*ORIGINAL PAPER*

# ON NEW EXACT TRAVELING WAVE SOLUTIONS OF THE HAMILTONIAN AMPLITUDE EQUATION

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*Manuscript received: 09.10.2024; Accepted paper: 12.12.2024;*
*Published online: 30.03.2025.*

***Abstract.*** *Nonlinear differential equations have an important place in mathematical physics. In this paper, the $G'/G^2$ expansion method is used to obtain new exact traveling wave solutions of the Hamiltonian amplitude equation that arise in the analysis of various problems in fluid mechanics and theoretical physics. All calculations in this study are made using the software program and the solutions obtained are substituted in the equations. The solutions obtained have important areas of use in the fields of mathematical physics and engineering.*

***Keywords:*** *Nonlinear differential equation; Hamiltonian amplitude equation; $G'/G^2$ expansion method.*

## 1. INTRODUCTION

One of the most important and productive areas of modern mathematical analysis is the understanding of systems with both linear and nonlinear partial differential equations and its consequences for all relevant scientific research fields. This is the most important reason for the development of nonlinear partial differential equations modeled to better understand various important physical situations.

Due to the nonlinearity of differential equations, it is difficult to determine exact solutions of nonlinear partial differential equations (PDEs). In recent years, with the development of symbolic computing software such as Maple, which allows us to perform complex calculations on a computer, different methods have been proposed, developed and extended to find exact solutions of nonlinear evolution equations.

In the literature, many efficient mathematical methods have been applied to obtain exact solutions for PDEs. Some of these methods are the Jacobi elliptic function method [1, 2], He's the semi-inverse method [3, 4], ansatz approximation method [4], G′/G expansion method [5, 6], 1/G expansion method [6], $G'/G^2$ expansion method [7-9], tanh-expansion method [10], homogeneous balance method [11], Sardar sub-equation method [12], $\exp(-\Phi(\varepsilon))$ method [9, 13], F expansion method [14], extended and improved G′/G method [15-18].

The Hamiltonian Amplitude Equation has an important place in physics [19,20]. It is a special case of the Schrödinger equation, one of the most important physical models describing the dynamics of soliton propagation in optical fiber theory [4,20,21]. This equation was derived from Hamiltonian mechanics and played an important role in the development of statistical mechanics and quantum mechanics. Hamiltonian mechanics can be used to study simple phenomena such as a bouncing ball or a pendulum. Apart from that, Hamiltonian

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mechanics is mainly used for more complex dynamical systems. Examples include the orbital motion of planets.

This study is analyzed in five chapters. In Section 1, partial differential equations and solution methods are mentioned in general terms and the space-time nonlinear Hamiltonian Amplitude equation is briefly explained. In Section 2, the solution of the Hamiltonian Amplitude equation G′/G$^2$ expansion method is described. In Section 3, the G′/G$^2$ expansion method is applied to the equation and new exact solutions are obtained. Section 4 presents results and discussion. The final results are given in Section 5.

## 2. DESCRIPTION OF THE G′/G$^2$ EXPANSION METHOD

Suppose we have a nonlinear partial differential equation (PDE) $u(x,t)$ in the form:

$$P(u,\frac{\partial u}{\partial x},\frac{\partial u}{\partial t},\frac{\partial^2 u}{\partial x^2},\frac{\partial^2 u}{\partial t^2},...)=0 \tag{1}$$

where *P* is a polynomial in its arguments. This section describes the general steps of the G′/G$^2$ expansion method.

**Step 1.** First the equation given by (1) is transformed into an ordinary differential equation with the traveling wave equation given as follows:

$$\begin{aligned} u(x,t) &= U(\epsilon) \\ \epsilon &= nx+lt, \end{aligned} \tag{2}$$

*n* and *l* are arbitrary constants. If the traveling wave equation (2) is applied to the equation (1), the ordinary differential equation of the following form is obtained.

$$F(U,\frac{dU}{d\epsilon},\frac{d^2U}{d\epsilon^2},\frac{d^3U}{d\epsilon^3},...)=0. \tag{3}$$

**Step 2.** If possible, integrate equation (3) term by term one or more times. The integral constant(s) can be set to zero

**Step 3.** Suppose that the solution to equation (3) is as follows:

$$U(\epsilon)=a_0+\sum_{i=1}^{N}\left[a_i\left(\frac{G'}{G^2}\right)^i+b_i\left(\frac{G'}{G^2}\right)^{-i}\right] \tag{4}$$

The $a_i$ and $b_i$ are the real coefficients to be calculated later. Let us assume that the function $G(\epsilon)$ satisfies the following second-order differential equation.

$$\left(\frac{G'}{G^2}\right)'=\mu+\lambda\left(\frac{G'}{G^2}\right)^2. \tag{5}$$

$\mu$ and $\lambda$ are the real coefficients to be calculated.

**Step 4.** The integer $N$ in equation (4) is found using the balancing method. It is achieved by balancing the highest-order linear term with the highest-order nonlinear term.

$$deg\left(\frac{d^qU}{d\epsilon^q}\right)=N+q$$
$$deg\left[U^r\left(\frac{d^qU}{d\epsilon^q}\right)^s\right]=Nr+s(q+N) \tag{6}$$

**Step 5.** The system of equations is obtained by using equation (5), substituting (4) into (3) and setting all powers of $G'/G^2$ equal to zero. With the help of Maple, Mathematica and other similar software, the system of equations is solved to determine the unknown coefficients $a_0,\ a_1$ and $b_1$.

**Step 6.** The equation (5) has three solutions which are trigonometric, hyperbolic, and rational as follows:

**Case 1.** If $\lambda\mu>0,$ trigonometric solution is:

$$\frac{G'}{G^2}=\sqrt{\frac{\mu}{\lambda}}\left(\frac{Acos(\sqrt{\mu\lambda}\epsilon)+Bsin(\sqrt{\mu\lambda}\epsilon)}{Bcos(\sqrt{\mu\lambda}\epsilon)-Asin(\sqrt{\mu\lambda}\epsilon)}\right), \tag{7}$$

**Case 2.** If $\lambda\mu<0,$ the hyperbolic solution is:

$$\frac{G'}{G^2}=-\frac{\sqrt{|\mu\lambda|}}{\lambda}\left(\frac{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)+B}{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)-B}\right), \tag{8}$$

**Case 3.** If $\lambda\neq 0,\ \mu=0$, rational solution is:

$$\frac{G'}{G^2}=-\frac{A}{\lambda(A\epsilon+B)}, \tag{9}$$

where $A$ and $B$ are arbitrary constants.

## 3. SOLUTIONS OF THE HAMILTONIAN AMPLITUDE EQUATION WITH $G'/G^2$ EXPANSION METHOD

In this section, the $G'/G^2$ expansion method described in the other section will be used to construct a solution for the Hamiltonian Amplitude equation. The Hamiltonian Amplitude Equation is given below [4]:

$$i\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial t^2}+2\eta|u|^2u-\alpha\frac{\partial^2 u}{\partial x\partial t}=0 \tag{10}$$

$\alpha<1,\ \eta\neq\pm1$ and $\alpha,\eta\in\mathbb{R}.$

Complex function in the equation $u(x,t) = U(\xi)e^{i\rho}$ the traveling wave equation that will be used to convert to an ordinary differential equation:

$$\begin{aligned} \xi &= mx - nt \\ \rho &= -kx + lt. \end{aligned} \tag{11}$$

The equation is converted to the ordinary differential equation (ODE) with the traveling wave equation and separated into imaginary and real:

$$\begin{aligned} &(\alpha mn + n^2)U'' + 2\eta U^3 - (\alpha kl + l^2 - k)U = 0, \\ &i(m - 2nl - \alpha ml + \alpha nk)U' = 0. \end{aligned} \tag{12}$$

When the imaginary part is regularized, the following equation is obtained:

$$n = \frac{1-\alpha l}{\alpha k + 2l} m. \tag{13}$$

The balancing method is applied to the real part of the equation (12). When the highest order linear term $U''$ and the highest order nonlinear term $U^3$ are balanced, $N = 1$ is found. Substituting in (4);

$$U(\xi) = a_0 + a_1\left(\frac{G'}{G^2}\right) + b_1\left(\frac{G'}{G^2}\right)^{-1} \tag{14}$$

The system of algebraic equations is constructed by substituting the equation (5) into (14) in the real part of (12) and all powers of G′/G$^2$ are equal to zero. The occurring algebraic system is solved with the aid of Maple to find the values of unknown constants $a_0$, $a_1$ and $b_1$.

$$\begin{aligned} &\left(\frac{G'}{G^2}\right)^{-3} : 2(\alpha mn + n^2)b_1\mu^2 + 2\eta b_1^3 = 0, \\ &\left(\frac{G'}{G^2}\right)^{-2} : 6\eta a_0 b_1^2 = 0, \\ &\left(\frac{G'}{G^2}\right)^{-1} : 2(\alpha mn + n^2)b_1\mu\lambda + 2\eta(3a_0^2 b_1 + 3a_1 b_1^2) - (\alpha kl + l^2 - k)b_1 = 0, \\ &\left(\frac{G'}{G^2}\right)^{0} : 2\eta(6a_0 a_1 b_1 + a_0^3) - (\alpha kl + l^2 - k)a_0 = 0, \\ &\left(\frac{G'}{G^2}\right)^{1} : 2(\alpha mn + n^2)a_1\mu\lambda + 2\eta(3a_1^2 b_1 + 3a_0 a_1^2) - (\alpha kl + l^2 - k)a_1 = 0, \\ &\left(\frac{G'}{G^2}\right)^{2} : 6\eta a_0 a_1 = 0, \\ &\left(\frac{G'}{G^2}\right)^{3} : 2(\alpha mn + n^2)a_1\lambda^2 + 2\eta a_1^3 = 0. \end{aligned} \tag{15}$$

Solving the system of algebraic equations (15), we obtain the following four results:

**Result 1.**

$$m=\frac{(-2\lambda\mu n^2+\alpha kl+l^2-k)}{2\alpha\lambda\mu n},\ a_0=0,$$
$$a_1=0,\ b_1=\pm\sqrt{-\frac{(\alpha\mu kl+\mu l^2-\mu k)}{2\lambda\eta}},\tag{16}$$
$$\eta<0,\ l=\frac{1}{\alpha},k=-\frac{2}{\alpha^2}.$$

**Result 2.**

$$m=\frac{(-2\lambda\mu n^2+\alpha kl+l^2-k)}{2\alpha\lambda\mu n},\ a_0=0,$$
$$a_1=\pm\sqrt{-\frac{(\alpha\lambda kl+\lambda l^2-\lambda k)}{2\mu\eta}},\ b_1=0,\tag{17}$$
$$\eta<0,\ l=\frac{1}{\alpha},k=-\frac{2}{\alpha^2}.$$

**Result 3.**

$$m=-\frac{(4\lambda\mu n^2+\alpha kl+l^2-k)}{4\lambda\mu\alpha n},\ a_0=0,$$
$$a_1=\pm\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{4\mu\eta}}\lambda,b_1=\pm\frac{\alpha kl+l^2-k}{2\eta\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{\mu\eta}}},\tag{18}$$
$$\eta<0,l=\frac{1}{\alpha},k=-\frac{2}{\alpha^2}.$$

**Result 4.**

$$m=\frac{(-8\lambda\mu n^2+\alpha kl+l^2-k)}{8\lambda\mu\alpha n},a_0=0,$$
$$a_1=\pm\sqrt{-\frac{(\alpha\lambda kl+\lambda l^2-\lambda k)}{8\mu\eta}},\ b_1=\pm\frac{\alpha kl+l^2-k}{4\eta\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{2\mu\eta}}},\tag{19}$$
$$\eta<0,l=\frac{1}{\alpha},k=-\frac{2}{\alpha^2}.$$

Substituting the obtained results in (7), (8), and (9), we get the traveling wave solutions of the Hamiltonian Amplitude equation.

**Solution 1.** Using (16), if $\lambda\mu>0,$ the trigonometric solution is:

$$U_1(\xi)=\left(\pm\sqrt{-\frac{(\alpha\mu kl+\mu l^2-\mu k)}{2\lambda\eta}}\left[\frac{Acos(\sqrt{\mu\lambda}\xi)+Bsin(\sqrt{\mu\lambda}\xi)}{Bcos(\sqrt{\mu\lambda}\xi)-Asin(\sqrt{\mu\lambda}\xi)}\right]^{-1}\right)e^{i\rho}.\tag{20}$$

If $\lambda\mu<0,$ the hyperbolic solution is:

$$U_2(\xi)=\left(\pm\sqrt{-\frac{(\alpha\mu kl+\mu l^2-\mu k)}{2\lambda\eta}}\left[-\frac{\sqrt{|\mu\lambda|}}{\lambda}\left(\frac{Asinh(2\sqrt{|\mu\lambda|}\xi)+Acosh(2\sqrt{|\mu\lambda|}\xi)+B}{Asinh(2\sqrt{|\mu\lambda|}\xi)+Acosh(2\sqrt{|\mu\lambda|}\xi)-B}\right)\right]^{-1}\right)e^{i\rho}. \quad (21)$$

If $\lambda\neq 0,\ \mu=0$, rational solution is:

$$U_3(\xi)=\left(\pm\sqrt{-\frac{\alpha\mu kl+\mu l^2-\mu k}{2\lambda\eta}}\left[-\frac{A}{\lambda(A\xi+B)}\right]^{-1}\right)e^{i\rho}. \quad (22)$$

In (20), (21) and (22), $\xi$ and $\rho$ as follows,

$$\xi=\left(\frac{-2\lambda\mu n^2+\alpha kl+l^2-k}{2\alpha\lambda\mu n}\right)x-nt,$$
$$\rho=\left(\frac{2}{\alpha^2}\right)x+\left(\frac{1}{\alpha}\right)t.$$

**Solution 2.** Based on (17), if $\lambda\mu>0$, the trigonometric solution is:

$$U_4(\xi)=\left(\pm\sqrt{-\frac{(\alpha\lambda kl+\lambda l^2-\lambda k)}{2\mu\eta}}\left[\sqrt{\frac{\mu}{\lambda}}\left(\frac{Acos(\sqrt{\mu\lambda}\xi)+Bsin(\sqrt{\mu\lambda}\xi)}{Bcos(\sqrt{\mu\lambda}\xi)-Asin(\sqrt{\mu\lambda}\xi)}\right)\right]\right)e^{i\rho}. \quad (23)$$

If $\lambda\mu<0$, the hyperbolic solution is:

$$U_5(\xi)=\left(\pm\sqrt{-\frac{(\alpha\lambda kl+\lambda l^2-\lambda k)}{2\mu\eta}}\left[-\frac{\sqrt{|\mu\lambda|}}{\lambda}\left(\frac{Asinh(2\sqrt{|\mu\lambda|}\xi)+Acosh(2\sqrt{|\mu\lambda|}\xi)+B}{Asinh(2\sqrt{|\mu\lambda|}\xi)+Acosh(2\sqrt{|\mu\lambda|}\xi)-B}\right)\right]\right)e^{i\rho}. \quad (24)$$

If $\lambda\neq 0,\ \mu=0$, rational solution is:

$$U_6(\xi)=\left(\pm\sqrt{-\frac{\alpha\lambda kl+\lambda l^2-\lambda k}{2\mu\eta}}\left[-\frac{A}{\lambda(A\xi+B)}\right]\right)e^{i\rho}. \quad (25)$$

In (23), (24) and (25), $\xi$ and $\rho$ as follows,

$$\xi=\left(\frac{-2\lambda\mu n^2+\alpha kl+l^2-k}{2\alpha\lambda\mu n}\right)x-nt,$$
$$\rho=\left(\frac{2}{\alpha^2}\right)x+\left(\frac{1}{\alpha}\right)t.$$

**Solution 3.** Considering (18), if $\lambda\mu>0$, the trigonometric solution is:

$$U_7=\Big(\pm\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{4\mu\eta}}\lambda\Big(\sqrt{\frac{\mu}{\lambda}}\big(\frac{Acos(\sqrt{\mu\lambda}\xi)+Bsin(\sqrt{\mu\lambda}\xi)}{Bcos(\sqrt{\mu\lambda}\xi)-Asin(\sqrt{\mu\lambda}\xi)}\Big)\Big) \quad (26)$$

$$\pm\frac{\alpha kl+l^2-k}{2\eta\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{\mu\eta}}}\Big(\sqrt{\frac{\mu}{\lambda}}\Big(\frac{Acos(\sqrt{\mu\lambda}\epsilon)+Bsin(\sqrt{\mu\lambda}\epsilon)}{Bcos(\sqrt{\mu\lambda}\epsilon)-Asin(\sqrt{\mu\lambda}\epsilon)}\Big)\Big)^{-1}\Big)e^{i\rho}.$$

If $\lambda\mu<0,$ the hyperbolic solution is:

$$U_8=\Big(\pm\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{4\mu\eta}}\lambda\Big(-\frac{\sqrt{|\mu\lambda|}}{\lambda}\Big(\frac{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)+B}{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)-B}\Big)\Big)$$
$$\pm\frac{\alpha kl+l^2-k}{2\eta\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{\mu\eta}}}\Big(-\frac{\sqrt{|\mu\lambda|}}{\lambda}\Big(\frac{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)+B}{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)-B}\Big)\Big)^{-1}\Big)e^{i\rho}. \quad (27)$$

If $\lambda\neq 0,\ \mu=0$, rational solution is:

$$U_9=\Big(\pm\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{4\mu\eta}}\lambda\Big(-\frac{A}{\lambda(A\xi+B)}\Big)$$
$$\pm\frac{\alpha kl+l^2-k}{2\eta\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{\mu\eta}}}\Big(-\frac{A}{\lambda(A\xi+B)}\Big)^{-1}\Big)e^{i\rho}. \quad (28)$$

In (26), (27) and (28), $\xi$ and $\rho$ as follows,

$$\xi=\Big(-\frac{(4\lambda\mu n^2+\alpha kl+l^2-k)}{4\lambda\mu\alpha n}\Big)x-nt,$$
$$\rho=\Big(\frac{2}{\alpha^2}\Big)x+\Big(\frac{1}{\alpha}\Big)t.$$

**Solution 4.** Based on (19), if $\lambda\mu>0,$ the trigonometric solution is:

$$U_{10}=\Big(\pm\sqrt{-\frac{(\alpha\lambda kl+\lambda l^2-\lambda k)}{8\mu\eta}}\Big(\sqrt{\frac{\mu}{\lambda}}\Big(\frac{Acos(\sqrt{\mu\lambda}\xi)+Bsin(\sqrt{\mu\lambda}\xi)}{Bcos(\sqrt{\mu\lambda}\xi)-Asin(\sqrt{\mu\lambda}\xi)}\Big)\Big)$$
$$\pm\frac{\alpha kl+l^2-k}{4\eta\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{2\mu\eta}}}\Big(\sqrt{\frac{\mu}{\lambda}}\Big(\frac{Acos(\sqrt{\mu\lambda}\epsilon)+Bsin(\sqrt{\mu\lambda}\epsilon)}{Bcos(\sqrt{\mu\lambda}\epsilon)-Asin(\sqrt{\mu\lambda}\epsilon)}\Big)\Big)^{-1}\Big)e^{i\rho}. \quad (29)$$

If $\lambda\mu<0,$ the hyperbolic solution is:

$$U_{11}=\Big(\pm\sqrt{-\frac{(\alpha\lambda kl+\lambda l^2-\lambda k)}{8\mu\eta}}\lambda\Big(-\frac{\sqrt{|\mu\lambda|}}{\lambda}\Big(\frac{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)+B}{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)-B}\Big)\Big)$$

$$\pm\frac{\alpha kl+l^2-k}{4\eta\sqrt{-\frac{(\lambda k-\lambda l^2-\alpha\lambda kl)}{2\mu\eta}}}\Big(-\frac{\sqrt{|\mu\lambda|}}{\lambda}\Big(\frac{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)+B}{Asinh(2\sqrt{|\mu\lambda|}\epsilon)+Acosh(2\sqrt{|\mu\lambda|}\epsilon)-B}\Big)\Big)^{-1}\Big)e^{i\rho}. \quad (30)$$

If $\lambda \neq 0,\ \mu = 0$, rational solution is:

$$U_{12} = \Big(\pm\sqrt{-\frac{(\alpha\lambda kl + \lambda l^2 - \lambda k)}{8\mu\eta}}\lambda\Big(-\frac{A}{\lambda(A\xi + B)}\Big) \pm \frac{\alpha kl + l^2 - k}{4\eta\sqrt{-\frac{(\lambda k - \lambda l^2 - \alpha\lambda kl)}{2\mu\eta}}}\Big(-\frac{A}{\lambda(A\xi + B)}\Big)^{-1}\Big)e^{i\rho}. \tag{31}$$

In (30), (31) and (32), $\xi$ and $\rho$ as follows,

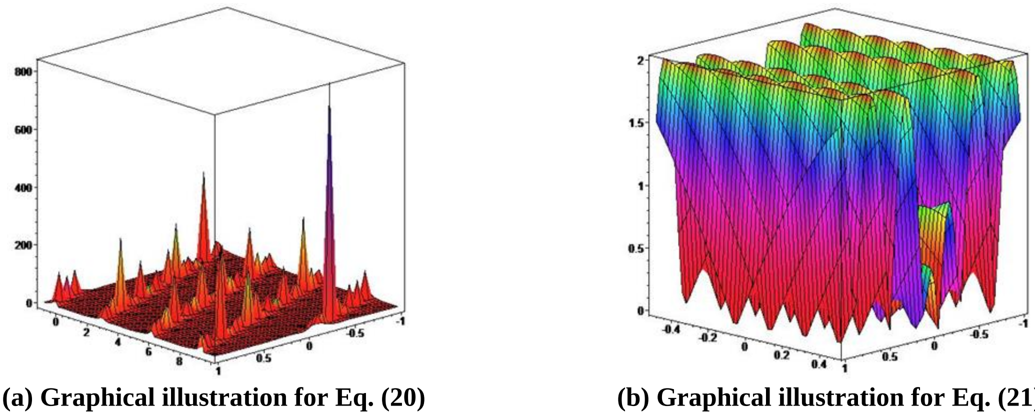
$$\xi = -\frac{(-8\lambda\mu n^2 + \alpha kl + l^2 - k)}{8\lambda\mu\alpha n}x - nt,$$

$$\rho = \Big(\frac{2}{\alpha^2}\Big)x + \Big(\frac{1}{\alpha}\Big)t.$$

## 4. RESULTS AND DISCUSSION

The newly exact solutions to the Hamiltonian amplitude problem are obtained in this study. To prevent singularities and trivial solutions, the solutions are found while adhering to the relevant constraints. Certain solutions have been illustrated graphically when specific values are allocated to the free parameters.

For Solution 1, the trigonometric results for (a) are plotted for $\lambda = \mu = n = A = B = 1,\ \eta = -0.5,\ \alpha = 0.5,\ l = 2,\ k = -8$ and the hyperbolic results for (b) are plotted for $\lambda = -1$, $\mu = n = A = B = 1,\ \alpha = \eta = 0.5,\ l = 2,\ k = -8$ in Fig. 1.

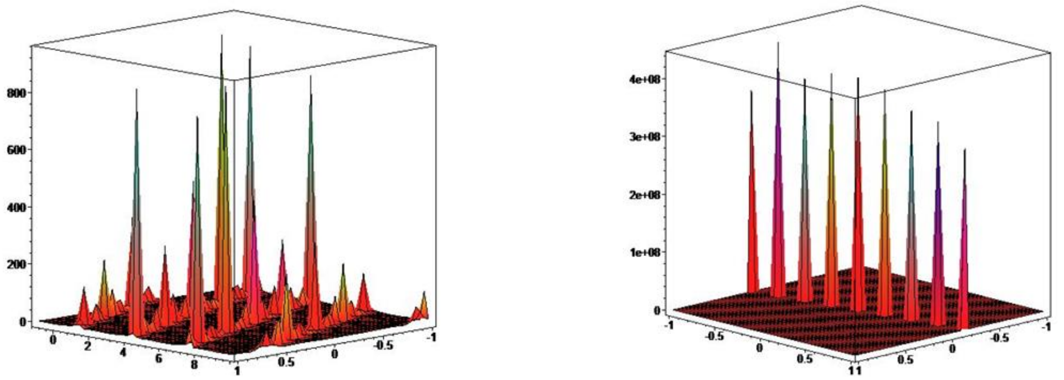


**(a) Graphical illustration for Eq. (20)** **(b) Graphical illustration for Eq. (21)**

**Figure 1. Graphical illustrations of Solution 1.**

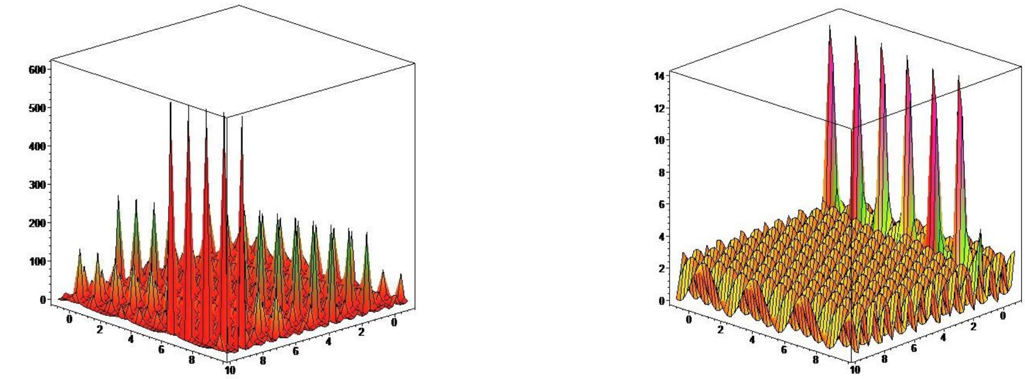
For Solution 2, the trigonometric results for (a) are plotted fo $\lambda = \mu = n = A = B = 1,\ \eta = -0.5,\ \alpha = 0.5,\ l = 2,\ k = -8$ and the hyperbolic results for (b) are plotted for $\lambda = -1$, $\mu = n = A = B = 1,\ \alpha = \eta = 0.5,\ l = 2,\ k = -8$ in Fig. 2.

In Solution 3, the trigonometric results for (a) are plotted for $\lambda = \mu = n = A = B = 1$ $\eta = -0.5,\ \alpha = 0.5,\ l = 2,\ k = -8$ and the hyperbolic results for (b) are plotted for $\lambda = -1,$ $\mu = n = A = B = 1,\ \alpha = \eta = -0.5,\ l = -2,\ k = -8$ in Fig. 3.



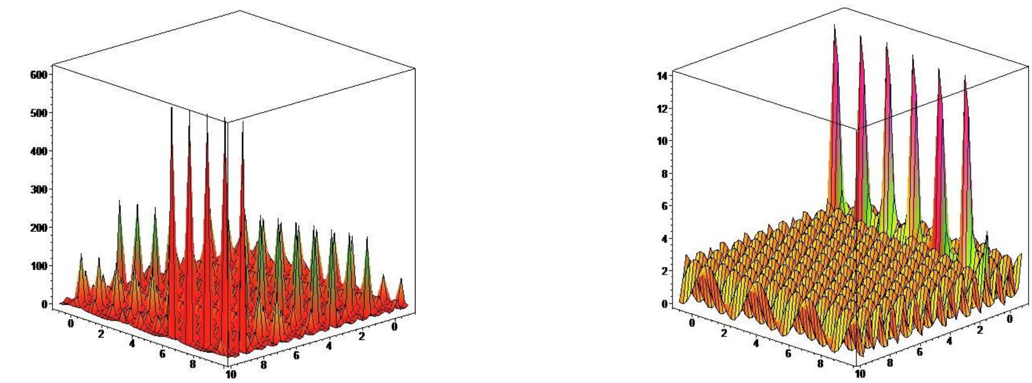
**(a) Graphical illustration for Eq. (23)** **(b) Graphical illustration for Eq. (24)**

**Figure 2. Graphical illustrations of Solution 2.**



**(a) Graphical illustration for Eq. (26)** **(b) Graphical illustration for Eq. (27)**

**Figure 3. Graphical illustrations of Solution 3.**



**(a) Graphical illustration for Eq. (29)** **(b) Graphical illustration for Eq. (30)**

**Figure 4. Graphical illustrations of Solution 4.**

In Solution 4, the trigonometric results for (a) are plotted for $\lambda = \mu = n = A = B = 1$ $\eta = -0.5,\ \alpha = 0.5,\ l = 2,\ k = -8$ and the hyperbolic results for (b) are plotted for $\lambda = -1,$ $\mu = n = A = B = 1,\ \alpha = \eta = 0.5,\ l = 2,\ k = -8$ in Fig. 4.

## 5. CONCLUSIONS

In this paper, using the $G'/G^2$ expansion method to obtain new exact traveling wave solutions of the new Hamiltonian amplitude equation. Various solutions of the equations using the $G'/G^2$ expansion method, including trigonometric, hyperbolic, and rational, have been extensively written. We anticipate that the results can be potentially useful for applications in mathematical physics and engineering. The adequacy of the $G'/G^2$ expansion method is clearly demonstrated by numerical simulation with graphical representation. The results are solved with the help of Maple, a symbolic computing system, to obtain understandable mathematical solutions. The results show that this method is a powerful tool for obtaining the exact solutions of complex nonlinear partial differential equations. It is concluded that the proposed method can be extended to solve nonlinear problems arising in soliton theory and other fields.

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