u(\xi)=\sum_{k=0}^{m} d_k\Phi(\xi)^k+\sum_{k=1}^{m} e_k\Phi(\xi)^{-k} \tag{3}$$

where m is a positive integer is found as the result of balancing the highest order linear term and the highest order nonlinear term found in the equation, the coefficients d_k and e_k are constants. $\Phi(\xi)=\left(\frac{G\prime}{G}\right)$ satisfies the following ordinary differential equation,

$$k_1GG''-k_2GG'-k_3(G')^2-k_4G^2=0. \tag{4}$$

Substituting solution (3) into Eq. (2) yields a set of algebraic equation for $\left(\frac{G\prime}{G}\right),\left(\frac{G\prime}{G}\right)^{-k}$, then, all coefficients of $\left(\frac{G\prime}{G}\right),\left(\frac{G\prime}{G}\right)^{-k}$, have to vanish. After this separated algebraic equation, we can found d_k, e_k ,k_1,k_2,k_3,k_4 and μ constants. The special solutions of Eq. (4) are as follows:

Family 1. When $k_2\neq 0, f=k_1-k_3$ and $s=k_2^2+4k_4(k_1-k_3)>0$, then

$$\Phi(\xi)=\frac{k_2}{2f}+\frac{\sqrt{s}}{2f}\frac{C_1 sinh\left(\frac{\sqrt{s}}{2k_1}\xi\right)+C_2 cosh\left(\frac{\sqrt{s}}{2k_1}\xi\right)}{C_1 cosh\left(\frac{\sqrt{s}}{2k_1}\xi\right)+C_2 sinh\left(\frac{\sqrt{s}}{2k_1}\xi\right)}.$$

Family 2. When $k_2\neq 0, f=k_1-k_3$ and $s=k_2^2+4k_4(k_1-k_3)<0$, then

$$\Phi(\xi)=\frac{k_2}{2f}+\frac{\sqrt{-s}}{2f}\frac{-C_1 sin\left(\frac{\sqrt{-s}}{2k_1}\xi\right)+C_2 cos\left(\frac{\sqrt{-s}}{2k_1}\xi\right)}{C_1 cos\left(\frac{\sqrt{-s}}{2k_1}\xi\right)+C_2 sin\left(\frac{\sqrt{-s}}{2k_1}\xi\right)}.$$

Family 3. When $k_2\neq 0, f=k_1-k_3$ and $s=k_2^2+4k_4(k_1-k_3)=0$, then

$$\Phi(\xi)=\frac{k_2}{2f}+\frac{C_2}{C_1+C_2\xi}.$$

Family 4. When $k_2=0, f=k_1-k_3$ and $g=fk_4>0$, then

$$\Phi(\xi)=\frac{\sqrt{g}}{f}\frac{C_1 sinh\left(\frac{\sqrt{g}}{k_1}\xi\right)+C_2 cosh\left(\frac{\sqrt{g}}{k_1}\xi\right)}{C_1 cosh\left(\frac{\sqrt{g}}{k_1}\xi\right)+C_2 sinh\left(\frac{\sqrt{g}}{k_1}\xi\right)}.$$

Family 5. When $k_2 = 0, f = k_1 - k_3$ and $g = fk_4 < 0$, then

$$\Phi(\xi) = \frac{\sqrt{-g}}{f} \frac{-C_1 sin\left(\frac{\sqrt{-g}}{k_1}\xi\right) + C_2 cos\left(\frac{\sqrt{-g}}{k_1}\xi\right)}{C_1 cos\left(\frac{\sqrt{-g}}{k_1}\xi\right) + C_2 sin\left(\frac{\sqrt{-g}}{k_1}\xi\right)}.$$

Family 6. When $k_4 = 0$ and $f = k_1 - k_3$, then

$$\Phi(\xi) = \frac{C_1 k_2^2 exp\left(\frac{-k_2}{k_1}\xi\right)}{fk_1 + C_1 k_1 k_2 exp\left(\frac{-k_2}{k_1}\xi\right)}.$$

Family 7. When $k_2 \neq 0$ and $f = k_1 - k_3 = 0$, then

$$\Phi(\xi) = -\frac{k_4}{k_2} + C_1 exp\left(\frac{k_2}{k_1}\xi\right).$$

Family 8. When $k_1 = k_3$, $k_2 = 0$ and $f = k_1 - k_3 = 0$, then

$$\Phi(\xi) = C_1 + \frac{k_4}{k_1}\xi.$$

Family 9. When $k_3 = 2k_1$, $k_2 = 0$ and $k_4 = 0$, then

$$\Phi(\xi) = -\frac{1}{C_1 + \left(\frac{k_3}{k_1} - 1\right)\xi}.$$

We consider the Sixth-order Ramani equation [33]

$$u_{xxxxxx} + 15u_x u_{xxxx} + 15u_{xx}u_{xxx} + 45u_x^2 u_{xx} - 5u_{xxxt} - 15u_x u_{xt} - 15u_t u_{xx} - 5u_{tt} = 0, \quad (5)$$

If $u(x,t) = u(\xi)$, $\xi = x - \mu t$ conversion is used, the (5) equation becomes the following ordinary differential equation,

$$u^{(6)} + 15u'u^{(4)} + 15u''u''' + 45(u')^2 u'' + 5\mu u^{(4)} + 30\mu u'u'' - 5\mu^2 u'' = 0, \quad (6)$$

When balancing $u^{(6)}$ with $u''u'''$ then gives *m=1*. The solution is as follows,

$$u(\xi) = d_0 + d_1\Phi(\xi) + e_1\Phi(\xi)^{-1} \quad (7)$$

If Eq. (7) is substituted in Eq. (6), we have a system of algebraic equations for $d_0, d_1, e_1, ,k_1, k_2, k_3, k_4$ and μ. These algebraic equations system are as follows

$$\frac{5\mu^2 e_1 k_2}{k_1} - \frac{30\mu e_1^2 k_2}{k_1} - \frac{45 e_1^3 k_2}{k_1} - \frac{5\mu e_1 k_2^3}{k_1^3} - \frac{30 e_1^2 k_2^3}{k_1^3} + \frac{45 d_1 e_1^2 k_2^3}{k_1^3} - \frac{e_1 k_2^5}{k_1^5} - \frac{5\mu^2 e_1 k_2 k_3}{k_1^2} +$$
$$\frac{60\mu e_1^2 k_2 k_3}{k_1^2} + \frac{135 e_1^3 k_2 k_3}{k_1^2} + \frac{5\mu e_1 k_2^3 k_3}{k_1^4} + \frac{60 e_1^2 k_2^3 k_3}{k_1^4} - \frac{45 d_1 e_1^2 k_2^3 k_3}{k_1^4} + \frac{e_1 k_2^5 k_3}{k_1^6} - \frac{30\mu e_1^2 k_2 k_3^2}{k_1^3} -$$

$$\frac{135e_1^3k_2k_3^2}{k_1^3}-\frac{30e_1^2k_2^3k_3^2}{k_1^5}+\frac{45e_1^3k_2k_3^3}{k_1^4}-\frac{5\mu^2d_1k_2k_4}{k_1^2}+\frac{40\mu e_1k_2k_4}{k_1^2}+\frac{150e_1^2k_2k_4}{k_1^2}-\frac{135d_1e_1^2k_2k_4}{k_1^2}+\frac{5\mu d_1k_2^3k_4}{k_1^4}+\cdots \tag{8}$$

If the system is solved, the coefficients are found as

Case 1

$$d_1\neq 0,\ e_1=0,\ k_3=k_1-\frac{d_1k_1}{2}, k_4=0, k_1k_2\neq 0,\ \mu=\frac{5k_2^2+3\sqrt{5}k_2^2}{10k_1^2}$$

Solution 1

$$u(x,t)=d_0+\frac{k_2}{k_1}+\frac{\left(Sinh\left[\frac{\sqrt{k_2^2}\left(10k_1^2x-(5+3\sqrt{5})k_2^2t\right)}{20k_1^3}\right]C_1+Cosh\left[\frac{\sqrt{k_2^2}\left(10k_1^2x-(5+3\sqrt{5})k_2^2t\right)}{20k_1^3}\right]C_2\right)\sqrt{k_2^2}}{\left(Cosh\left[\frac{\sqrt{k_2^2}\left(10k_1^2x-(5+3\sqrt{5})k_2^2t\right)}{20k_1^3}\right]C_1+Sinh\left[\frac{\sqrt{k_2^2}\left(10k_1^2x-(5+3\sqrt{5})k_2^2t\right)}{20k_1^3}\right]C_2\right)k_1} \tag{9}$$

Case 2

$$k_2=0,\ k_3=\frac{1}{2}(2k_1-d_1k_1),\ k_4=\frac{e_1k_1}{2},\ e_1\neq 0,\ \mu=\frac{2}{5}\left(5d_1e_1+3\sqrt{5}d_1e_1\right), k_1\neq 0$$

Solution 2

$$u(x,t)=d_0+\frac{\left(\mathrm{Cos}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{-d_1e_1k_1^2}}{5k_1}\right]C_1^2+2\mathrm{Sin}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{-d_1e_1k_1^2}}{5k_1}\right]C_1C_2-\mathrm{Cos}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{-d_1e_1k_1^2}}{5k_1}\right]C_2^2\right)\sqrt{-d_1e_1k_1^2}}{\left(\mathrm{Sin}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{-d_1e_1k_1^2}}{10k_1}\right]C_1-\mathrm{Cos}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{-d_1e_1k_1^2}}{10k_1}\right]C_2\right)\left(\mathrm{Cos}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{-d_1e_1k_1^2}}{10k_1}\right]C_1+\mathrm{Sin}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{-d_1e_1k_1^2}}{10k_1}\right]C_2\right)k_1} \tag{10}$$

$$u(x,t)=d_0+\frac{\left(\mathrm{Cosh}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{d_1e_1k_1^2}}{5k_1}\right]C_1^2+2\mathrm{Sinh}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{d_1e_1k_1^2}}{5k_1}\right]C_1C_2+\mathrm{Cosh}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{d_1e_1k_1^2}}{5k_1}\right]C_2^2\right)\sqrt{d_1e_1k_1^2}}{\left(\mathrm{Sinh}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{d_1e_1k_1^2}}{10k_1}\right]C_1+\mathrm{Cosh}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{d_1e_1k_1^2}}{10k_1}\right]C_2\right)\left(\mathrm{Cosh}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{d_1e_1k_1^2}}{10k_1}\right]C_1+\mathrm{Sinh}\left[\frac{(5x-2(5+3\sqrt{5})td_1e_1)\sqrt{d_1e_1k_1^2}}{10k_1}\right]C_2\right)k_1}$$

(11)

Case 3

$$d_1=d_1,\quad e_1\neq 0,\quad k_2=0,\quad k_3=k_1,\quad k_1\neq 0, k_4=\frac{e_1k_1}{2},\ \mu=\frac{3}{2}\left(d_1e_1+\sqrt{2}d_1e_1\right)$$

Solution3

$$u(x,t) = d_0 + C_1 d_1 + \frac{1}{4} e_1 \left(2d_1 x - 3\left(1+\sqrt{2}\right) d_1^2 e_1 t + \frac{16}{4C_1 + e_1\left(2x - 3\left(1+\sqrt{2}\right) d_1 e_1 t\right)} \right) \tag{12}$$

Case4

$$d_1 = 0,\ e_1 \neq 0,\ k_2 = 0,\ k_4 = 0,\ k_1 \neq 0,\ \mu = \frac{30 e_1 k_1^4 - 30 e_1 k_1^3 k_3 - \sqrt{15}\sqrt{120 e_1^2 k_1^8 - 240 e_1^2 k_1^7 k_3 + 120 e_1^2 k_1^6 k_3^2}}{10 k_1^4}$$

Solution4

$$u(x,t) = d_0 - e_1(x + C_1 + 3e_1 t) - \frac{3\sqrt{2} e_1 \sqrt{e_1^2 k_1^8}\, t}{k_1^4} \tag{13}$$

Case 5

$$d_1 \neq 0,\ e_1 = 0,\ k_2 = 0,\ k_3 = k_1,\ k_1 \neq 0,\ \mu = -\left(\frac{-30 d_1 k_1^3 k_4 + 30\sqrt{2}\sqrt{d_1^2 k_1^6 k_4^2}}{10 k_1^4} \right)$$

Solution 5

$$u(x,t) = d_0 + d_1 \left(C_1 + \frac{k_4\left(k_1^4 x - 3 d_1 k_1^3 k_4 t + 3\sqrt{2}\sqrt{d_1^2 k_1^6 k_4^2}\, t \right)}{k_1^5} \right). \tag{14}$$

We consider the new Konno-Oono equation system [29]

$$\begin{aligned} v_t + 2uu_x &= 0 \\ u_{xt} - 2vu &= 0. \end{aligned} \tag{15}$$

If $u(x,t) = u(\xi),\ \xi = x - \mu t$ conversion is used, the (15) equation becomes the following ordinary differential equation,

$$\begin{aligned} -\mu v' + 2uu' &= 0, \\ -\mu u'' - 2vu &= 0. \end{aligned} \tag{16}$$

When balancing v' with uu' and u'' with uv then gives $m_1 = 1$ and $m_2 = 2$. The solutions are as follows,

$$\begin{aligned} u(\xi) &= d_0 + d_1 \Phi(\xi) + e_1 \Phi(\xi)^{-1} \\ v(\xi) &= f_0 + f_1 \Phi(\xi) + f_2 \Phi(\xi)^2 + g_1 \Phi(\xi)^{-1} + g_2 \Phi(\xi)^{-2} \end{aligned} \tag{17}$$

If Eq. (17) is substituted in Eq. (16), we have a system of algebraic equations for $d_0, d_1, e_1,\ f_0, f_1, f_2, g_1, g_2, k_1, k_2, k_3, k_4$ and μ. These algebraic equations system are as follows

$$2d_0e_1-\mu g_1-\frac{2d_0e_1k_3}{k_1}+\frac{\mu g_1k_3}{k_1}+\frac{2d_0d_1k_4}{k_1}-\frac{\mu f_1k_4}{k_1}=0,-\frac{2e_1^2k_4}{k_1}+\frac{2\mu g_2k_4}{k_1}=0,-\frac{2e_1^2k_2}{k_1}+\frac{2\mu g_2k_2}{k_1}-\frac{2d_0e_1k_4}{k_1}+\frac{\mu g_1k_4}{k_1}=0,\ 2e_1^2-2\mu g_2-\frac{2d_0e_1k_2}{k_1}+\frac{\mu g_1k_2}{k_1}-\frac{2e_1^2k_3}{k_1}+\frac{2\mu g_2k_3}{k_1}=0\cdots \tag{18}$$

If the system is solved, the coefficients are found as

Case 1

$$d_1=\frac{2(-d_0k_1+d_0k_3)}{k_2},\ e_1=0, f_0=0, k_4=0, k_2\neq 0, k_1\neq 0, d_0\neq 0, g_1=0, g_2=0, f_1=\frac{2id_0(-k_1+k_3)}{k_1}, f_2=\frac{d_1f_1}{2d_0},\ \mu=\frac{d_1^2}{f_2}$$

Solution 1

$$u(x,t)=-\frac{\left(Sinh\left[\frac{\sqrt{k_2^2}(2id_0k_1t+k_2x)}{2k_1k_2}\right]C_1+Cosh\left[\frac{\sqrt{k_2^2}(2id_0k_1t+k_2x)}{2k_1k_2}\right]C_2\right)d_0\sqrt{k_2^2}}{\left(Cosh\left[\frac{\sqrt{k_2^2}(2id_0k_1t+k_2x)}{2k_1k_2}\right]C_1+Sinh\left[\frac{\sqrt{k_2^2}(2id_0k_1t+k_2x)}{2k_1k_2}\right]C_2\right)k_2} \tag{19}$$

$$v(x,t)=-\frac{i(C_1^2-C_2^2)d_0k_2}{2\left(Cosh\left[\frac{\sqrt{k_2^2}(2id_0k_1t+k_2x)}{2k_1k_2}\right]C_1+Sinh\left[\frac{\sqrt{k_2^2}(2id_0k_1t+k_2x)}{2k_1k_2}\right]C_2\right)^2k_1} \tag{20}$$

Case 2

$$d_0=0, d_1=0, f_1=0, f_2=0, g_1=0, k_2=0, e_1\neq 0, k_3=\frac{e_1k_1+if_0k_1}{e_1}, f_0\neq 0, k_4=\frac{g_2(-k_1+k_3)}{f_0}, k_4\neq 0, \mu=-\frac{g_2k_1^2}{k_4^2}$$

Solution 2

$$u(x,t)=\frac{i\left(\text{Cos}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{-\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_1+\text{Sin}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{-\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_2\right)f_0k_1}{\left(\text{Sin}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{-\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_1-\text{Cos}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{-\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_2\right)\sqrt{-\frac{f_0g_2k_1^2}{e_1^2}}} \tag{21}$$

$$v(x,t)=\frac{(C_1^2+C_2^2)f_0}{\left(\mathrm{Sin}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{-\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_1-\mathrm{Cos}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{-\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_2\right)^2} \tag{22}$$

$$u(x,t)=-\frac{i\left(\mathrm{Cosh}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_1+\mathrm{Sinh}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_2\right)f_0k_1}{\left(\mathrm{Sinh}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_1+\mathrm{Cosh}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_2\right)\sqrt{\frac{f_0g_2k_1^2}{e_1^2}}} \tag{23}$$

$$v(x,t)=\frac{(-C_1^2+C_2^2)f_0}{\left(\mathrm{Sinh}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_1+\mathrm{Cosh}\left[\frac{\left(x-\frac{te_1^2}{g_2}\right)\sqrt{\frac{f_0g_2k_1^2}{e_1^2}}}{k_1}\right]C_2\right)^2} \tag{24}$$

Case 3

$$e_1=\frac{2d_0k_4}{k_2}, f_0=0,\ k_2\neq 0, k_1=k_3, d_0\neq 0, d_1=0, g_2\neq 0, f_1=0, f_2=0, k_3\neq 0, g_2=\frac{e_1g_1}{2d_0}, g_1=\frac{2id_0k_4}{k_3}, \mu=\frac{e_1^2}{g_2}$$

Solution 3

$$u(x,t)=d_0\left(1+\frac{2k_4}{e^{2id_0t+\frac{k_2x}{k_3}}C_1k_2-k_4}\right) \tag{25}$$

$$v(x,t)=\frac{2ie^{2id_0t+\frac{k_2x}{k_3}}C_1d_0k_2^2k_4}{k_3\left(-e^{2id_0t+\frac{k_2x}{k_3}}C_1k_2+k_4\right)^2} \tag{26}$$

Case 4

$e_1=0, f_0=0,\ k_2=0, k_4=0,\ d_0=0,\ f_1=0, f_2\neq 0, k_1\neq 0, g_1=0,\ g_2=0,\ f_2=\frac{id_1(-k_1+k_3)}{k_1},\ \mu=\frac{d_1^2}{f_2}$

Solution 4

$$u(x,t)=-\frac{d_1}{x+C_1+id_1t} \tag{27}$$

$$v(x,t) = \frac{id_1}{(x+C_1+id_1t)^2} \tag{28}$$

Case 5

$$f_0 = 0,\ k_2 = 0,\ d_0 = 0, d_1 = 0,\ k_1 = k_3,\ f_1 = 0,\ f_2 = 0,\ k_3 \neq 0, g_1 = 0,\ g_2 \neq 0,\ g_2 = \frac{ie_1k_4}{k_3},\ \mu = \frac{e_1^2}{g_2}$$

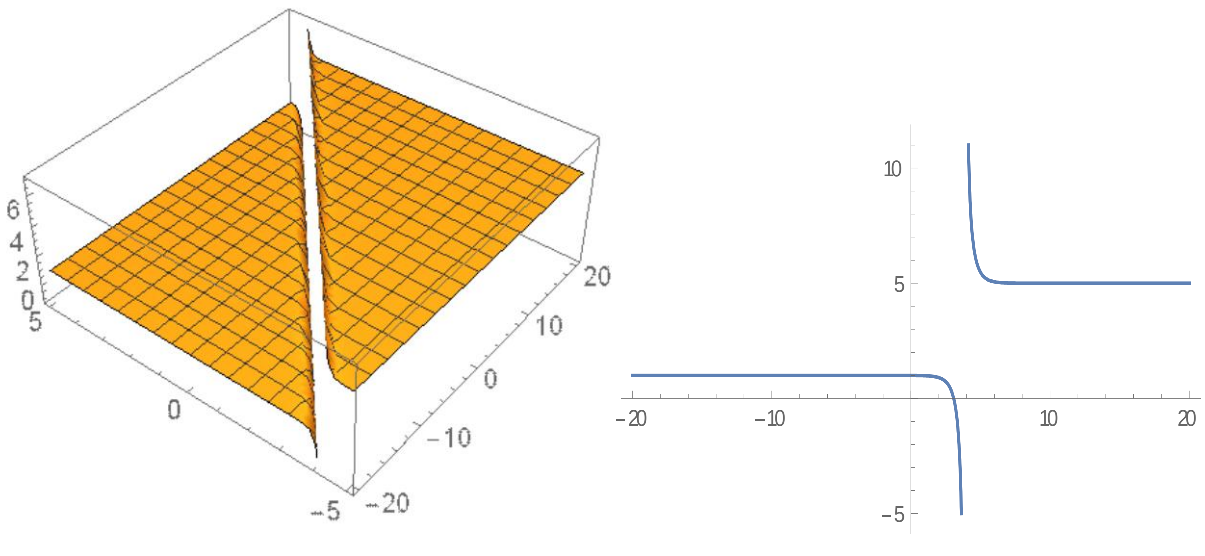
Solution 5

$$u(x,t) = -\frac{e_1}{C_1+ie_1t+\frac{k_4x}{k_1}} \tag{29}$$

$$v(x,t) = \frac{ie_1k_1k_4}{(C_1k_1+ie_1k_1t+k_4x)^2} \tag{30}$$

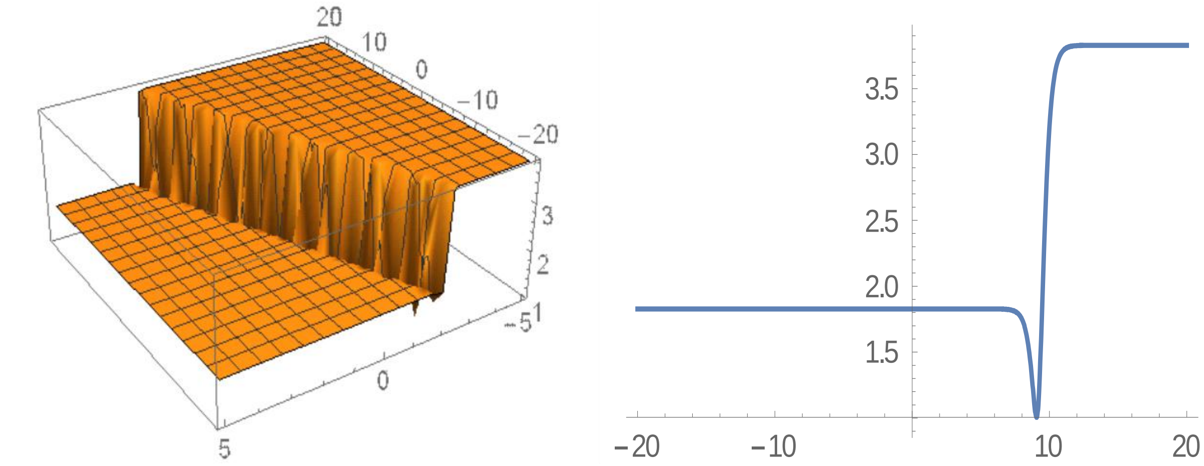
3. GRAPHS AND NUMERICAL EXPLANATIONS OF SOME SOLUTIONS

The graphical performance of found some solutions are demonstrated Figs. 1-7. These figures have the following physical explanations.



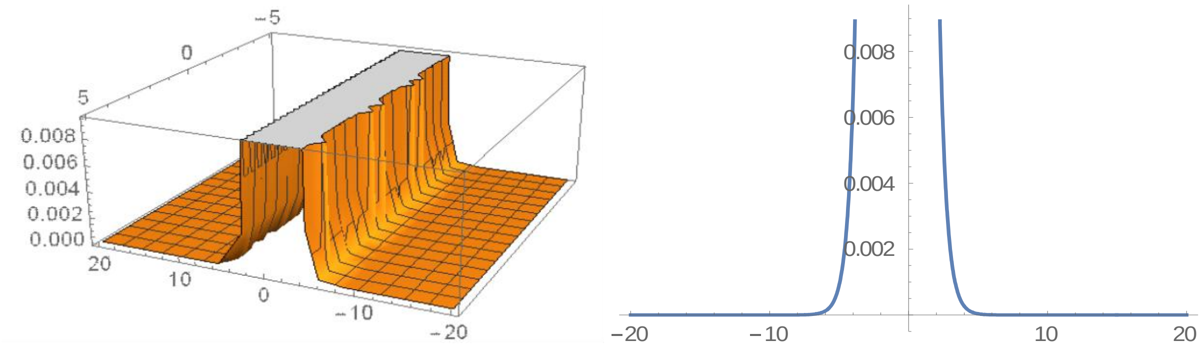
a) b)

Figure 1. a) The 3 Dimensional surfaces of Eq. (9) for $k_1 = 1, k_2 = 2, d_0 = 1, C_1 = 2, C_2 = 3$. b) The 2 Dimensional surfaces of Eq. (9) for $k_1 = 1, k_2 = 2, d_0 = 1, C_1 = 2, C_2 = 3$ and $t = 1$.



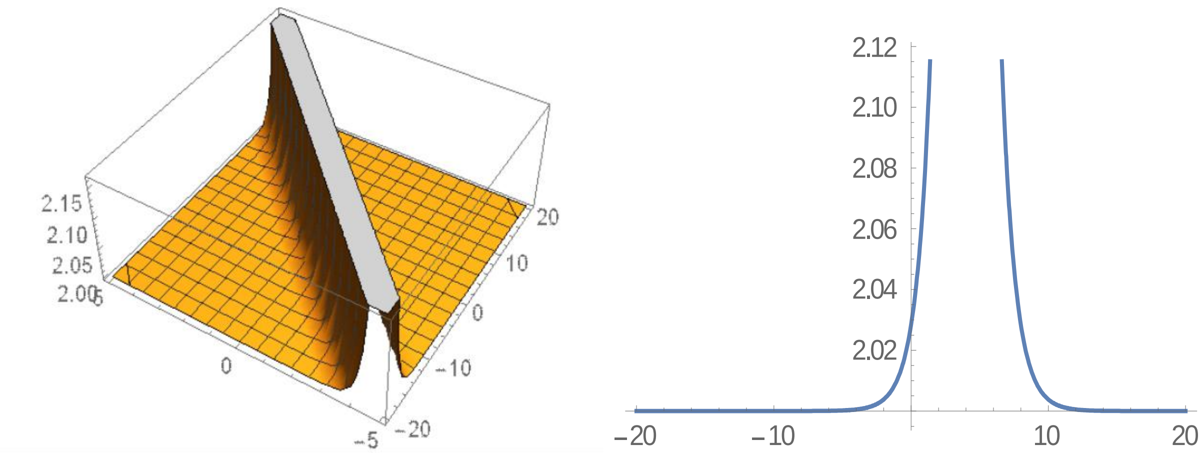
a) **b)**

Figure 2. a) The 3 Dimensional surfaces of Abs(Eq. (10)) for $k_1 = 2, d_1 = 2, d_0 = 1, e_1 = 1, C_1 = 3, C_2 = 2$. b) The 2 Dimensional surfaces of Abs(Eq. (10)) for $k_1 = 2, d_1 = 2, d_0 = 1, e_1 = 1, C_1 = 3, C_2 = 2$ and $t = 1$.



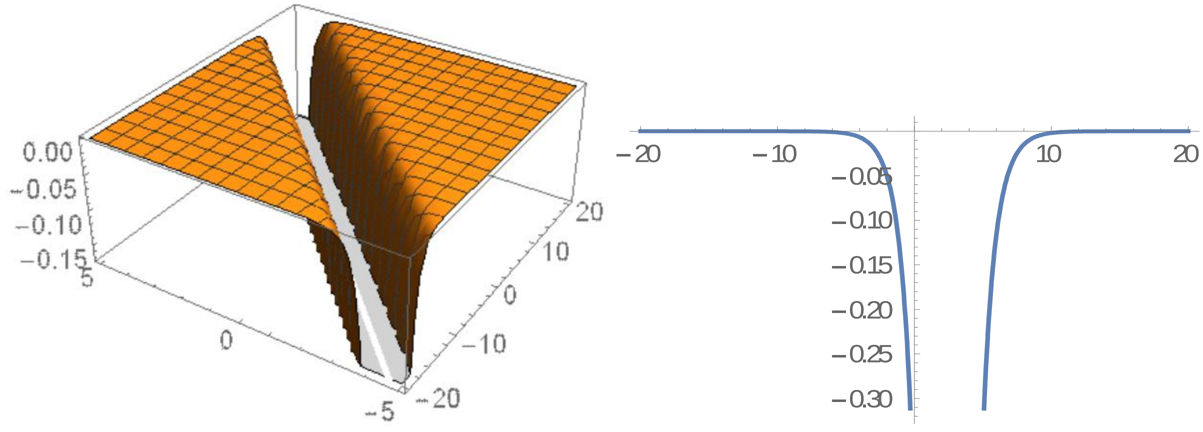
a) **b)**

Figure 3. a) The 3 Dimensional surfaces of Abs(Eq. (20)) for $k_1 = 1, k_2 = 2, d_0 = 1, C_1 = 2, C_2 = 3$. b) The 2 Dimensional surfaces of Abs(Eq. (20)) for $k_1 = 1, k_2 = 2, d_0 = 1, C_1 = 2, C_2 = 3$ and $t = 1$.



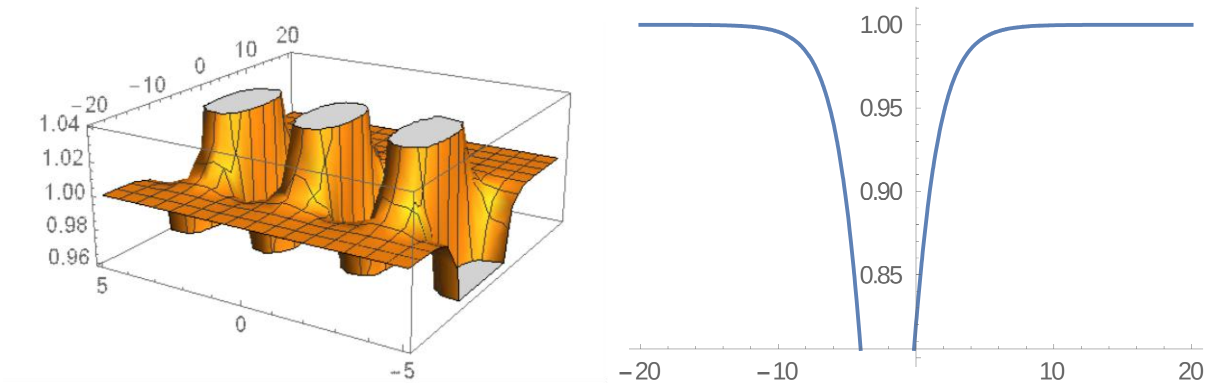
a) **b)**

Figure 4. a) The 3 Dimensional surfaces of Abs(Eq. (21)) for $k_1 = 2, e_1 = 2, f_0 = 1, g_2 = 1, C_1 = 3, C_2 = 2$. b) The 2 Dimensional surfaces of Abs(Eq. (21)) for $k_1 = 2, e_1 = 2, f_0 = 1, g_2 = 1, C_1 = 3, C_2 = 2$ and $t = 1$.



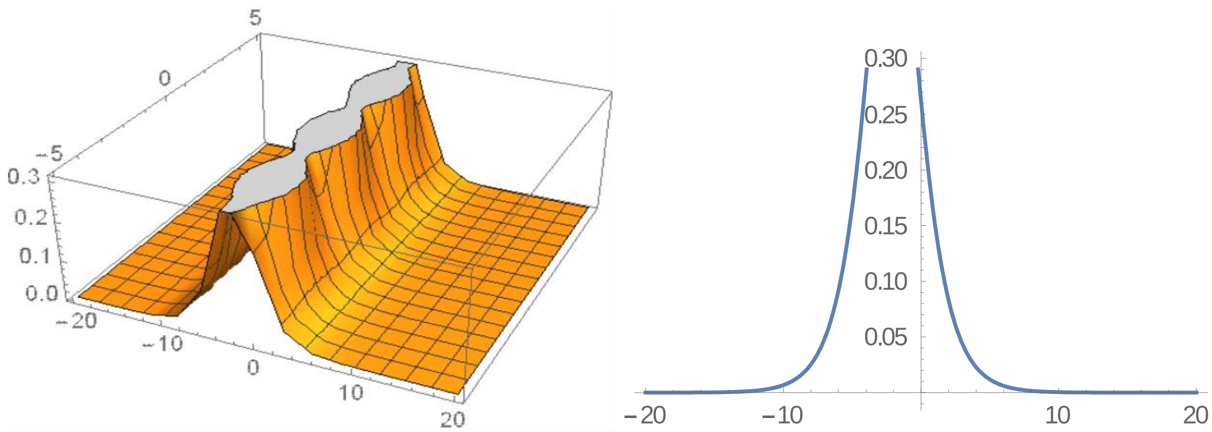
a) **b)**

Figure 5. a) The 3 Dimensional surfaces of Eq. (24) for $k_1 = 2, e_1 = 2, f_0 = 1, g_2 = 1, C_1 = 3, C_2 = 2$. b) The 2 Dimensional surfaces of Eq. (24) for $k_1 = 2, e_1 = 2, f_0 = 1, g_2 = 1, C_1 = 3, C_2 = 2$ and $t = 1$.



a) **b)**

Figure 6. a) The 3 Dimensional surfaces of Abs(Eq. (25)) for $k_2 = 2, k_3 = 3, k_4 = 1, d_0 = 1, C_1 = 2$. b) The 2 Dimensional surfaces of Abs(Eq. (25)) for $k_2 = 2, k_3 = 3, k_4 = 1, d_0 = 1, C_1 = 2$ and $t = 1$.



a) **b)**

Figure 7. a) The 3 Dimensional surfaces of Abs(Eq. (26)) for $k_2 = 2, k_3 = 3, k_4 = 1, d_0 = 1, C_1 = 2$. b) The 2 Dimensional surfaces of Abs(Eq. (26)) for $k_2 = 2, k_3 = 3, k_4 = 1, d_0 = 1, C_1 = 2$ and $t = 1$.

Sixth-order Ramani equation: The shapes of Eqs.(9)-(10) are represented in Figs. 1-2 within the interval $-20 \leq x \leq 20, -5 \leq t \leq 5$. New Konno-Oono equation system: The shapes of Eqs.(20)-(21)-(24)-(25)-(26) are represented in Figs. 3-7 within the interval $-20 \leq x \leq 20, -5 \leq t \leq 5$.

4. CONCLUSIONS

We used the generalized $\left(\frac{G'}{G}\right)$ – expansion method to find the some soliton wave solution sixth-order Ramani equation, new Konno-Oono equation system. The solutions found as a result of the application of the method are hyperbolic function solutions, trigonometric function solutions, exponential function solutions and rational functional solutions.

The accuracy of these solutions was seen by using Mathematica 11.2 computer program. In addition, graphs of some solutions and numerical explanations of these graphs were given. Many nonlinear partial differential equations and system of equations can be solved using this method.

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