ORIGINAL PAPER

NANO Λ_g-CLOSED SETS IN AN IDEAL NANOTOPOLOGICAL SPACE

ERULAN VEERAMALAI[1], ILANGOVAN RAJASEKARAN[2],
MALAISAMY NAVANEETHAKRISHNAN[1]

Manuscript received: 09.08.2024; Accepted paper: 25.04.2025;
Published online: 30.06.2025.

Abstract. *The notion of nano Λ_g-closed sets is introduced in an ideal nanotopological space. Characterizations and properties of Λ_g-nI-closed sets and Λ_g-nI-open sets are given. A characterization of normal spaces is given in terms of Λ_g-nI-open sets. Also, it is established that an Λ_g-nI-closed subset of a nano $\mathfrak{J}$-compact space is nano $\mathfrak{J}$-compact compact.*

Keywords: *Λ_g-nI-closed sets; Λ_g-nI-open sets; nano $\mathfrak{J}$-compact.*

1. INTRODUCTION AND PRELIMINARIES

In 2017, Rajasekaran et al. [1] introduced the notion of nano Λ-sets in nanotopological spaces. A nano Λ-set is a set H that is equal to its kernel (= saturated set), i.e., the intersection of all open supersets of H. Further, they have introduced and investigated the notion of nano Λ-closed sets by involving nano Λ-sets and closed sets. Rajasekaran et al [2] have introduced and investigated the notion of Λ_g-closed sets in nanotopological spaces and established several properties of such sets. An ideal I [3] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

1. $A \in I$ and $B \subseteq A$ imply $B \in I$ and
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a topological space (X, τ) with an ideal I on X. If $\wp(X)$ is the family of all subsets of X, a set operator $(.)^\star: \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X: U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau: x \in U\}$ [4]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [5] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space, denoted by (X, τ, I). We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$.

Some new notions in the concept of ideal nano topological spaces were introduced by Parimala et al. [6].

Definition 1.1. [7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the

[1] Kamaraj College, PG and Research Department of Mathematics, 628003 Thoothukudi, Tamil Nadu, India.
E-mails: veeramalaiveera2@gmail.com; navaneethan65@yahoo.co.in.
[2] Tirunelveli Dakshina Mara Nadar Sangam College, PG and Research Department of Mathematics, 627113 Tirunelveli, Tamil Nadu, India. E-mail: sekarmelakkal@gmail.com.

https://doi.org/10.46939/J.Sci.Arts-25.2-a06

same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects that can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U} \{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects that can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U} \{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects that can be classified neither as X nor as not $-X$ with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 1.2. [8] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:
1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nanotopology with respect to X and $(U, \tau_R(X))$ is called the nanotopological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

Throughout the paper, we denote a nanotopological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $n\text{-}int(A)$ and $n\text{-}cl(A)$, respectively.

Definition 1.3. A subset A of a space $(U, \mathcal{N})$ is called
1. nano semi-open [8] if $A \subseteq n\text{-}cl(n\text{-}int(A))$,
2. nano pre-open [8] if $A \subseteq n\text{-}int(n\text{-}cl(A))$.

The complements of the above-mentioned sets are called their respective closed sets.

Definition 1.4. A subset A of a space $(U, \mathcal{N})$ is called a nano generalized closed set (briefly ng-closed) [9] if $n\text{-}cl(A) \subseteq B$, whenever $A \subseteq B$ and B is n-open.The complement of ng-closed is called ng-open.

A space $(U, \mathcal{N})$ with an ideal I on U is called [6] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [6] the family of n-open sets containing x.

Definition 1.5. [6] Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A \subseteq U$, $A^\star_n(I, \mathcal{N}) = \{x \in U: G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A^\star_n$ for $A^\star_n(I, \mathcal{N})$.
Here, an ideal nanotopological space $(U, \mathcal{N}, I)$ is mentioned as a space.

Theorem 1.6. [6] Let $(U, \mathcal{N}, I)$ be a space and A and B be subsets of U. Then
1. $A \subseteq B \Rightarrow A^\star_n \subseteq B^\star_n$,
2. $A^\star_n = n\text{-}cl(A^\star_n) \subseteq n\text{-}cl(A)$($A^\star_n$ is a n-closed subset of $n\text{-}cl(A)$),
3. $(A^\star_n)^\star_n \subseteq A^\star_n$,

4. $(A \cup B)_n^\star = A_n^\star \cup B_n^\star$,
5. $V \in \mathcal{N} \Rightarrow V \cap A_n^\star = V \cap (V \cap A)_n^\star \subseteq (V \cap A)_n^\star$,
6. $J \in I \Rightarrow (A \cup J)_n^\star = A_n^\star = (A - J)_n^\star$.

Theorem 1.7. [6] Let $(U, \mathcal{N}, I)$ be a space with an ideal I and $A \subseteq A_n^\star$, then $A_n^\star = n\text{-}cl(A_n^\star) = n\text{-}cl(A)$.

Definition 1.8. [6] Let $(U, \mathcal{N}, I)$ be a space. The set operator $n\text{-}cl^\star$ called a nano $\star$-closure is defined by $n\text{-}cl^\star(A) = A \cup A_n^\star$ for $A \subseteq U$.

It can be easily observed that $n\text{-}cl^\star(A) \subseteq n\text{-}cl(A)$.

Theorem 1.9. [6] In a space $(U, \mathcal{N}, I)$, if A and B are subsets of U, then the following results are true for the set operator $n\text{-}cl^\star$.
1. $A \subseteq n\text{-}cl^\star(A)$,
2. $n\text{-}cl^\star(\phi) = \phi$ and $n\text{-}cl^\star(U) = U$,
3. If $A \subseteq B$, then $n\text{-}cl^\star(A) \subseteq n\text{-}cl^\star(B)$,
4. $n\text{-}cl^\star(A) \cup n\text{-}cl^\star(B) = n\text{-}cl^\star(A \cup B)$,
5. $n\text{-}cl^\star(n\text{-}cl^\star(A)) = n\text{-}cl^\star(A)$.

Definition 1.10. [6] A subset A of a space $(U, \mathcal{N}, I)$ is said to be nano-I-open (briefly nI-open) if $A \subseteq n\text{-}int(A_n^\star)$.

Definition 1.11. [10] A subset A of a space $(U, \mathcal{N}, I)$ is $n \star$-dense in itself (resp. $n \star$-closed) if $A \subseteq A_n^\star$ (resp. $A_n^\star \subseteq A$).

Definition 1.12. A subset A of a space $(U, \mathcal{N}, I)$ is called a nano I_g-closed (briefly nI_g-closed) [10] if $A_n^\star \subseteq B$ whenever $A \subseteq B$ and B is n-open. The complement of a nI_g-closed set is said to be nI_g-open.

Definition 1.13 An ideal I in a space $(U, \mathcal{N}, I)$ is said to be
1. $\mathcal{N}$-codense ideal [6] if $\mathcal{N} \cap I = \{\phi\}$.
2. completely $\mathcal{N}$-codense ideal [11] if $n\text{-}PO(U) \cap I = \{\phi\}$, where $n\text{-}PO(U)$ is the family of all np-open sets in $(U, \mathcal{N})$.

2. ON Λ_g-CLOSED SETS

Definition 2.1. A subset H of U in an ideal nanotopological space $(U, \mathcal{N}, I)$ is said to be
1. Λ_g-nI-closed if $H_n^\star \subseteq K$ whenever $H \subseteq K$ and K is $n\lambda$-open.
2. Λ_g-nI-open if its complement is Λ_g-nI-closed.

Proposition 2.2. In an ideal nanotopological space $(U, \mathcal{N}, I)$, for a subset H of U the following statements are hold;
1. If Λ_g-nI-closed set then nI_g-closed,
2. If $n\Lambda_g$-closed set ([2], Definition 3.3) then nI_g-closed.

Proof:
1. It follows from the fact that every n-open set is $n\lambda$-open.
2. It follows from the Definitions of $n\Lambda_g$-closed and nI_g-closed.

Remark 2.3. The converse of Proposition 2.2 need not be true, as seen from the following Example.

Example 2.4. Let $U = \{r_1, r_2, r_3\}$ with $U/R = \{\{r_3\}, \{r_1, r_2\}\}$ and $X = \{r_1, r_2\}$. Then $\mathcal{N} = \{\phi, \{r_1, r_2\}, U\}$. Let be an ideal $I = \{\phi\}$. In the ideal nanotopological space $(U, \mathcal{N}, I)$, then the subset $\{r_1, r_3\}$ is nI_g-closed but not Λ_g-nI-closed and $n\Lambda_g$-closed.

The following Theorem gives characterizations of Λ_g-nI-closed sets.

Theorem 2.5. Let H be a Λ_g-nI-closed subset of U in an ideal nanotopological space $(U, \mathcal{N}, I)$. Then the following results are equivalent;
1. H is Λ_g-nI-closed,
2. n-$cl^\star(H) \subseteq O$ whenever $H \subseteq O$ and O is $n\lambda$-open in U,
3. n-$cl^\star(H) - H$ contains no nonempty $n\lambda$-closed set,
4. $H_n^\star - H$ contains no nonempty $n\lambda$-closed set.

Proof: (1) $\Rightarrow$ (2) Let $H \subseteq O$ where O is $n\lambda$-open in U. Since H is Λ_g-nI-closed, $H_n^\star \subseteq O$ and so n-$cl^\star(H) = H \cup H_n^\star \subseteq O$.

(2) $\Rightarrow$ (3) Let K be a $n\lambda$-closed subset such that $K \subseteq n$-$cl^\star(H) - H$. Then $K \subseteq n$-$cl^\star(H)$. Also $K \subseteq cl^\star(H) - H \subseteq U - H$ and hence $H \subseteq U - K$ where $U - K$ is $n\lambda$-open. By (2) n-$cl^\star(H) \subseteq U - K$ and so $K \subseteq U - n$-$cl^\star(H)$. Thus $K \subseteq n$-$cl^\star(H) \cap U - n$-$cl^\star(H) = \phi$.

(3) $\Rightarrow$ (4) $H_n^\star - H = H \cup H_n^\star - H = n$-$cl^\star(H) - H$ which has no nonempty $n\lambda$-closed subset by (3).

(4) $\Rightarrow$ (1) Let $H \subseteq O$ where O is $n\lambda$-open. Then $U - O \subseteq U - H$ and so $H_n^\star \cap (U - O) \subseteq H_n^\star \cap (U - H) = H_n^\star - H$. Since $H_n^\star$ is always a n-closed subset and $U - O$ is $n\lambda$-closed, $H_n^\star \cap (U - O)$ is a $n\lambda$-closed set contained in $H_n^\star - H$ and hence $H_n^\star \cap (U - O) = \phi$. by (4). Thus $H_n^\star \subseteq O$ and H is Λ_g-nI-closed.

Proposition 2.6. For a subset H of U in an ideal nanotopological space $(U, \mathcal{N}, I)$, each $n\star$-closed set is Λ_g-nI-closed.

Proof: Let H be a $n\star$-closed. To prove H is Λ_g-nI-closed, let G be any $n\lambda$-open set such that $H \subseteq G$. Since H is $n\star$-closed, $H_n^\star \subseteq H \subseteq G$. Thus H is Λ_g-nI-closed.

Remark 2.7. The converse of Proposition 2.6 is need not be true as seen from the following Example.

Example 2.8. Let $U = \{r_1, r_2, r_3\}$ with $U/R = \{\{r_1\}, \{r_2, r_3\}\}$ and $X = \{r_1\}$. Then $\mathcal{N} = \{\phi, \{r_1\}, U\}$. Let be an ideal $I = \{\phi\}$. In the ideal nanotopological space $(U, \mathcal{N}, I)$, then the subset $\{r_2\}$ is Λ_g-nI-closed set but not $n\star$-closed.

Theorem 2.9. In an ideal nanotopological space $(U, \mathcal{N}, I)$, for each $H \in I$, H is Λ_g-nI-closed.

Proof: Let $H \in I$ and let $H \subseteq G$ where G is $n\lambda$-open. Since $H \in I, H_n^\star = \phi \subseteq G$. It follows that H is Λ_g-nI-closed.

Theorem 2.10. If $(U, \mathcal{N}, I)$ is an ideal nanotopological space, then $H_n^\star$ is always Λ_g-nI-closed for each subset H of U.

Proof: Let $H_n^\star \subseteq G$ where G is $n\lambda$-open. Since $(H_n^\star)_n^\star \subseteq H_n^\star$, we have $(H_n^\star)_n^\star \subseteq G$. Hence $H_n^\star$ is Λ_g-nI-closed.

Theorem 2.11. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space. If for a subset H of U then each Λ_g-nI-closed, $n\lambda$-open set is $n\star$-closed.

Proof: Let H be Λ_g-nI-closed and $n\lambda$-open. We have $H \subseteq H$ where H is $n\lambda$-open. Since H is Λ_g-nI-closed, $H_n^\star \subseteq H$. This proves that H is $n\star$-closed.

Definition 2.12. An ideal nanotopological space $(U, \mathcal{N}, I)$ is said to be a nano T_I-space if every nI_g-closed subset of U is a $n\star$-closed set.

Lemma 2.13. If $(U, \mathcal{N}, I)$ is a nano T_1-space and H is an nI_g-closed set, then H is a $n\star$-closed set.

Corollary 2.14. If $(U, \mathcal{N}, I)$ is a nano T_I-space and H is a Λ_g-nI-closed set, then H is $n\star$-closed set.

Proof: By assumption H is Λ_g-nI-closed in $(U, \mathcal{N}, I)$ and so by Proposition 2.2, H is nI_g-closed. Since $(U, \mathcal{N}, I)$ is a nano T_I-space, by Definition 2.12, H is $n\star$-closed.

Corollary 2.15. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space and H be a Λ_g-nI-closed set. Then the following results are equivalent;
1. H is $n\star$-closed,
2. n-$cl^\star(H) - H$ is a $n\lambda$-closed set,
3. $H_n^\star - H$ is a $n\lambda$-closed set.

Proof: (1) $\Rightarrow$ (2) By (1) H is $n\star$-closed. Hence $H_n^\star \subseteq H$ and n-$cl^\star(H) - H = (H \cup H_n^\star) - H = \phi$ which implies that $n\lambda$-closed.
(2) $\Rightarrow$ (3) $H_n^\star - H = H \cup H_n^\star - H = n$-$cl^\star(H) - H$ which implies that $n\lambda$-closed by (2).
(3) $\Rightarrow$ (1) Since H is Λ_g-nI-closed, by Theorem 2.5 $H_n^\star - H$ contains no non-empty $n\lambda$-closed set. By assumption (3) $H_n^\star - H$ is $n\lambda$-closed and hence $H_n^\star - H = \phi$. Which proves that $H_n^\star \subseteq H$ and H is $n\star$-closed.

Proposition 2.16. In an ideal nanotopological space $(U, \mathcal{N}, I)$, for a subset H of U, each $n\Lambda_g$-closed set is a Λ_g-nI-closed set.

Proof: Let H be a $n\Lambda_g$-closed set. Let G be any $n\lambda$-open set such that $H \subseteq G$. Since H is $n\Lambda_g$-closed, n-$cl(H) \subseteq G$. So, $H^\star \subseteq n$-$cl(H) \subseteq G$ and hence H is Λ_g-nI-closed.

Remark 2.17. The converse of Proposition 2.16 need not be true, as seen from the following Example.

Example 2.18. Let $U = \{r_1, r_2, r_3\}$ with $U/R = \{\{r_1\}, \{r_2, r_3\}\}$ and $X = \{r_1\}$. Then $\mathcal{N} = \{\phi, \{r_1\}, U\}$. Let be an ideal $I = \{\phi, \{r_1\}, \{r_2\}, \{r_1, r_2\}\}$. In the ideal nanotopological space $(U, \mathcal{N}, I)$, then the subset $\{r_1\}$ is a Λ_g-nI-closed set but not $n\Lambda_g$-closed.

Theorem 2.19. If $(U, \mathcal{N}, I)$ is an ideal nanotopological space and H is a $n\star$-dense in itself, Λ_g-nI-closed subset of U, then H is $n\Lambda_g$-closed.

Proof: Let $H \subseteq G$ where G is $n\lambda$-open. Since H is Λ_g-nI-closed, $H_n^\star \subseteq G$. As H is $n\star$-dense in itself, n-$cl(H) = H_n^\star$. Thus n-$cl(H) \subseteq G$ and hence H is $n\Lambda_g$-closed.

Corollary 2.20. If $(U, \mathcal{N}, I)$ is any ideal nanotopological space where $I = \{\phi\}$, then H is Λ_g-nI-closed if and only if H is $n\Lambda_g$-closed.

Proof: In $(U, \mathcal{N}, I)$, if $I = \{\phi\}$ then $H_n^\star = n$-$cl(H)$ for the subset H. H is Λ_g-nI-closed $\Leftrightarrow H_n^\star \subseteq G$ whenever $H \subseteq G$ and G is $n\lambda$-open $\Leftrightarrow n$-$cl(H) \subseteq G$ whenever $H \subseteq G$ and G is $n\lambda$-open $\Leftrightarrow H$ is $n\Lambda_g$-closed.

Corollary 2.21. In an ideal nanotopological space $(U, \mathcal{N}, I)$ where I is $\mathcal{N}$-codense, if H is a ns-open set and Λ_g-nI-closed subset of U, then H is $n\Lambda_g$-closed.

Proof: H is $n\star$-dense in itself. By Theorem 2.19, H is $n\Lambda_g$-closed.

Remark 2.22. In an ideal nanotopological space $(U, \mathcal{N}, I)$, the family of ng-closed sets and the family of Λ_g-nI-closed sets are independent of each other as shown in the following Example.

Example 2.23.
1. In Example 2.4, the subset $\{r_1, r_3\}$ is ng-closed set but not Λ_g-nI-closed.
2. In Example 2.18, the subset $\{r_1\}$ is Λ_g-nI-closed set but not ng-closed.

Remark 2.24. These relations are shown in the following diagram.

n-closed → $n\Lambda g$-closed → I_g-closed
↓ ↓↑
$n\star$-closed → Λ_g-nI-open ↮ ng-closed

Theorem 2.25. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space and $H \subseteq U$. Then H is Λ_g-nI-closed if and only if $H = K - S$ where K is $n\star$-closed and S contains no nonempty $n\lambda$-closed set.

Proof: If H is $n\Lambda_g$-closed, then by Theorem 2.5 (4), $S = H_n^\star - H$ contains no nonempty $n\lambda$-closed set. If $K = n$-$cl^\star(H)$, then K is $n\star$-closed such that $K - S = (H \cup H_n^\star) - (H_n^\star - H) = (H \cup H_n^\star) \cap (H_n^\star \cap H^c)^c = (H \cup H_n^\star) \cap ((H_n^\star)^c \cup H) = (H \cup H_n^\star) \cap (H \cup (H_n^\star)^c) = H \cup (H_n^\star \cap (H_n^\star)^c) = H$.

Conversely, suppose $H = K - S$ where K is $n\star$-closed and S contains no nonempty $n\lambda$-closed set. Let G be a $n\lambda$-open set such that $H \subseteq G$. Then $K - S \subseteq G$ which implies that $K \cap (U - G) \subseteq S$. Now $H \subseteq K$ and $K_n^\star \subseteq K$ then $H_n^\star \subseteq K_n^\star$ and so $H_n^\star \cap (U - G) \subseteq K_n^\star \cap (U - G) \subseteq K \cap (U - G) \subseteq S$. Since $H_n^\star \cap (U - G)$ is $n\lambda$-closed, by hypothesis $H_n^\star \cap (U - G) = \phi$ and so $H_n^\star \subseteq G$. Thus H is Λ_g-nI-closed.

Theorem 2.26. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space and $H \subseteq U$. If $H \subseteq K \subseteq H_n^\star$, then $H_n^\star = K_n^\star$ and K is $n\star$-dense in itself.

Proof: Since $H \subseteq K$, then $H_n^\star \subseteq K_n^\star$ and since $K \subseteq H_n^\star$, then $K_n^\star \subseteq (H_n^\star)_n^\star \subseteq H_n^\star$. Therefore $H_n^\star = K_n^\star$ and $K \subseteq H_n^\star \subseteq K_n^\star$.

Theorem 2.27. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space. If H and K are subsets of U such that $H \subseteq K \subseteq n\text{-}cl^\star(H)$ and H is Λ_g-nI-closed, then K is Λ_g-nI-closed.

Proof: Since H is Λ_g-nI-closed, then by Theorem 2.5(3), $n\text{-}cl^\star(H) - H$ contains no nonempty $n\lambda$-closed set. But $n\text{-}cl^\star(K) - K \subseteq n\text{-}cl^\star(H) - H$ and so $n\text{-}cl^\star(K) - K$ contains no nonempty $n\lambda$-closed set. This proves that K is Λ_g-nI-closed.

Corollary 2.28. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space. If H and K are subsets of U such that $H \subseteq K \subseteq H_n^\star$ and H is Λ_g-nI-closed, then H and K are $n\Lambda_g$-closed sets.

Proof: Let H and K be subsets of U such that $H \subseteq K \subseteq H_n^\star$. Then $H \subseteq K \subseteq H_n^\star \subseteq n\text{-}cl^\star(H)$. Since H is Λ_g-nI-closed, by Theorem 2.27, K is Λ_g-nI-closed. Since $H \subseteq K \subseteq H_n^\star$, we have $H_n^\star = K_n^\star$. Hence $H \subseteq H_n^\star$ and $K \subseteq K_n^\star$. Thus H is $n\star$-dense in itself and H is $n\star$-dense in itself and by Theorem 2.19, H and K are $n\Lambda_g$-closed.

The characterization of Λ_g-nI-open sets are given in the following Theorem.

Theorem 2.29. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space and $H \subseteq U$. Then H is Λ_g-nI-open if and only if $K \subseteq n\text{-}int^\star(H)$ whenever K is $n\lambda$-closed and $K \subseteq H$.

Proof: Suppose H is Λ_g-nI-open. If K is $n\lambda$-closed and $K \subseteq H$, then $U - H \subseteq U - K$ and so $n\text{-}cl^\star(U - H) \subseteq U - K$ by Theorem 2.5(2). Therefore $K \subseteq U - n\text{-}cl^\star(U - H) = n\text{-}int^\star(H)$. Hence $K \subseteq n\text{-}int^\star(H)$.

Conversely, suppose the condition holds. Let G be a $n\lambda$-open set such that $U - H \subseteq G$. Then $U - G \subseteq H$ and so $U - G \subseteq n\text{-}int^\star(H)$. Therefore $n\text{-}cl^\star(U - H) \subseteq G$. By Theorem 2.5(2), $U - H$ is Λ_g-nI-closed. Which proves that H is Λ_g-nI-open.

Corollary 2.30. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space and $H \subseteq U$. If H is Λ_g-nI-open, then $K \subseteq int^\star(H)$ whenever K is n-closed and $K \subseteq H$.

The properties of Λ_g-nI-closed sets are given in the following Theorem.

Theorem 2.31. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space and $H \subseteq U$. If H is Λ_g-nI-open and $int^\star(H) \subseteq K \subseteq H$, then K is Λ_g-nI-open.

Proof: Since $int^\star(H) \subseteq K \subseteq H$, we have $U - H \subseteq U - K \subseteq U - int^\star(H) = n\text{-}cl^\star(U - H)$. By assumption H is Λ_g-nI-open and so $U - H$ is Λ_g-nI-closed. Hence, by Theorem 2.27, $U - H$ is Λ_g-nI-closed and K is Λ_g-nI-open.

The following Theorem gives a characterization of Λ_g-nI-closed sets in terms of Λ_g-nI-open sets.

Theorem 2.32. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space and $H \subseteq U$. Then the following results are equivalent.
1. H is Λ_g-nI-closed,
2. $H \cup (U - H_n^\star)$ is Λ_g-nI-closed,
3. $H_n^\star - H$ is Λ_g-nI-open.

Proof: (1)⇒(2) Let G be any $n\lambda$-open set such that $H \cup (U - H_n^\star) \subseteq G$. Then $G^c \subseteq [A \cup (U - H_n^\star)]^c = [H \cup (H_n^\star)^c]^c = H_n^\star \cap H^c = H_n^\star - H$ where G^c is $n\lambda$-closed. Since H is Λ_g-nI-closed, by Theorem 2.5(4), $G^c = \phi$ and $U = G$. Thus U is the only $n\lambda$-open set containing $H \cup (U - H_n^\star)$ and hence $H \cup (U - H_n^\star)$ is Λ_g-nI-closed.

(2)⇒(3) $(H_n^\star - H)^c = (H_n^\star \cap H^c)^c = H \cup (H_n^\star)^c = H \cup (U - H_n^\star)$ which is Λ_g-nI-closed by (2). This proves that $H_n^\star - H$ is Λ_g-nI-open.

(3) ⇒ (1) Since $H_n^\star - H$ is Λ_g-nI-open, $(H_n^\star - H)^c = H \cup (H_n^\star)^c$ is Λ_g-nI-closed. Hence by Theorem 2.5(4) $(H \cup (H_n^\star)^c)_n^\star - (H \cup (H_n^\star)^c)$ contains no nonempty $n\lambda$-closed subset. But $(H \cup (H_n^\star)^c)_n^\star - (H \cup (H_n^\star)^c) = (H \cup (H_n^\star)^c)_n^\star \cap (H \cup (H_n^\star)^c)^c = (H \cup (H_n^\star)^c)_n^\star \cap (H_n^\star \cup H^c) = (H_n^\star \cup ((H_n^\star)^c)_n^\star) \cap (H_n^\star \cap H^c) = H_n^\star \cap H^c = H_n^\star - H$. Thus $H_n^\star - H$ has no nonempty $n\lambda$-closed subset. Hence by Theorem 2.5(4), H is Λ_g-nI-closed.

Theorem 2.33. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space. Then every subset of U is nano Λ_g-nI-closed if and only if every $n\lambda$-open set is $n\star$-closed.

Proof: Suppose every subset of U is Λ_g-nI-closed. Let G be $n\lambda$-open in U. Then $G \subseteq G$ and G is Λ_g-nI-closed by assumption implies $G_n^\star \subseteq G$. Hence G is $n\star$-closed.

Conversely, let $H \subseteq U$ and G be $n\lambda$-open such that $H \subseteq G$. Since G is $n\star$-closed by assumption, we have $H_n^\star \subseteq G_n^\star \subseteq G$. Thus H is Λ_g-nI-closed.

The following Theorem gives a characterization of normal spaces in terms of Λ_g-nI-open sets.

Theorem 2.34. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space where I is completely $\mathcal{N}$-codense. Then the following results are equivalent;
1. U is normal,
2. For any disjoint n-closed sets H and K, there exist disjoint Λ_g-nI-open sets E and F such that $H \subseteq E$ and $K \subseteq F$,
3. For any n-closed set H and n-open set F containing H, there exists an Λ_g-nI-open set E such that $H \subseteq E \subseteq n\text{-}cl^\star(E) \subseteq F$.

Proof: (1)⇒(2) The proof follows from the fact that every n-open set is Λ_g-nI-open.

(2)⇒(3) Suppose H is n-closed and F is a n-open set containing H. Since H and $U-F$ are disjoint n-closed sets, there exist disjoint Λ_g-nI-open sets E and S such that $H \subseteq E$ and $U-F \subseteq S$. Since $U-F$ is $n\lambda$-closed and S is Λ_g-nI-open, $U-F \subseteq n\text{-}int^{\star}(S)$. Then $U - n\text{-}int^{\star}(S) \subseteq F$. Again $F \cap S = \phi$ which implies that $E \cap n\text{-}int^{\star}(S) = \phi$ and so $E \subseteq U - n\text{-}int^{\star}(S)$. Then $n\text{-}cl^{\star}(E) \subseteq U - n\text{-}int^{\star}(S) \subseteq F$ and thus E is the required Λ_g-nI-open sets with $H \subseteq E \subseteq n\text{-}cl^{\star}(E) \subseteq F$.

(3)⇒(1) Let H and K be two disjoint n-closed subsets of U. Then H is a n-closed set and $U-K$ an n-open set containing H. By hypothesis, there exists a Λ_g-nI-open set E such that $H \subseteq E \subseteq n\text{-}cl^{\star}(E) \subseteq U-K$. Since E is Λ_g-nI-open and H is $n\lambda$-closed we have $H \subseteq n\text{-}int^{\star}(E)$. Since I is completely $\mathcal{N}$-codense ideal, $\mathcal{N}^{\star} \subseteq \mathcal{N}^{\alpha}$ and so $n\text{-}int^{\star}(E)$ and $U - n\text{-}cl^{\star}(E) \in \mathcal{N}^{\alpha}$. Hence $H \subseteq n\text{-}int^{\star}(E) \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(n\text{-}int^{\star}(E)))) = G$ and $K \subseteq U - n\text{-}cl^{\star}(E) \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(U - n\text{-}cl^{\star}(E)))) = H$. O and C are the required disjoint n-open sets containing H and K respectively, which proves (1).

Definition 2.35. A subset H of an ideal nanotopological space $(U, \mathcal{N}, I)$ is called a nano $\Lambda_{g\alpha}$-closed set if $n\text{-}cl_{\alpha}(H) \subseteq G$ whenever $H \subseteq G$ and G is $n\lambda$-open. The complement of nano $\Lambda_{g\alpha}$-closed set is called a nano $\Lambda_{g\alpha}$-open.

If $I = \mathcal{T}$, it is not difficult to see that Λ_g-nI-closed sets coincide with nano $\Lambda_{g\alpha}$-closed sets and so we have the following Corollary.

Corollary 2.36. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space where $I = \mathcal{T}$. Then the following statements are equivalent;
1. U is normal,
2. For any disjoint n-closed sets H and K, there exists a disjoint nano $\Lambda_{g\alpha}$-open sets X and Y such that $H \subseteq X$ and $K \subseteq Y$,
3. For any n-closed set H and n-open set Y containing H, there exists a nano $\Lambda_{g\alpha}$-open set X such that $H \subseteq X \subseteq n\text{-}cl_{\alpha}(X) \subseteq Y$.

Definition 2.37. A subset H of an ideal nanotopological space is said to be nI-compact or compact modulo nI if for every n-open cover $\{X_\alpha | \alpha \in \Delta\}$ of H, there exists a finite subset Δ_0 of Δ such that $H - \bigcup\{X_\alpha | \alpha \in \Delta_0\} \in nI$. The space $(U, \mathcal{N}, I)$ is nI-compact if X is nI-compact as a subset.

Corollary 2.38. Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space for a subset H of U. If H is a Λ_g-nI-closed subset of U, then H is nI-compact.

Proof: The proof follows from the fact that every Λ_g-nI-closed is nI_g-closed.

4. CONCLUSIONS

This paper introduces a new class of sets in an ideal nanotopological space, called nano Λ_g-closed sets. Suitable examples are provided together with the characterizations and features of Λ_g-nI-closed sets and Λ_g-nI-open sets. Normal spaces are characterized in terms of Λ_g-nI-open sets. Furthermore, it is proven that a nano $\mathcal{J}$-compact space is nano $\mathcal{J}$-compact when its subset is Λ_g-nI- closed.

REFERENCES

[1] Rajasekaran, I., Nethaji, O., *Journal of New Theory*, **16**, 52, 2017.
[2] Rajasekaran, I., Nethaji, O., *Journal of New Theory*, **17**, 38, 2017.
[3] Vaidyanathaswamy, R., *Set Topology*, Chelsea Publishing Company, New York, 1946.
[4] Kuratowski, K., *Topology*, Vol. I. Academic Press, New York, 1966.
[5] Vaidyanathaswamy, R., *Proceedings - Mathematical Sciences*, **20**, 51, 1945.
[6] Parimala, M., Jafari, S., *Eurasian Bulletin of Mathematics*, **1**, 85, 2018.
[7] Pawlak, Z., *International Journal of Computer and Information Sciences*, **11**, 341, 1982.
[8] Lellis Thivagar, M., Carmel Richard, *International Journal of Mathematics and Statistics Invention*, **1**, 31, 2013.
[9] Bhuvaneshwari, K., Mythili Gnanapriya, K., *International Journal of Scientific and Research Publications*, **4**, 1, 2014.
[10] Parimala, M., Jafari, S., Murali, S., *Annales Universitatis Scientiarium Budapestinensis, Sectio Mathematica*, **60**, 3, 2017.
[11] Lellis Thivagar, M., Kavitha, J., *Missouri Journal of Mathematical Sciences*, **29**, 80, 2017.