ORIGINAL PAPER

BINARY πg-LOCALLY CLOSED SETS

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Manuscript received: 13.01.2024; Accepted paper: 16.08.2024;
Published online: 30.09.2024.

Abstract. *This paper aims to introduce some new locally closed sets called binary πg-locally closed sets, $b\pi glc^{\star}$-set, $b\pi glc^{\star\star}$-set and the relations between them are studied in a binary topological space. The concepts of $b\pi g$-submaximal space and their related properties have been introduced. Also a characterization of a $b\pi g$-submaximal space is found,* and *suitable examples are given.*

Keywords: *$b\pi g$-locally closed set; $b\pi glc^{\star}$-set; $b\pi glc^{\star\star}$-set.*

1. INTRODUCTION AND PRELIMINARIES

In 2011, S.Nithyanantha Jothi and P.Thangavelu [1] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$. In this paper we introduce binary πg-locally closed sets in a binary topological space and discuss some of their properties.

Let X and Y be any two nonempty sets. A binary topology [1] from X to Y is a binary structure $\mathcal{M} \subseteq \mathbb{P}(X) \times \mathbb{P}(Y)$ that satisfies the axioms namely

1.(ϕ, ϕ) and $(X, Y) \in \mathcal{M}$,

2.$(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$, and

3.If $\{(A_\alpha, B_\alpha): \alpha \in \delta\}$ is a family of members of $\mathcal{M}$, then $(\bigcup_{\alpha\in\delta} A_\alpha, \bigcup_{\alpha\in\delta} B_\alpha) \in \mathcal{M}$.

If $\mathcal{M}$ is a binary topology from X to Y then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binarytopological space $(X, Y, \mathcal{M})$. If $Y = X$ then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary topological space.

Definition 1.1.[1] Let X and Y be any two nonempty sets and let (A, B) and $(\mathrm{C}, \mathrm{D}) \in \mathbb{P}(\mathrm{X}) \times \mathbb{P}(\mathrm{Y})$. We say that $(\mathrm{A}, \mathrm{B}) \subseteq (\mathrm{C}, \mathrm{D})$ if $\mathrm{A} \subseteq \mathrm{C}$ and $\mathrm{B} \subseteq \mathrm{D}$.

Definition 1.2.[1] Let $(\mathrm{X}, \mathrm{Y}, \mathcal{M})$ be a binary topological space and $\mathrm{A} \subseteq \mathrm{X}$, $\mathrm{B} \subseteq \mathrm{Y}$. Then (A, B) is called binary closed in $(\mathrm{X}, \mathrm{Y}, \mathcal{M})$ if $(\mathrm{X} \backslash \mathrm{A}, \mathrm{Y} \backslash \mathrm{B}) \in \mathcal{M}$.

Proposition 1.3.[1] Let $(\mathrm{X}, \mathrm{Y}, \mathcal{M})$ be a binary topological space and $(\mathrm{A}, \mathrm{B}) \subseteq (\mathrm{X}, \mathrm{Y})$.

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https://doi.org/10.46939/J.Sci.Arts-24.3-a09

Let $(A,B)^{1*} = \cap\{A_\alpha: (A_\alpha, B_\alpha)$ is binary closed and $(A,B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A,B)^{2*} = \cap\{B_\alpha: (A_\alpha, B_\alpha)$ is binary closed and $(A,B) \subseteq (A_\alpha, B_\alpha)\}$. Then $((A,B)^{1*}, (A,B)^{2*})$ is binary closed and $(A,B) \subseteq ((A,B)^{1*}, (A,B)^{2*})$.

Proposition 1.4.[1] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1*} = \cup\{A_\alpha: (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $(A, B)^{2*} = \cup\{B_\alpha: (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.

Definition 1.5.[1] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of (A, B), denoted by b-cl(A, B) in the binary space $(X, Y, \mathcal{M})$ where $(A, B) \subseteq (X, Y)$.

Definition 1.6.[1] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ defined in proposition 1.4 is called the binary interior of of (A, B), denoted by b-int(A, B). Here $((A, B)^{1*}, (A, B)^{2*})$ is binary open and $((A, B)^{1*}, (A, B)^{2*}) \subseteq (A, B)$.

Definition 1.7.[1] Let $(X, Y, \mathcal{M})$ be a binary topological space and let $(x, y) \subseteq (X, Y)$. The binary open set (A, B) is said to be a binary neighborhood of (x, y) if $x \in A$ and $y \in B$.

Proposition 1.8.[1] Let $(A, B) \subseteq (C, D) \subseteq (X, Y)$ and $(X, Y, \mathcal{M})$ be a binary topological space.

Then, the following statements hold:
1.$b\text{-}int(A,B) \subseteq (A,B)$.
2.If (A,B) is binary open, then $b\text{-}int(A,B) = (A,B)$.
3.$b\text{-}int(A,B) \subseteq b\text{-}int(C,D)$.
4.$b\text{-}int(b\text{-}int(A,B)) = b\text{-}int(A,B)$.
5.$(A,B) \subseteq b\text{-}cl(A,B)$.
6.If (A,B) is binary closed, then $b\text{-}cl(A,B) = (A,B)$.
7.$b\text{-}cl(A,B) \subseteq b\text{-}cl(C,D)$.
8.$b\text{-}cl(b\text{-}cl(A,B)) = b\text{-}cl(A,B)$.

Definition 1.9.A subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$ is called
1.a binary semi-open set [2] if $(A, B) \subseteq$ b-cl(b-int(A, B)).
2.a binary regular open set [3] if $(A, B) =$ b-int(b-cl(A, B)).
3.a binary π-open [4] if the finite union of binary regular-open sets.

Definition 1.10.A subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$ is called
1.a binary g-closed set [5] if $b\text{-}cl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary open.
2.a binary πg-closed [4] if $b\text{-}cl(A,B) \subseteq (U,V)$, whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary π-open.

Definition 1.11.[6] A subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$ is called
1.binary locally closed if $(A,B) = (E,F) \cap (G,H)$ where (E,F) is binary open and (G,H) is binary closed in (X,Y).
2.binary generalized locally closed (briefly $bglc$) if $(A,B) = (E,F) \cap (G,H)$ where (E,F) is binary g-open and (G,H) is binary g-closed in (X,Y).

2. BINARY πg-LOCALLY CLOSED SETS

Definition 2.1. A subset (A, B) of $(X, Y, \mathcal{M})$ is said to be binary πg-locally closed (bπg-lc) if $(A, B) = (G, H) \cap (U, V)$ where (G, H) is bπg-open and (U, V) is bπg-closed in $(X, Y, \mathcal{M})$.

Definition 2.2. A subset (A, B) of $(X, Y, \mathcal{M})$ is called bπg-lc* if there exists a bπg-open set (G, H) and a binary closed set (U, V) of $(X, Y, \mathcal{M})$ such that $(A, B) = (G, H) \cap (U, V)$.

Definition 2.3. A subset (A, B) of $(X, Y, \mathcal{M})$ is called bπg-lc** if there exists an binary open set (G, H) and a bπg-closed set (U, V) of $(X, Y, \mathcal{M})$ such that $(A, B) = (G, H) \cap (U, V)$.

The collection of all $b\pi g$-locally closed (resp. $b\pi g$-$lc^\star$, $b\pi g$-$lc^{\star\star}$) sets of a space $(X, Y, \mathcal{M})$ will be denoted by $B\pi GLC(X, Y)$ (resp. $B\pi GLC^\star(X, Y)$, $B\pi GLC^{\star\star}(X, Y)$).From the above definitions we have the following results.

Remark 2.4.
1.Every binary locally closed set is $b\pi g$-lc.
2.Every $b\pi g$-$lc^\star$-set is $b\pi g$-lc.
3.Every binary locally closed set is $b\pi g$-$lc^\star$ and $b\pi g$-$lc^{\star\star}$.

However the converses of the above are not true may be seen by the following Examples.

Example 2.5. Let $X = \{1,2\}, Y = \{a, b\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{b\}), (\{1\}, \{a\}), (\{1\}, Y), (X, Y)\}$.
Then $BLC(X, Y) = \{(\phi, \phi), (\phi, \{b\}), (\{1\}, \{a\}), (\{1\}, Y), (\{2\}, \phi), (\{2\}, \{b\}), (X, \{a\}), (X, Y)\}$.
$B\pi GLC(X, Y) = \mathbb{P}(X) \times \mathbb{P}(Y)$. $B\pi GLC^\star(X, Y) =$
$\{(\phi, \phi), (\phi, \{a\}), (\phi, \{b\}), (\phi, Y), (\{1\}, \phi), (\{1\}, \{a\}), (\{1\}, \{b\}), (\{1\}, Y), (\{2\}, \phi), (\{2\}, \{b\}), (X, \{a\}),$
$(X, Y)\}$. $B\pi GLC^{\star\star}(X, Y) = \mathbb{P}(X) \times \mathbb{P}(Y)$.

Theorem 2.6. For a subset (A, B) of $(X, Y, \mathcal{M})$ the following are equivalent:
1.$(A, B) \in B\pi GLC^\star(X, Y)$.
2.$(A, B) = (J, K) \cap b\text{-}cl(A, B)$ for some $b\pi g$-open set (J, K).
3.$b\text{-}cl(A, B) - (A, B)$ is $b\pi g$-closed.
4.$(A, B) \cup ((X, Y) - b\text{-}cl(A, B))$ is $b\pi g$-open.

Proof: (1) ⇒ (2): Let $(A, B) \in B\pi GLC^\star(X, Y)$. Then there exists a $b\pi g$-open set (J, K) and a binary closed set (U, V) such that $(A, B) = (J, K) \cap (U, V)$. Since $(A, B) \subseteq (J, K)$ and $(A, B) \subseteq b\text{-}cl(A, B)$ we have $(A, B) \subseteq (J, K) \cap b\text{-}cl(A, B)$. Conversely, since $b\text{-}cl(A, B) \subseteq (U, V)$, $(J, K) \cap b\text{-}cl(A, B) \subseteq (J, K) \cap (U, V) = (A, B)$ which implies that $(A, B) = (J, K) \cap b\text{-}cl(A, B)$.

(2) ⇒ (1): Since (J, K) is $b\pi g$-open and $b\text{-}cl(A, B)$ is binary closed $(J, K) \cap b\text{-}cl(A, B) \in B\pi GLC^\star(X, Y)$.

(3) ⇒ (4): Let $(U, V) = b\text{-}cl(A, B) - (A, B)$. Then (U, V) is $b\pi g$-closed by the assumption and $(X, Y) - (U, V) = (X, Y) \cap (b\text{-}cl(A, B) - (A, B))^c = (A, B) \cup ((X, Y) - b\text{-}cl(A, B))$. But $(X, Y) - (U, V)$ is $b\pi g$-open. This shows that $(A, B) \cup ((X, Y) - b\text{-}cl(A, B))$ is $b\pi g$-open.

(4) ⇒ (3): Let $(E, F) = (A, B) \cup ((X, Y) - b\text{-}cl(A, B))$. Then (E, F) is $b\pi g$-open. This implies that $(X, Y) - (E, F)$ is $b\pi g$-closed and $(X, Y) - (E, F) = (X, Y) - ((A, B) \cup ((X, Y) - b\text{-}cl(A, B))) = b\text{-}cl(A, B) \cap ((X, Y) - (A, B)) = b\text{-}cl(A, B) - (A, B)$. Thus $b\text{-}cl(A, B) - (A, B)$ is $b\pi g$-closed.

(4) ⇒ (2): Let $(E,F) = (A,B) \cup ((X,Y) - b\text{-}cl(A,B))$. Then (E,F) is $b\pi g$-open. Hence we prove that $(A,B) = (E,F) \cap b\text{-}cl(A,B)$ for some $b\pi g$-open set (E,F). $(E,F) \cap b\text{-}cl(A,B) = ((A,B) \cup ((X,Y) - b\text{-}cl(A,B))) \cap b\text{-}cl(A,B) = (b\text{-}cl(A,B) \cap (A,B)) \cup (b\text{-}cl(A,B) \cap (X,Y) - b\text{-}cl(A,B)) = (A,B) \cup (\phi,\phi) = (A,B)$. Therefore $(A,B) = (E,F) \cap b\text{-}cl(A,B)$.

(2) ⇒ (4): Let $(A,B) = (J,K) \cap b\text{-}cl(A,B)$ for some $b\pi g$-open set (J,K). Then we prove that $(A,B) \cup ((X,Y) - b\text{-}cl(A,B))$ is $b\pi g$-open. $(A,B) \cup ((X,Y) - b\text{-}cl(A,B)) = ((J,K) \cap b\text{-}cl(A,B)) \cup ((X,Y) - b\text{-}cl(A,B)) = (J,K) \cap (b\text{-}cl(A,B) \cup (X,Y) - b\text{-}cl(A,B)) = (J,K) \cap (X,Y) = (J,K)$ which is $b\pi g$-open. Thus $(A,B) \cup ((X,Y) - b\text{-}cl(A,B))$ is $b\pi g$-open.

Definition 2.7. A binary topological space $(X, Y, \mathcal{M})$ is called bπg-submaximal if every binary dense subset is bπg-open.

Theorem 2.8. A binary topological space $(X, Y, \mathcal{M})$ is bπg-submaximal if and only if $\mathbb{P}(X) \times \mathbb{P}(Y) = B\pi GLC^{\star}(X, Y)$.

Proof: Necessity: Let $(A,B) \in \mathbb{P}(X) \times \mathbb{P}(Y)$ and let $(E,F) = (A,B) \cup ((X,Y) - b\text{-}cl(A,B))$. Then (E,F) is $b\pi g$-open and $b\text{-}cl(E,F) = b\text{-}cl(A,B) \cup ((X,Y) - b\text{-}cl(A,B)) = (X,Y)$. This implies that (E,F) is a binary dense subset of (X,Y). By the above Theorem $(A,B) \in B\pi GLC^{\star}(X,Y)$. Therefore, $\mathbb{P}(X) \times \mathbb{P}(Y) = B\pi GLC^{\star}(X,Y)$.

Sufficiency: Let (A,B) be a binary dense subset of $(X,Y,\mathcal{M})$. Then $(A,B) \cup ((X,Y) - b\text{-}cl(A,B)) = (A,B) \Rightarrow (A,B) \in B\pi GLC^{\star}(X,Y)$ and (A,B) is $b\pi g$-open. This proves that (X,Y) is $b\pi g$-submaximal.

Remark 2.9. It follows from definitions that if $(X, Y, \mathcal{M})$ is bg-submaximal, then it is bπg-submaximal. But the converse is not true as seen by the following Example.

Example 2.10 Let $X = \{a, b\}$, $Y = \{1,2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{1\}), (\{b\}, \{1\}), (X, \{1\}), (X, Y)\}$. Then the binary dense sets are $(\phi, \{1\}), (\phi, Y), (\{a\}, \{1\}), (\{a\}, Y), (\{b\}, \{1\}), (\{b\}, Y), (X, \{1\}), (X, Y)$, bg-open sets are $(\phi, \phi), (\phi, \{1\}), (\{a\}, \phi), (\{a\}, \{1\}), (\{b\}, \phi), (\{b\}, \{1\}), (X, \phi), (X, \{1\}), (X, Y)$ and bπg-open sets are $\mathbb{P}(X) \times \mathbb{P}(Y)$. Then it is bπg-submaximal but not bg-submaximal.

Theorem 2.11. For a subset (A, B) of $(X, Y, \mathcal{M})$ if $(A, B) \in B\pi GLC^{\star\star}(X, Y)$ then there exists an binary open set (S, T) such that $(A, B) = (S, T) \cap$ bπg-cl(A, B) where bπg-cl(A, B) is the bπg-closure of (A, B).

Proof: Let $(A, B) \in B\pi GLC^{\star\star}(X, Y)$. Then there exists an binary open set (S, T) and a bπg-closed set (G, H) such that $(A, B) = (S, T) \cap (G, H)$. Since $(A, B) \subseteq (S, T)$ and $(A, B) \subseteq$ bπg-cl(A, B), we have $(A, B) \subseteq (S, T) \cap$ bπg-cl(A, B).

Conversely since bπg-cl$(A, B) \subseteq (G, H)$, we have $(S, T) \cap$ bπg-cl$(A, B) \subseteq (S, T) \cap (G, H) = (A, B)$. Thus $(A, B) = (S, T) \cap$ bπg-cl(A, B).

Theorem 2.12. Let (A, B) and (C, D) be subsets of $(X, Y, \mathcal{M})$. If $(A, B) \in B\pi GLC^{\star}(X, Y)$ and $(C, D) \in B\pi GLC^{\star}(X, Y)$ then $(A, B) \cap (C, D) \in B\pi GLC^{\star}(X, Y)$.

Proof: Let (A,B) and $(C,D) \in B\pi GLC^{\star}(X,Y)$. Then there exist $b\pi g$-open sets (S,T) and (U,V) such that $(A,B) = (S,T) \cap b\text{-}cl(A,B)$ and $(C,D) = (U,V) \cap b\text{-}cl(C,D)$.

Therefore $(A,B) \cap (C,D) = (S,T) \cap b\text{-}cl(A,B) \cap (U,V) \cap b\text{-}cl(C,D) = (S,T) \cap (U,V) \cap b\text{-}cl(A,B) \cap b\text{-}cl(C,D)$ where $(S,T) \cap (U,V)$ is $b\pi g$-open and $b\text{-}cl(A,B)$ and $b\text{-}cl(C,D)$ is binary closed. This shows that $(A,B) \cap (C,D) \in B\pi GLC^{\star}(X,Y)$.

Theorem 2.13. If $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star\star}(\mathrm{X},\mathrm{Y})$ and (C,D) is binary open, then $(\mathrm{A},\mathrm{B}) \cap (\mathrm{C},\mathrm{D}) \in \mathrm{B\pi GLC}^{\star\star}(\mathrm{X},\mathrm{Y})$.

*Proof.*Let $(A,B) \in B\pi GLC^{\star\star}(X,Y)$. Then there exists a binary open set (J,K) and a $b\pi g$-closed set (G,H) such that $(A,B) = (J,K) \cap (G,H)$. So $(A,B) \cap (C,D) = (J,K) \cap (G,H) \cap (C,D) = (J,K) \cap (C,D) \cap (G,H)$. This proves that $(A,B) \cap (C,D) \in B\pi GLC^{\star\star}(X,Y)$.

Theorem 2.14.*If $(A,B) \in B\pi GLC(X,Y)$ and (C,D) is $b\pi g$-open, then $(A,B) \cap (C,D) \in B\pi GLC(X,Y)$.*

*Proof:*Let $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}(\mathrm{X},\mathrm{Y})$. Then $(\mathrm{A},\mathrm{B}) = (\mathrm{J},\mathrm{K}) \cap (\mathrm{G},\mathrm{H})$ where (J,K) is bπg-open and (G,H) is bπg-closed. So $(\mathrm{A},\mathrm{B}) \cap (\mathrm{C},\mathrm{D}) = (\mathrm{J},\mathrm{K}) \cap (\mathrm{G},\mathrm{H}) \cap (\mathrm{C},\mathrm{D}) = (\mathrm{J},\mathrm{K}) \cap (\mathrm{C},\mathrm{D}) \cap (\mathrm{G},\mathrm{H})$. This implies that $(\mathrm{A},\mathrm{B}) \cap (\mathrm{C},\mathrm{D}) \in \mathrm{B\pi GLC}(\mathrm{X},\mathrm{Y})$.

Theorem 2.15. If $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{X},\mathrm{Y})$ and (C,D) is bπg-closed bπ-open subset of (X,Y), then $(\mathrm{A},\mathrm{B}) \cap (\mathrm{C},\mathrm{D}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{X},\mathrm{Y})$.

*Proof:*Let $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{X},\mathrm{Y})$. Then $(\mathrm{A},\mathrm{B}) = (\mathrm{J},\mathrm{K}) \cap (\mathrm{G},\mathrm{H})$ where (J,K) is bπg-open and (G,H) is binary closed. $(\mathrm{A},\mathrm{B}) \cap (\mathrm{C},\mathrm{D}) = (\mathrm{J},\mathrm{K}) \cap ((\mathrm{G},\mathrm{H}) \cap (\mathrm{C},\mathrm{D}))$ where (J,K) is bπg-open and $(\mathrm{G},\mathrm{H}) \cap (\mathrm{C},\mathrm{D})$ is binary closed. Hence $(\mathrm{A},\mathrm{B}) \cap (\mathrm{C},\mathrm{D}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{X},\mathrm{Y})$.

Theorem 2.16. Let (A,B) and (U,V) be subsets of $(\mathrm{X},\mathrm{Y},\mathcal{M})$ and let $(\mathrm{A},\mathrm{B}) \subseteq (\mathrm{U},\mathrm{V})$. If (U,V) is bπg-open in $(\mathrm{X},\mathrm{Y},\mathcal{M})$ and $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$, then $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{X},\mathrm{Y},\mathcal{M})$.

*Proof:*Suppose (A,B) is bπglc*-set, then there exists a bπg-open set (J,K) of $(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$ such that $(\mathrm{A},\mathrm{B}) = (\mathrm{J},\mathrm{K}) \cap \mathrm{b\text{-}cl}_{(\mathrm{U},\mathrm{V})}(\mathrm{A},\mathrm{B})$. But $\mathrm{b\text{-}cl}_{(\mathrm{U},\mathrm{V})}(\mathrm{A},\mathrm{B}) = (\mathrm{U},\mathrm{V}) \cap \mathrm{b\text{-}cl}(\mathrm{A},\mathrm{B})$. Therefore, $(\mathrm{A},\mathrm{B}) = (\mathrm{J},\mathrm{K}) \cap (\mathrm{U},\mathrm{V}) \cap \mathrm{b\text{-}cl}(\mathrm{A},\mathrm{B})$ where $(\mathrm{J},\mathrm{K}) \cap (\mathrm{U},\mathrm{V})$ is bπg-open. Thus $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{X},\mathrm{Y},\mathcal{M})$.

Theorem 2.17. If (U,V) is bπg-closed, bπ-open set in $(\mathrm{X},\mathrm{Y},\mathcal{M})$ and $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$ then $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{X},\mathrm{Y})$.

*Proof:*Let $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$. Then $(\mathrm{A},\mathrm{B}) = (\mathrm{J},\mathrm{K}) \cap (\mathrm{G},\mathrm{H})$ where (J,K) is bπg-open and (G,H) is binary closed in $(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$. Since (G,H) is binary closed in $(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$, $(\mathrm{G},\mathrm{H}) = (\mathrm{C},\mathrm{D}) \cap (\mathrm{U},\mathrm{V})$ for some binary closed set (C,D) of $(\mathrm{X},\mathrm{Y},\mathcal{M})$. Therefore $(\mathrm{A},\mathrm{B}) = (\mathrm{J},\mathrm{K}) \cap (\mathrm{C},\mathrm{D}) \cap (\mathrm{U},\mathrm{V})$. Then $(\mathrm{C},\mathrm{D}) \cap (\mathrm{U},\mathrm{V})$ is binary closed. Hence $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}^{\star}(\mathrm{X},\mathrm{Y})$.

Theorem 2.18. If (U,V) is binary closed and binary open in $(\mathrm{X},\mathrm{Y},\mathcal{M})$ and $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$, then $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}(\mathrm{X},\mathrm{Y})$.

*Proof:*Let $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$. Then there exists a bπg-open set (J,K) and a bπg-closed set (G,H) of $(\mathrm{U},\mathrm{V},\mathcal{M}/(\mathrm{U},\mathrm{V}))$ such that $(\mathrm{A},\mathrm{B}) = (\mathrm{J},\mathrm{K}) \cap (\mathrm{G},\mathrm{H})$. Then by the above Theorem $(\mathrm{A},\mathrm{B}) \in \mathrm{B\pi GLC}(\mathrm{X},\mathrm{Y})$.

Theorem 2.19.*If (U,V) is $b\pi g$-closed, bπ-open subset of (X,Y) and $(A,B) \in B\pi GLC^{\star\star}(U,V,\mathcal{M}/(U,V))$, then $(A,B) \in B\pi GLC^{\star\star}(X,Y)$.*

*Proof:*Let $(A, B) \in B\pi GLC^{**}(U, V, \mathcal{M}/(U, V))$. Then $(A, B) = (J, K) \cap (G, H)$ where (J, K) is binary open and (G, H) is bπg-closed in $(U, V, \mathcal{M}/(U, V))$. Since (U, V) is bπg-closed bπ-open subset of $(X, Y, \mathcal{M})$, then (G, H) is bπg-closed in $(X, Y, \mathcal{M})$. Therefore $(A, B) \in B\pi GLC^{**}(X, Y)$.

Theorem 2.20. If (A, B) is bπg-open and (C, D) is binary open, then $(A, B) \cap (C, D)$ is bπg-open

*Proof:*Let (A, B) be bπg-open. Then b-int$(A, B) \supseteq (G, H)$ whenever $(A, B) \supseteq (G, H)$ and (G, H) is bπ-closed set. Suppose $(A, B) \cap (C, D) \supseteq (G, H)$, then we prove that b-int$((A, B) \cap (C, D)) \supseteq (G, H)$. Since (C, D) is binary open, b-int$(C, D) = (C, D) \supseteq (G, H)$. Therefore by assumptions b-int$((A, B) \cap (C, D)) =$ b-int$(A, B) \cap$ b-int$(C, D) \supseteq (G, H)$. This proves that $(A, B) \cap (C, D)$ is bπg-open.

Theorem 2.21. Suppose that the collection of all bπg-open sets of $(X, Y, \mathcal{M})$ is binary closed under finite unions. Let $(A, B) \in B\pi GLC^{\star}(X, Y)$ and $(C, D) \in B\pi GLC^{\star}(X, Y)$. If (A, B) and (C, D) are separated, then $(A, B) \cup (C, D) \in B\pi GLC^{\star}(X, Y)$.

Proof:Let (A, B), $(C, D) \in B\pi GLC^{\star}(X, Y)$. Then there exist bπg-open sets (J, K) and (S, T) of $(X, Y, \mathcal{M})$ such that $(A, B) = (J, K) \cap$ b-cl(A, B) and $(C, D) = (S, T) \cap$ b-cl(C, D). Put $(P, Q) = (J, K) \cap ((X, Y) -$ b-cl$(C, D))$ and $(U, V) = (S, T) \cap ((X, Y) -$ b-cl$(A, B))$. Then (P, Q) and (U, V) are bπg-open sets and $(A, B) = (P, Q) \cap$ b-cl(A, B) and $(C, D) = (U, V) \cap$ b-cl(C, D). Also $(P, Q) \cap$ b-cl$(C, D) = (\phi, \phi)$ and $(U, V) \cap$ b-cl$(A, B) = (\phi, \phi)$. Hence it follows that (P, Q) and (U, V) are bπg-open sets of $(X, Y, \mathcal{M})$. Therefore $(A, B) \cup (C, D) = ((P, Q) \cap$ b-cl$(A, B)) \cup ((U, V) \cap$ b-cl$(C, D)) = (P, Q) \cup (U, V) \cap$ b-cl$(A, B) \cup$ b-cl(C, D). Here $(P, Q) \cup (U, V)$ is bπg-open by assumption. Thus $(A, B) \cup (C, D) \in B\pi GLC^{\star}(X, Y)$.

4. CONCLUSION

The main aim of this paper is to introduce and study the concepts of binary πg-locally closed sets in a binary topological space and discussed some of their properties with suitable examples are given.

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