ORIGINAL PAPER

AN EFFECTIVE TECHNIQUE OF $exp(-\phi(\xi))$- EXPANSION METHOD FOR THE SCHAMEL-BURGERS EQUATION

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Abstract. *The Schamel-Burgers equation, producing the shock-type traveling waves in magnificent physical cases, has lots of potential for analyzing ion-acoustic waves in plasma physics and fluid dynamics. Scientists have worked for a long time to explore the traveling wave solutions of such equations. Thus, in this article, some new traveling wave solutions of the Schamel-Burgers equation, different from those found in the literature, are generated. For this aim, the* $exp(-\phi(\xi))$*- expansion method is implemented. We also provide the solutions through two- and three-dimensional figures. Generally, exact traveling wave solutions will be useful in the theoretical and numerical study of the nonlinear evolution equations. The obtained results are very supportive, which ensures a more effective mathematical instrument for examining exact traveling wave solutions of the nonlinear equations arising in the recent area of applied sciences and engineering*

Keywords: $exp(-\phi(\xi))$ *expansion method; Schamel–Burgers equation; travelling wave solutions.*

1. INTRODUCTION

Schamel-Burgers equation is a special form of following Schamel-Korteweg-de Vries (SKdV) equation,

$$u_t + (Au^{1/2} + Bu)_x + Cu_{xxx} = 0\,, AB \neq 0 \tag{1}$$

which identifies the nonlinear interaction of ion-acoustic waves when electron capturing is present [1], has the form

$$u_t + Au^{1/2}u_x - Bu_{xx} = 0 \tag{2}$$

where *A*, *B* and *C* are constants and match up to the activation trapping, convection, and dispersion coefficients, respectively [2, 3]. In particular Eq. (2) defines the nonlinear interaction of ion-acoustic waves in plasma physics by including a qquasi-potentialeffect [4]. In the presence of solitary waves, the S–KdV model is used to simulate the effect of surface for deepwater [5]. Several important techniques have been built to generate exact solutions of the S-KdV equation such as extended unified method [4], modified Kudrayshov approach [5], Lie symmetry approach [6], (G'/G) expansion method [7,8], exp function method [1, 9], extended Kudryashov method [10], Khater and the modified Khater methods

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[11], improved version of Tanh method [12], etc. The S-KdV equation turns into Korteweg de-Vries equation (when $A = 0$) [13-18]

$$u_t + Buu_x + Cu_{xxx} = 0, \tag{3}$$

and the Schamel equation (when $B = 0$)

$$u_t + Au^{1/2}u_x + Cu_{xxx} = 0. \tag{4}$$

The Schamel equation, which was derived by Hans Schamel, and widely used in the field of plasma physics, serves as a mathematical model for studying nonlinear wave phenomena, particularly the dynamics of ion acoustic waves in plasmas [19–26]. Eq.(4) was solved by some existing methods in [7, 10, 26-30]. Schamel Burgers equation generally exists in nonlinear dynamics, plasma physics, and optical fiber. The obtained exact solutions are significant for revealing the most complex physical phenomena or exploring new phenomena. Besides, these solutions assist for those doing numerical calculations to evaluate the correctness of their results. It is noted that this method changes the given difficult problems which are solved easily. Since Eq. (2) has a very extensive implementation area, it is significant to look for new exact solutions to the equation. In literature, some scientists have studied this model analytically such as the tangent hyperbolic method [31-35] extended generalized (G'/G) expansion method [2], generalized-improved (G'/G) and and generalized (G'/G) expansion methods [3].

The implementation of the $exp(-\phi(\xi))$- expansion method in this work highlights our principal objective and shows its capacity to deal with nonlinear equations, permitting it to be utilized to solve several types of nonlinearity models. So, we begin in Sections 2 and 3, introducing mathematical analysis and methodology of the $exp(-\phi(\xi))$- expansion method, respectively. In Section 4, several new families of analytical solutions of the equation has been obtained using the $exp(-\phi(\xi))$- expansion method. Section 5 addresses a graphical illustration of a few of these solutions. Conclusions complete this study.

2. $exp(-\phi(\xi))$-EXPANSION METHOD: MATHEMATICAL ANALYSIS OF THE MODEL

We implement the $exp\left(-\phi(\xi)\right)$- expansion method [36] to derive exact solutions of the equation. To obtain the traveling wave solution of Eq. (2), we employ the following transformation:

$$u(x, t) = \Omega(\xi), \tag{5}$$

where

$$\xi = \kappa x - t\omega \tag{6}$$

where ω represents the speed of the traveling wave. With the help of equations (5) and (6), we can rewrite Eq. (2) as:

$$\Omega'(\xi)\left(A\kappa\sqrt{\Omega(\xi)} - \omega\right) - B\kappa^2\Omega''(\xi) = 0. \tag{7}$$

Now, we can take the transformation

$$\Omega(\xi) = h[(\xi)]^\wedge 2, \tag{8}$$

the (7) can be writen as

$$-2h(\xi)h'^2(h(\xi)h''(\xi)+h'^2)=0. \tag{9}$$

Then, we can solve (9) to find the solution (2).

3. METHODOLOGY OF THE $exp(-\phi(\xi))$- EXPANSION METHOD

Take into consideration that a nonlinear partial differential (NLPDE) equation has the form

$$P\left(u,u_t,u_x,u_y,u_z,u_{xt},u_{xx},\dots\right)=0. \tag{10}$$

Step 1: Joining the independent variables x and t into one variable ξ. The traveling wave transformations (5) and (6) allows us to reduct (10) to the following ordinary differential equation (ODE):

$$F(h',h'',h''',\dots)=0. \tag{11}$$

Step 2: Assume traveling wave solution of (11) is written as a finite series:

$$\mathrm{h}(\xi)=\sum_{\mathrm{i}=0}^{\mathrm{N}}\mathrm{f_i}(\exp\left(-\mathrm{i}\emptyset(\xi)\right)), \tag{12}$$

where $\mathrm{f_i}(0\le \mathrm{i}\le \mathrm{N})$ are constants, $f_N\neq 0$ and $\Phi=\Phi(\xi)$ procures following ODE,

$$\emptyset'(\xi)=\exp\left(-\emptyset(\xi)\right)+\mu\exp\left(\emptyset(\xi)\right)+\lambda. \tag{13}$$

Equality (13) has the following solutions:

Group 1: when $\lambda^2-4\mu>0,\mu\neq 0,$

$$\emptyset(\xi)=\ln\left(\frac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}{2\mu}\right), \tag{14}$$

$$\emptyset(\xi)=\ln\left(\frac{-\sqrt{(\lambda^2-4\mu)}\coth\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}{2\mu}\right), \tag{15}$$

where E is a constant of integration.

Group 2: when $\lambda^2-4\mu<0,\mu\neq 0,$

$$\emptyset(\xi)=\ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}{2\mu}\right), \tag{16}$$

$$\emptyset(\xi)=\ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\cot\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}{2\mu}\right). \tag{17}$$

Group 3: when $\lambda^2-4\mu>0,\mu=0,\lambda\neq 0,$

$$\emptyset(\boldsymbol{\xi}) = -\ln\left(\frac{\lambda}{\exp\big(\lambda(\xi+E)\big)-1}\right). \tag{18}$$

Group 4: when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

$$\emptyset(\xi) = \ln(-\frac{2(\lambda(\xi+E)+2}{\lambda^2(\xi+E)}). \tag{19}$$

Group 5: when $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0,$

$$\emptyset(\xi) = \ln(\xi+E). \tag{20}$$

Step 3: N is obtained by balancing the highest order derivative term with the highest power nonlinear term in (12). We substitute (12) into (9), all coefficients of the same power of $exp(-\phi(\xi))$ is taken zero. This leads a system of algebraic equations for $f_0, f_1, \dots, f_N$, κ, ω, which can be solved by Mathematica program.

4. IMPLEMENTION OF THE $\boldsymbol{exp(-\phi(\xi))}$-EXPANSION METHOD

Applying the balance principle to (9) between the terms hh'' and h^2h' yields $3N + 1 = 2N + 2$, which implies that $N = 1$. Using (12), we can express the solution to (9) as follows:

$$h(\xi) = \sum_{i=0}^{1} f_i\big(\exp\big(-i\emptyset(\xi)\big)\big). \tag{21}$$

By substituting Eq. (21) for Eq. (9) and equating coefficients of like powers of $exp(-\phi(\xi))$ to zero, following system is obtained:

$$-2Af_0^2f_1\kappa\mu - 2Bf_0f_1\kappa^2\lambda\mu - 2Bf_1^2\kappa^2\mu^2 + 2f_0f_1\mu\omega = 0,$$
$$-2Af_0^2f_1\kappa\lambda - 4Af_0f_1^2\kappa\mu - 2Bf_0f_1\kappa^2\lambda^2 - 6Bf_1^2\kappa^2\lambda\mu - 4Bf_0f_1\kappa^2\mu + 2f_0f_1\lambda\omega + 2f_1^2\mu\omega = 0,$$
$$-4Af_0f_1^2\kappa\lambda - 2Af_1^3\kappa\mu - 2Af_0^2f_1\kappa - 4Bf_1^2\kappa^2\lambda^2 - 6Bf_0f_1\kappa^2\lambda - 8Bf_1^2\kappa^2\mu + 2f_1^2\lambda\omega + 2f_0f_1\omega = 0,$$
$$-2Af_1^3\kappa\lambda - 4Af_0f_1^2\kappa - 10Bf_1^2\kappa^2\lambda - 4Bf_0f_1\kappa^2 + 2f_1^2\omega = 0,$$
$$-2Af_1^3\kappa - 6Bf_1^2\kappa^2 = 0.$$

By solving aforementioned set of equations using the Mathematica program, following sets of solution was obtained:

Set 1:

$$\kappa = -\frac{\sqrt[4]{-1}\sqrt{\omega}}{\sqrt[4]{16B^2\mu - 4B^2\lambda^2}},$$
$$f_0 = \frac{3(\frac{\sqrt[4]{-1}\sqrt{2}B\lambda\omega^{3/2}}{\sqrt[4]{-B^2(\lambda^2-4\mu)}} + \frac{4(-1)^{3/4}\sqrt{2}B^2\mu\omega^{3/2}}{\big(-B^2(\lambda^2-4\mu)\big)^{3/4}} - \frac{(-1)^{3/4}\sqrt{2}B^2\lambda^2\omega^{3/2}}{(-B^2(\lambda^2-4\mu))^{3/4}}}{4A\omega} \tag{22}$$

$$f_1 = \frac{3\sqrt[4]{-1}B\sqrt{\omega}}{A\sqrt[4]{16B^2\mu - 4B^2\lambda^2}}.$$

By substituting Eqs. (22) to (21) with Eqs. (8), (5) and (6), respectively, solutions of Eq. (2) is obtained in the following way:

Group 1: when $\lambda^2 - 4\mu > 0, \mu \neq 0,$

$$u_1(x,t) = (f_0 + f_1(\frac{2\mu}{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)-\lambda}))^2,$$
$$u_2(x,t) = (f_0 + f_1(\frac{2\mu}{-\sqrt{(\lambda^2-4\mu)}\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)-\lambda}))^2. \quad (23)$$

Group 2: when $\lambda^2 - 4\mu < 0, \mu \neq 0,$

$$u_3(x,t) = (f_0 + f_1(\frac{2\mu}{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)\right)-\lambda}))^2,$$
$$u_4(x,t) = (f_0 + f_1(\frac{2\mu}{\sqrt{(4\mu-\lambda^2)}\cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)\right)-\lambda}))^2 \quad (24)$$

Group 3: when $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \neq 0,$

$$u_5(x,t) = (f_0 + f_1(\tfrac{\lambda}{\exp(\lambda(\xi+E))-1}))^2. \quad (25)$$

Group 4: when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

$$u_6(x,t) = (f_0 + f_1(-\tfrac{\lambda^2(\xi+E)}{2(\lambda(\xi+E)+2}))^2. \quad (26)$$

Set 2:

$$\kappa = \frac{\sqrt[4]{-1}\sqrt{\omega}}{\sqrt[4]{16B^2\mu-4B^2\lambda^2}},$$
$$f_0 = \frac{3\left(-\frac{\sqrt[4]{-1}B\lambda\omega^{3/2}}{\sqrt[4]{16B^2\mu-4B^2\lambda^2}}+\frac{2(-1)^{3/4}B^2\lambda^2\omega^{3/2}}{(16B^2\mu-4B^2\lambda^2)^{3/4}}-\frac{8(-1)^{3/4}B^2\mu\omega^{3/2}}{(16B^2\mu-4B^2\lambda^2)^{3/4}}\right)}{2A\omega}, \quad (27)$$
$$f_1 = -\frac{3\sqrt[4]{-1}B\sqrt{\omega}}{\sqrt{2}A\sqrt[4]{-B^2(\lambda^2-4\mu)}}.$$

By substituting Eqs. (27) to (21) with Eqs. (8), (5) and (6), respectively, solutions of Eq. (2) are obtained in the following way:

Group 1: An Effective Technique of ... Khalıd K. Alı and Seydı Battal Gazı Karakoc$\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$u_1(x,t) = \left(f_0 + f_1\left(\frac{2\mu}{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}\right)\right)^2,$$
$$u_2(x,t) = \left(f_0 + f_1\left(\frac{2\mu}{-\sqrt{(\lambda^2-4\mu)}\coth\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}\right)\right)^2. \tag{28}$$

Group 2: when $\lambda^2 - 4\mu < 0, \mu \neq 0$,

$$u_3(x,t) = \left(f_0 + f_1\left(\frac{2\mu}{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}\right)\right)^2,$$
$$u_4(x,t) = \left(f_0 + f_1\left(\frac{2\mu}{\sqrt{(4\mu-\lambda^2)}\cot\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}\right)\right)^2. \tag{29}$$

Group 3: when $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \neq 0$,

$$u_5(x,t) = \left(f_0 + f_1\left(\frac{\lambda}{\exp(\lambda(\xi+E))-1}\right)\right)^2. \tag{30}$$

Group 4: when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$u_6(x,t) = \left(f_0 + f_1\left(-\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E))+2}\right)\right)^2. \tag{31}$$

Set 3:

$$\kappa = -\frac{(-1)^{3/4}\sqrt{\omega}}{\sqrt[4]{16B^2\mu-B^2\lambda^2}},$$
$$f_0 = \frac{3\left(\frac{(-1)^{3/4}B\lambda\omega^{3/2}}{\sqrt[4]{16B^2\mu-4B^2\lambda^2}} - \frac{2\sqrt[4]{-1}B^2\lambda^2\omega^{3/2}}{(16B^2\mu-4B^2\lambda^2)^{3/4}} + \frac{8\sqrt[4]{-1}B^2\mu\omega^{3/2}}{(16B^2\mu-4B^2\lambda^2)^{3/4}}\right)}{2A\omega}, \tag{32}$$
$$f_1 = \frac{3(-1)^{3/4}B\sqrt{\omega}}{A\sqrt[4]{16B^2\mu-4B^2\lambda^2}}.$$

By substituting Eqs. (32) to (21) with Eqs. (8), (5) and (6), respectively, solutions of Eq. (2) is obtained in the following way:

Group 1: when $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$u_1(x,t) = \left(f_0 + f_1\left(\frac{2\mu}{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}\right)\right)^2,$$
$$u_2(x,t) = \left(f_0 + f_1\left(\frac{2\mu}{-\sqrt{(\lambda^2-4\mu)}\coth\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}\right)\right)^2. \tag{33}$$

Group 2: when $\lambda^2 - 4\mu < 0, \mu \neq 0$,

$$
\begin{aligned}
u_3(x,t) &= (f_0 + f_1(\frac{2\mu}{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}))^2,\\
u_4(x,t) &= (f_0 + f_1(\frac{2\mu}{\sqrt{(4\mu-\lambda^2)}\cot\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}))^2.
\end{aligned}
\tag{34}
$$

Group 3: when $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \neq 0$,

$$
u_5(x,t) = (f_0 + f_1(\frac{\lambda}{\exp(\lambda(\xi+E))-1}))^2. \tag{35}
$$

Group 4: when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$
u_6(x,t) = (f_0 + f_1(-\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E))+2}))^2. \tag{36}
$$

Set 4:

$$
\begin{aligned}
\kappa &= \frac{(-1)^{3/4}\sqrt{\omega}}{\sqrt[4]{16B^2\mu-4B^2\lambda^2}},\\
f_0 &= \frac{3\left(\frac{\sqrt[4]{-1}\sqrt{2}B^2\lambda^2\omega^{3/2}}{(-B^2(\lambda^2-4\mu)^{3/4}}-\frac{(-1)^{3/4}\sqrt{2}B\omega^{3/2}}{\sqrt[4]{-B^2(\lambda^2-4\mu)}}+\frac{4\sqrt[4]{-1}\sqrt{2}B^2\mu\omega^{3/2}}{(-B^2(\lambda^2-4\mu))^{3/4}}\right)}{4A\omega},\\
f_1 &= -\frac{3(-1)^{3/4}B\sqrt{\omega}}{\sqrt{2}A\sqrt[4]{-B^2(\lambda^2-4\mu)}}.
\end{aligned}
\tag{37}
$$

By substituting Eqs. (37) to (21) with Eqs. (8), (5) and (6), respectively, solutions of Eq. (2) is obtained in the following way:

Group 1: when $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$
\begin{aligned}
u_1(x,t) &= (f_0 + f_1(\frac{2\mu}{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}))^2,\\
u_1(x,t) &= (f_0 + f_1(\frac{2\mu}{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}))^2,
\end{aligned}
\tag{38}
$$

Group 2: when $\lambda^2 - 4\mu < 0, \mu \neq 0$,

$$
\begin{aligned}
u_3(x,t) &= (f_0 + f_1(\frac{2\mu}{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}))^2,\\
u_4(x,t) &= (f_0 + f_1(\frac{2\mu}{\sqrt{(4\mu-\lambda^2)}\cot\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}))^2.
\end{aligned}
\tag{39}
$$

Group 3: when $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \neq 0$

$$
u_5(x,t) = (f_0 + f_1(\frac{\lambda}{\exp(\lambda(\xi+E))-1}))^2. \tag{40}
$$

Group 4: when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$

$$u_6(x,t) = (f_0 + f_1(-\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E))+2}))^2. \quad (41)$$

5. GRAPHICAL ILLUSTRATIONS

We utilize the analytical approach in this study to explore the equations we derived. To enhance the clarity of our outcomes, we present a collection of two-dimensional and threedimensional figures that depict some of the analytical solutions we obtained. Specifically, we illustrated the analytical solutions for Eqs. (24), (28), (34) and (38) in Figs. 1-4, respectively, under various parameter configurations. These figures visually represent the solutions behavior and demonstrate how the different parameters influence the system. By analyzing these figures, we gain insight into the problem dynamics and the impact of the parameters on the solutions' overall behavior. Our outcomes demonstrate the efficacy of our approach and provide valuable insights into the solutions' behavior and properties.

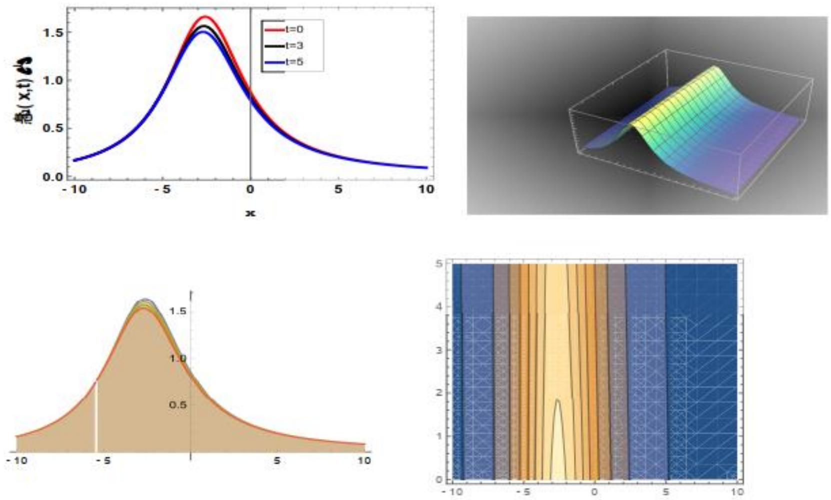

Figure 1. Graph of (24) for *A* = 0.6, *B* = 0.7, *λ* = 0.01, *μ* = 0.01, *ω* = 0.005, *E* = 0.

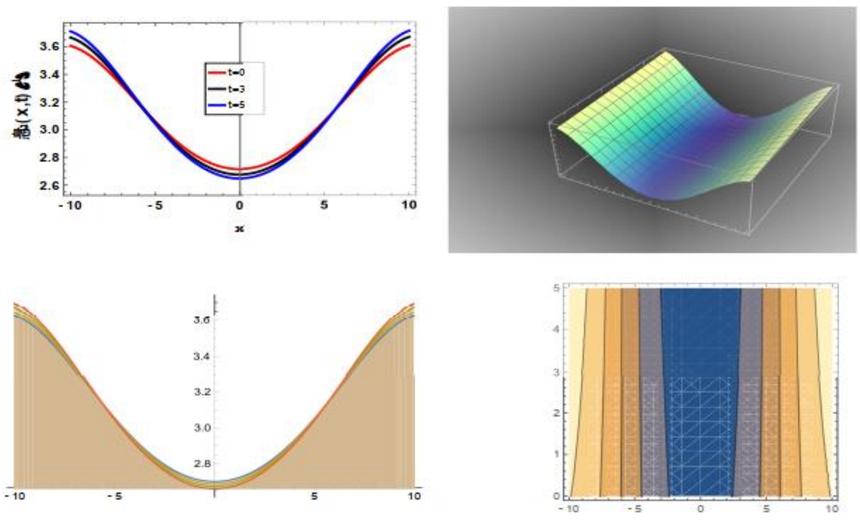

Figure 2. Graph of (28) for *A* = 0.1, *B* = 0.2, *λ* = 0.4, *μ* = 0.01, *ω* = 0.1, *E* = 0.

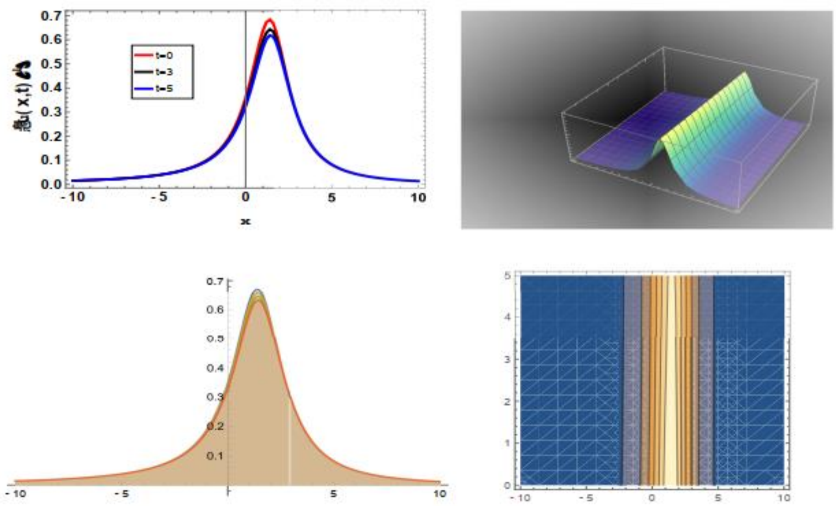

Figure 3. Graph of (34) for *A* = 0.5, *B* = 0.2, *λ* = 0.01, *μ* = 0.01, *ω* = 0.005, *E* = 0.

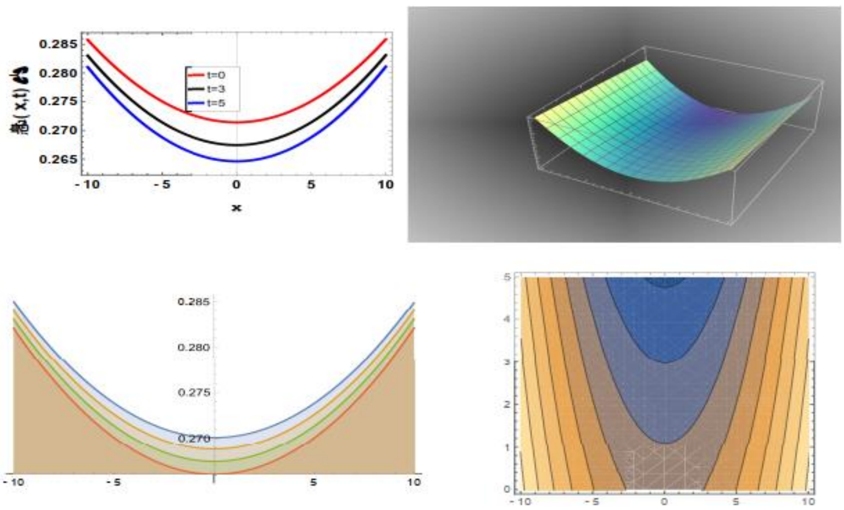

Figure 4. Graph of (38) for *A* = 0.4, *B* = 0.2, *λ* = 0.4, *μ* = 0.01, *ω* = 0.07, *E* = 0.

6. CONCLUSION

In this study, we propose an analytic solution procedure to obtain some new traveling wave solutions of well known Schamel-Burgers equation which is popular for studying ionacoustic waves in plasma physics and fluid dynamics. For this goal, $exp(-\phi(\xi))$-expansion method is successfully implemented. The obtained solutions from $exp(-\phi(\xi))$-expansion methods are plotted graphically to check the dynamical behavior of the solutions. The presented exact solutions provided here may describe various new characteristics of waves and then may be useful in the theoretical studies of the considered equation. From this, the proposed technique is extremely influential and competent and can be further applied to various NLPDEs.

REFERENCES

[1] Lee, J., Sakthivel, R., *Reports on Mathematical Physics*, **68**(2), 153, 2011.

[2] Mohanty, S.K., Kumary, S., Dekaz, M.K., Dev, A.N., *International Journal of Modern Physics B*, **35** (31), 1, 2021.
[3] Mohanty, S.K., Kravchenko, O.V., Dev, A.N., *Results in Physics*, **33,** 105124, 2022.
[4] Abdel-Gawad, H. I., Tantawy, M., *Information Sciences Letters*, **3**, 103, 2014.
[5] Tariq, K.U., Rezazadeh, H., Zubair, M., Osman, M.S., Akinyemi, L., *Journal of Applied and Computational Mathematics*, **8**(114), 1, 2022.
[6] Giresunlu, I.B., Ozkan, Y.S., Yasar, E., *Mathematical Methods in the Applied Sciences*, **40**(11), 3927, 2017.
[7] Kangalgil, F., *Journal of the Egyptian Mathematical Society*, **24**, 526, 2016.
[8] Donmez, O., Daghan, D., *Süleyman Demirel University Journal of Natural and Applied Sciences*, **21**(1), 208, 2017.
[9] Pinar, Z., Yildirim, A., Mohyud-Din, S.T., Oguz, K.F., Djabrailov, S., Biswas, A., *Studies in Nonlinear Sciences*, **3**(3), 102, 2012.
[10] Hassan, M.M., Shehata, A.R., Abdel-Daym, M.S., *Assiut University Journal of Multidisciplinary Scientific Research*, **51**(2), 115, 2022.
[11] Tariq, K.U., Inc, Mustafa, Yepez-Martinez, H., Khater, M. M. A., *Journal of Ocean Engineering and Science*, 10.1016/j.joes.2022.05.025, 2022.
[12] Aziz, F., Asif, A., Bint-e-Munir, F., *Chaos, Solitons and Fractals*, **134,** 109737, 2020.
[13] Korteweg, D.J., de Vries, G., *Philosophical Magazine*, **39**, 422, 1895.
[14] Zabusky, N.J., *Nonlinear Partial Differential Equations*, Academic Press, 223, 1967.
[15] Karakoc, S.B.G., *Statistics, Optimization and Information Computing*, **6**, 609, 2018.
[16] Bashan, A., Esen, A., *Numerical Methods for Partial Differential Equations*, **37**(2), 1561, 2021.
[17] Karakoc, S.B.G., *Malaya Journal of Matematik*, **6**(4), 835, 2018.
[18] Karakoc, S.B.G., Saha, A., Sucu, D., *Chinese Journal of Physics*, **68**, 605, 2020.
[19] Schamel, H., *Plasma Physics*, **14**(10), 905, 1972.
[20] Schamel, H., *Journal of Plasma Physics*, **9**(3), 377, 1973.
[21] Williams, G., Verheest, F., Hellberg, M.A., Anowar, M.G.M., Kourakis, I.A., *Physics of Plasmas*, **21**(9), 092103, 2014.
[22] Mushtaq, A., Shah, H.A., *Physics of Plasmas*, **13**, 012303, 2006.
[23] Saha, A., Chatterjee, P., *European Physical Journal Plus*, **130**(11), 222, 2015.
[24] Saha, A., Chatterjee, P., *European Physical Journal D*, **69**(9), 203, 2015.
[25] Chowdhury, S., Mandi, L., Chatterjee, P., *Physics of Plasmas*, **25**, 042112, 2018.
[26] Flamarion, M.V., Pelinovsky, E., Didenkulova, E., *Chaos, Solitons and Fractals*, **174**, 113870, 2023.
[27] Ghanbari, B., Akgul, A., *Physica Scripta*, **95**(7), 075201, 2020.
[28] Daghan, D., Donmez, O., *Journal of Applied Mechanics and Technical Physics*, **59**, 389, 2018.
[29] Taha, W.M., Noorani, M., Hashim, I., *Abstract and Applied Analysis*, **2013**, 1, 2013.
[30] Mohanty, S.K., Dev, A.N., *Recent Trends in Applied Mathematics. Lecture Notes in Mechanical Engineering - Select Proceedings of AMSE 2019*, 109, 2021.
[31] Dev, A.N., Deka, M.K., Sarma, J., Adhikary, N.C., *Journal of the Korean Physical Society*, **67**, 339, 2015.
[32] Deka, M. K., Dev, A.N., *Plasma Physics Reports*, **44**, 965, 2018.
[33] Duha, S.S., Shikha, B., Mamun, A.A., *Pramana*, **77**(2), 357, 2011.
[34] Sumi, R.A., Tasnim, I., Anowor, M.G.M., Mamun, A.A., *Journal of Plasma Physics*, **85**, 1, 2019.
[35] Parkes, E.J., *Applied Mathematics and Computation*, **217**, 1759, 2010.
[36] Ali, K.K., Hadhoud, A.R., *Results in Physics*, **17**, 103137, 2020.