ORIGINAL PAPER

A NEW DIFFERENTIAL TRANSFORMATION SCHEME FOR SOLVING MANY-INTERVAL BOUNDARY VALUE PROBLEMS WITH ADDITIONAL TRANSMISSION CONDITIONS

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Abstract. *The main goal of this paper is to investigate a new class of boundary value problems (BVPs) that are defined on a finite number of separated intervals, and the solutions defined in these intervals are connected with transmission conditions. We will call such problems as many interval boundary value transmission problems (MIBVTPs). It is obvious that MIBVTPs are much more complicated to solve than classical single-interval BVPs. In this paper, we propose a new generalization of the well-known differential transform method (DTM) to solve MIBVTPs (for the proposed generalization of DTM, we will use the abbreviation GDTM). Two MIBVTPs are solved by the proposed GDTM. The calculated approximate solutions are compared with the exact solutions graphically. The results obtained demonstrate that the GDTM is an efficient method for solving MIBVTPs.*

Keywords: *Differential transformation method; boundary value problems; transmission conditions; approximate solution; exact solution.*

1. INTRODUCTION

Recently, there has been heightened interest in boundary value problems for many-interval differential equations. Such problems have arisen after the development of new and interesting applications in physics [1-6]. Various approximation methods, such as the Adomian Decomposition method, the Euler method, the shooting method, the finite difference method, the predictor correction method, the variational iteration method, the differential transformation method, the homotopy perturbation method, etc. and their different modifications and generalizations are becoming increasingly important interest due to the complexity of finding exact solutions to many initial and/or BVPs for differential equations of various types. Particularly, DTM was used to obtain numerical or analytical solutions not only for differential equations, but also for difference equations, integral equations, and integro-differential equations (see [7-13] and references cited therein). For example, in [14] Hassan uses one-dimensional DTM to obtain eigenvalues and eigenfunctions for second and fourth-order differential equations and uses two-dimensional DTM to solve first and second-order partial differential equations (PDEs). There are many studies on the applications of DTM to various concrete problems arising in physics and engineering.

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In [15] Bert used DTM to study of steady-state heat transfer in triangular fins with constant characteristics. In [16], by considering variable specific heat coefficients were solved two nonlinear heat transfer problems. In [17], a new scheme was developed by combining the methods DTM and ADM (Adomian Decomposition Method) for finding approximate solutions to PDEs for which inhomogeneous Dirichlet-type boundary conditions are specified. In [18], Li et.al investigated the existence of solutions to the Sturm-Liouville equation with random momenta. In [19], Mukhtarov and Yücel investigated the eigenvalues and eigenfunctions of two-interval Sturm-Liouville problems, which arise in modeling many problems of physics and other areas of the natural sciences. In [20] and [21], Mukhtarov et. al propose a new generalization of DTM (the so-called α-parameterized DTM) to find an approximate solution of many non-classical BVPs. The well-known DTM is based on Taylor's expansion to find approximate or exact solutions in polynomial form. DTM was first defined by Zhou [22] in the research of electrical circuits. In [23], solutions of some differential equations have been investigated using the alpha-parametrized differential transform method. In this work, we generalized the standard DTM to solve many-interval BVPs with additional transmission conditions. To illustrate the proposed generalization of DTM, we have solved two many-interval boundary value transmission problems (MIBVTPs).

2. THE DEFINITION AND BASIC PROPERTIES OF THE TRANSFORMATION METHOD

Let $\mathrm{v} = \mathrm{v}(\mathrm{t})$ any analytic function in the neighborhood of the point $t = t_0$ and let

$$v(t) = \sum_{l=0}^{\infty} V_l(t_0)\,(t - t_0)^l$$

be a Taylor expansion of the function $v(t)$, where

$$V_l(t_0) = \frac{1}{l!}\left[\frac{d^l}{dt^l}v(t)\right]_{t=t_0}$$

are the Taylor coefficients.

Definition 2.1 The sequence $V_{t_0} = (V_{t_0}(0),\ V_{t_0}(1), \dots)$ is called the differential transformation of the analytic function $v(t)$ at the point $t = t_0$ and is denoted by $T_{t_0}(v)$.

Definition 2.2 The differential inverse transformation of the sequence $V_{t_0} = (V_{t_0}(0),$ $V_{t_0}(1), \dots)$ is said to be the series $\sum_{l=0}^{\infty} V_{t_0}(l)\,(t - t_0)^l$ and is denoted by $T_{t_0}^{-1}(V_{t_0})$.

It is clear that $T_{t_0}^{-1}\left(T_{t_0}(v)\right) = v$. Here $v(t)$ is called the original function and the sequence $V_{t_0} = T_{t_0}(v)$ is called the T-transform of $v(t)$. From the definition of the T-transform, it follows easily the following properties:

i. $T_{t_0}(a_1 + a_2) = T_{t_0}(a_1) + T_{t_0}(a_2)$

ii. $T_{t_0}(\gamma \mathrm{b}) = \gamma T_{t_0}(\mathrm{b})$ for any $\gamma \in \mathrm{R}$

iii. If $T_{t_0}(\mathrm{v}) = \left(V_{t_0}(\mathrm{l})\right)$ then $T_{t_0}\left(\frac{d^n}{\mathrm{dt}^n}v\right) = \left((\mathrm{l}+1)(l+2)\dots(l+n)V_{t_0}(\mathrm{l}+\mathrm{n})\right)$ $\mathrm{n} \in \mathrm{N}$.

iv.If $T_{t_0}(\text{f}) = \left(f_{t_0}(\text{l})\right)$ and $T_{t_0}(\text{g}) = \left(g_{t_0}(\text{l})\right)$ then $T_{t_0}(\text{fg}) = \left(f_{t_0}(\text{l})\right) * \left(g_{t_0}(\text{l})\right)$ where $\left(f_{t_0}(\text{l})\right) * \left(g_{t_0}(\text{l})\right)$ is the convolution of the sequences $\left(f_{t_0}(\text{l})\right)$ and $\left(g_{t_0}(\text{l})\right)$.

In a real application, the differential inverse transform $T_{t_0}^{-1}(V_{t_0}(l))$ is defined by a finite sum

$$T_{t_0}^{-1}\left(V_{t_0}(l)\right) = \sum_{l=0}^{s} V_{t_0}(l)(t - t_0)^l$$

for sufficiently large s.

3. THE ALGORITHM OF THE GENERALIZED DTM IN SOLVING MANY INTERVAL TRANSMISSION PROBLEMS

To show that the GDTM can be applied not only to standard BVPs but also to MIBVTPs, let us solve two MIBVTPs by using a generalized DTM.

Example 1. (Application of GDTM to MIBVTP) Let us consider the differential equation, which is defined on three separate intervals and given by

$$y''(t) - \left(1 + \frac{1}{t+2}\right)y'(t) + \frac{1}{t+2}y(t) = 0, \quad t \in [0,1) \cup (1,2) \cup (2,3] \tag{1}$$

subject to the boundary conditions specified at the end-points $t = 0$ and $t = 3$, given by

$$y(0) = 6, \quad y(3) = 3e^2 \tag{2}$$

and additional transmission conditions specified at the common end-points $t = 1$ and $t = 2$, given by

$$y(1+0) = \frac{5}{8}y(1-0), \quad y'(1+0) = y'(1-0$$

and

$$y(2+0) = \frac{3e}{5+e}y(2-0), \qquad y'(2+0) = \frac{3e}{1+e}y'(2-0)$$

respectively.

Let $T_0(l)$, $T_1(l)$ and $T_2(l)$ be the differential transform of the original function $y(t)$ at the points $t = 0$, $t = 1$ and $t = 2$, respectively. If we apply differential transformation to equation (1) in the interval [0,1) with $t_0 = 0$, then we find that

$$T_0(y_1, l+2) = \frac{-1}{2(l+1)(l+2)}[(l-3)(l+1)T_0(y_1, l+1) + (1-l)T_0(y_1, l)] \tag{3}$$

Now, applying the differential inverse transform, we have

$$y_1(t) = T_0(y_1, 0) + T_0(y_1, 1)t + T_0(y_1, 2)t^2 + T_0(y_1, 3)t^3 + \cdots$$

where $y_1(t)$ denotes the restriction of $y(t)$ on the left interval [0,1). The first boundary condition $y(0) = 6$ yields

$$T_0(y_1, 0) = 6.$$

Put

$$T_{0}(y_1,1)=A$$

By using the recurrence formula (3), we can calculate the other terms of the sequence $(T_{0}(y_1,l))$ as follows.

$$T_{0}(y_1,2)=\frac{3}{4}(A-2)$$
$$T_{0}(y_1,3)=\frac{1}{3}T_{0}(y_1,2)$$
$$T_{0}(y_1,4)=\frac{1}{12}T_{0}(y_1,2)$$
$$T_{0}(y_1,5)=\frac{1}{60}T_{0}(y_1,2)$$
$$T_{0}(y_1,6)=\frac{1}{360}T_{0}(y_1,2)$$
$$T_{0}(y_1,7)=\frac{1}{2520}T_{0}(y_1,2),\ \ldots.$$

Thus, we have the following formula for the solution that is defined on the first interval [0,1).

$$y_1(t)=T_{0}(y_1,0)+T_{0}(y_1,1)t+T_{0}(y_1,2)t^2+T_{0}(y_1,3)t^3+\cdots$$
$$=6+At+T_{0}(y_1,2)t^2+\frac{1}{3}T_{0}(y_1,2)t^3+\frac{1}{12}T_{0}(y_1,2)t^4+\frac{1}{60}T_{0}(y_1,2)t^5+\cdots$$
$$=6+At+T_{0}(y_1,2)\left(t^2+\frac{1}{3}t^3+\frac{1}{12}t^4+\frac{1}{60}t^5+\cdots\right)$$
$$=6+At+\frac{3}{2}(A-2)(e^t-t-1)$$

Using the boundary conditions (2) we obtain A=2. Secondly, let us get the solution defined in the second interval (1, 2). If the differential transform method is applied to the differential equation (1) in the around of the point $t_0=1$, we have

$$T_{1}(y_2,l+2)=\frac{-1}{3(l+1)(l+2)}[(l-4)(l+1)T_{1}(y_2,l+1)+(1-l)T_{1}(y_2,l)] \quad (4)$$

where $y_2(t)$ denotes the restriction of $y(t)$ on the second interval (1,2).

By applying the differential inverse transform in the second interval (1,2) we have

$$y_2(t)=T_{1}(y_2,0)+T_{1}(y_2,1)(t-1)+T_{1}(y_2,2)(t-1)^2+T_{1}(y_2,3)(t-1)^3+\cdots$$
$$y_2(1)=T_{1}(y_2,0)=\frac{5}{8}y_1(1)$$
$$T_{1}(y_2,0)=\frac{5}{8}\left[6+A+\frac{3}{2}(A-2)(e-2)\right]$$

Let $T_{1}(y_2,1)=B$. By using the recurrence formula (4), we have

$$T_{1}(y_2,2)=\frac{2}{3}B-\frac{1}{6}T_{1}(y_2,0)$$
$$T_{1}(y_2,3)=\frac{1}{3}T_{1}(y_2,2)$$
$$T_{1}(y_2,4)=\frac{1}{12}T_{1}(y_2,2)$$

$$T_1(y_2,5)=\frac{1}{60}T_1(y_2,2)$$
$$T_1(y_2,6)=\frac{1}{360}T_1(y_2,2)$$
$$T_1(y_2,7)=\frac{1}{2520}T_1(y_2,2),\ \ldots.$$

Consequently

$$\begin{aligned} y_2(t) &= T_1(y_2,0)+T_1(y_2,1)(t-1)+T_1(y_2,2)(t-1)^2+T_1(y_2,3)(t-1)^3+\cdots \\ &= T_1(y_2,0)+B(t-1)+\left[\frac{2}{3}B-\frac{1}{6}Y(y_2,0)\right]2(e^{t-1}-t) \\ &= \frac{5}{8}\left[6+A+\frac{3}{2}(A-2)(e-2)\right]+B(t-1) \\ &\quad +\left[\frac{2}{3}B-\frac{5}{48}\left[6+A+\frac{3}{2}(A-2)(e-2)\right]\right]2(e^{t-1}-t) \end{aligned}$$

Finally, let us get the solution for the problem in the interval (2, 3]. If the transform method is applied to the differential equation (1) in the interval (2,3] at the point $t_0=2$, then we have the following recurrence formula

$$T_2(y_3,l+2)=\frac{-1}{4(l+1)(l+2)}\left[(l-5)(l+1)T_2(y_3,l+1)+(1-l)T_2(y_3,l)\right]$$

where $y_3(t)$ denotes the restriction of $y(t)$ on the interval (2,3]. Applying the differential inverse transformation gives

$$y_3(t)=T_2(y_3,0)+T_2(y_3,1)(t-2)+T_2(y_3,2)(t-2)^2+T_2(y_3,3)(t-2)^3+\cdots$$

where

$$y_3(2)=T_2(y_3,0)=\frac{3e}{5+e}y_2(2)$$

$$\begin{aligned} T_2(y_3,0)=\frac{3e}{5+e}\Bigg[&\frac{5}{8}\left(6+A+\frac{3}{2}(A-2)(e-2)\right)+B \\ &+\left(\frac{2}{3}B-\frac{5}{48}\left[6+A+\frac{3}{2}(A-2)(e-2)\right]\right)2(e-2)\Bigg] \end{aligned}$$

$$T_2(y_3,1)=\frac{3e}{1+e}\left[B+\left(\frac{2}{3}B-\frac{5}{48}\left[6+A+\frac{3}{2}(A-2)(e-2)\right]\right)2(e-1)\right]$$
$$T_2(y_3,2)=\frac{5}{8}T_2(y_3,1)-\frac{1}{8}T_2(y_3,0)$$
$$T_2(y_3,3)=\frac{1}{3}T_2(y_3,2)$$
$$T_2(y_3,4)=\frac{1}{12}T_2(y_3,2)$$
$$T_2(y_3,5)=\frac{1}{60}T_2(y_3,2)$$
$$T_2(y_3,6)=\frac{1}{360}T_2(y_3,2)$$
$$T_2(y_3,7)=\frac{1}{2520}T_2(y_3,2),\ \ldots.$$

By using transmission conditions, we find $A+\frac{3}{2}(A-2)(e-1)=B$ (Fig. 1). So,

$$y_3(t)= T_2(y_3,0)+T_2(y_3,1)(t-2)+\left[\frac{5}{8}T_2(y_3,1)-\frac{1}{8}T_2(y_3,0)\right][2(e^{t-2}-t+1)]$$

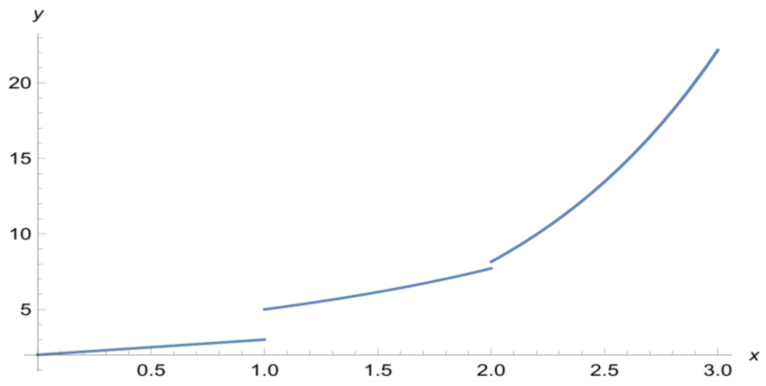

Figure 1. Graph of the approximate solution.

We can show that the exact solution of this MIBVTP is (Fig. 2)

$$y(t)=\begin{cases}2t+6 \text{ for} & t\in[0,1)\\ t+e^{t-1}+3 \text{ for} & t\in(1,2)\\ 3e^{t-1} \text{ for} & t\in(2,3].\end{cases}$$

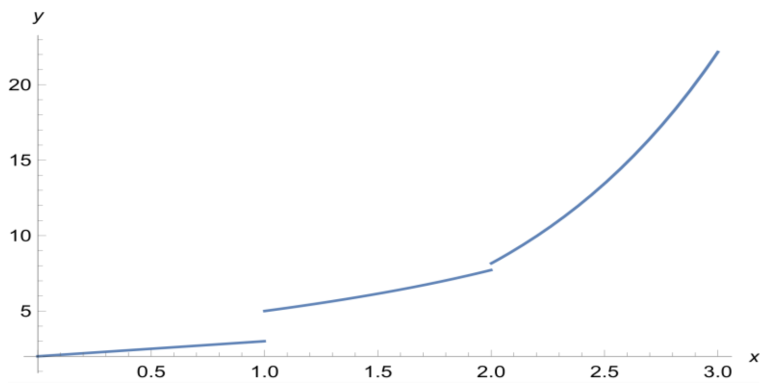

Figure 2. Graph of the exact solution.

Example 2. Application of GDTM to initial value problem with additional transmission conditions.

Now, let us consider the differential equation,

$$y''(t)-\frac{(t+3)}{2(t+1)}y'(t)+\frac{1}{2(t+1)}y(t)=0, \qquad t\in[0,1)\cup(1,2)\cup(2,3] \tag{5}$$

subject to the initial conditions at the end-points $t=0$ and $t=3$, given by

$$y(0)=3,\ y'(0)=1 \tag{6}$$

and additional transmission conditions at the common end-points $t=1$ and $t=2$, given by

$$y(1+0)=\frac{1}{3}y(1-0)+5e^{\frac{-1}{2}},\ y'(1+0)=\frac{5}{8}e^{\frac{-1}{2}}y(1-0)+\frac{1}{3} \tag{7}$$

and

$$y(2+0) = y(2-0) + 3e - \frac{20}{3}, \qquad y'(2+0) = \frac{9e}{40} y(2-0) \tag{8}$$

respectively.

Let $T_0(l)$, $T_1(l)$ and $T_2(l)$ be the differential transform of the original function $y(t)$ at the points $t = 0$, $t = 1$ and $t = 2$, respectively. If we apply differential transformation to equation (5) in the interval [0,1) with $t_0 = 0$, then we find that

$$T_0(y_1, l+2) = \frac{-1}{2(l+1)(l+2)} [(2l-3)(l+1) T_0(y_1, l+1) + (1-l) T_0(y_1, l)] \tag{9}$$

where $y_1(t)$ denotes the restriction of $y(t)$ on the left interval [0,1).

Now, applying the differential inverse transform, we have

$$y_1(t) = T_0(y_1, 0) + T_0(y_1, 1)t + T_0(y_1, 2)t^2 + T_0(y_1, 3)t^3 + \cdots$$

The first boundary condition $y(0) = 3$ yields

$$T_0(y_1, 0) = 3$$

Put

$$T_0(y_1, 1) = B.$$

By using the recurrence formula (8), we can calculate the other terms of the sequence $(T_0(y_1, l))$ as follows.

$$T_0(y_1, 2) = \tfrac{-1}{4}((-3) T_0(y_1, 1) + T_0(y_1, 0)),$$
$$T_0(y_1, 3) = \tfrac{1}{6} T_0(y_1, 2),$$
$$T_0(y_1, 4) = \tfrac{1}{48} T_0(y_1, 2),$$
$$T_0(y_1, 5) = \tfrac{1}{480} T_0(y_1, 2),$$
$$T_0(y_1, 6) = \tfrac{1}{5760} T_0(y_1, 2), \ldots.$$

Then we have the following approximation of the first solution $y_1(t)$:

$$\begin{aligned} y_1(t) &= T_0(y_1, 0) + T_0(y_1, 1)t + T_0(y_1, 2)t^2 + T_0(y_1, 3)t^3 + \cdots \\ &= T_0(y_1, 0) + T_0(y_1, 1)t + T_0(y_1, 2) 8(e^{\frac{t}{2}} - \frac{t}{2} - \frac{1}{2}) \\ &= 3 + Bx + \frac{3}{4}(B-1) 8(e^{\frac{t}{2}} - \frac{t}{2} - \frac{1}{2}) \end{aligned}$$

Using the initial condition $y'(0) = 1$, we find $B = 1$. Secondly, if we apply differential transformation to the equation (5) in the second interval (1,2) with $t_0 = 1$, then we find that

$$T_1(y_2, l+2) = \frac{-1}{4(l+1)(l+2)} [(2l-4)(l+1) T_1(y_2, l+1) + (1-l) T_1(y_2, l)] \tag{10}$$

Now, applying the differential inverse transform, we have

$$y_2(t) = T_1(y_2, 0) + T_1(y_2, 1)t + T_1(y_2, 2)t^2 + T_1(y_2, 3)t^3 + \cdots$$

where $y_2(t)$ denotes the restriction of $y(t)$ on the interval (1,2). By using the recurrence formula (9), we can calculate the other terms of the sequence $(T_1(y_2, l))$ as follows.

$$T_1(y_2,2)=\frac{-1}{8}((-4)T_1(y_2,1)+T_1(y_2,0)),$$
$$T_1(y_2,3)=\frac{1}{6}T_1(y_2,2),$$
$$T_1(y_2,4)=\frac{1}{48}T_1(y_2,2),$$
$$T_1(y_2,5)=\frac{1}{480}T_1(y_2,2),$$
$$T_1(y_2,6)=\frac{1}{5760}T_1(y_2,2), \dots.$$

Then, we have the following approximation of the second solution:

$$\begin{aligned} y_2(t) &= T_1(y_2,0)+T_1(y_2,1)(t-1)+T_1(y_2,2)(t-1)^2+T_1(y_2,3)(t-1)^3 \\ &= T_1(y_2,0)+T_1(y_2,1)(t-1)+T_1(y_2,2)8(e^{\frac{t-1}{2}}-\frac{t}{2}-\frac{1}{2}) \end{aligned}$$

Using the transmission condition (7), we have

$$T_1(y_2,0)=\frac{4}{3}+5e^{-\frac{1}{2}}$$
$$T_1(y_2,1)=\frac{1}{3}+5e^{-\frac{1}{2}}$$

So, the second solution can be written as follows:

$$y_2(t)=\frac{4}{3}+5e^{-\frac{1}{2}}+\left(\frac{1}{3}+5e^{-\frac{1}{2}}\right)(t-1)+\frac{15}{8}e^{-\frac{1}{2}}8\left(e^{\frac{t-1}{2}}-\frac{t}{2}-\frac{1}{2}\right).$$

Finally, let us get the solution in the third interval (2, 3]. If the transform method is applied to the differential equation (5) in the interval (2,3] at the point $t_0=2$, then we have

$$T_2(y_3,l+2)=\frac{-1}{6(l+1)(l+2)}[(2l-5)(l+1)T_2(y_3,l+1)+(1-l)T_2(y_3,l)]$$

where $y_3(t)$ denotes the restriction of $y(t)$ on the interval (2,3]. Applying the inverse transformation gives

$$y_3(t)=T_2(y_3,0)+T_2(y_3,1)(t-2)+T_2(y_3,2)(t-2)^2+T_2(y_3,3)(t-2)^3+\cdots$$

where

$$T_2(y_3,2)=\frac{5}{12}T_2(y_3,1)-\frac{1}{12}T_2(y_3,0)$$
$$T_2(y_3,3)=\frac{1}{6}T_2(y_3,2)$$
$$T_2(y_3,4)=\frac{1}{48}T_2(y_3,2)$$
$$T_2(y_3,5)=\frac{1}{480}T_2(y_3,2),\dots$$

Consequently

$$y_3(t)=T_2(y_3,0)+T_2(y_3,1)(t-2)+T_2(y_3,2)8(e^{\frac{t-2}{2}}-1-\frac{t-2}{2})$$

Using transmission condition (7), we have

$$T_2(y_3,0) = 3e$$
$$T_2(y_3,1) = \frac{3e}{2}$$

So, the third solution can be written as follows (Fig. 3):

$$y_3(t) = 3e + \frac{3e}{2}(t-2) + 3e\left(e^{\frac{t-2}{2}} - \frac{t-2}{2} - 1\right).$$

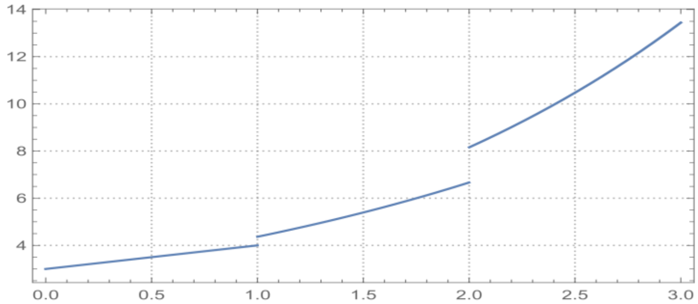

Figure 3. Graph of the approximate solution.

We can show that the exact solution of this problem is (Fig. 4)

$$y(t) = \begin{cases} t+3 \text{ for} & t \in [0,1) \\ \frac{t}{3} + 5e^{\frac{t}{2}-1} + 1 \text{ for} & t \in (1,2) \\ 3e^{\frac{t}{2}} \text{ for} & t \in (2,3]. \end{cases}$$

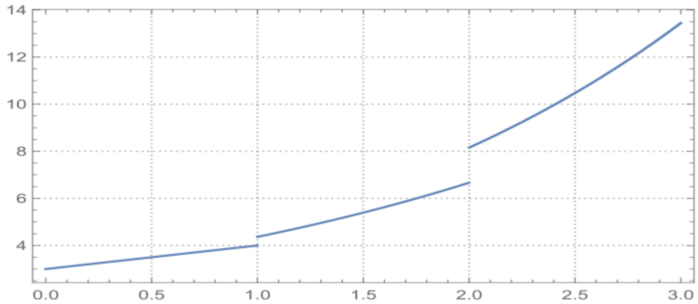

Figure 4. Graph of the exact solution.

4. CONCLUSIONS

The classical approximation methods, as well as the well-known differential transformation method, were intended for solving single-interval initial and/or boundary value problems without transmission conditions. In this work, we have generalized the classical DTM in such a way that it can be applied not only to problems defined on a single interval but also to problems that are defined on two or more separated intervals. In this case, additional transmission conditions are imposed on the points of interaction (i.e., on the common endpoints of the separated intervals). The demonstrate the applicability of the proposed GDTM, we have solved both the initial-value and boundary-value problems for three-interval differential equations under additional transmission conditions that are imposed on the common ends of the intervals. In addition, the obtained GDTM solutions were graphically compared with the exact solutions to show the effectiveness of the proposed method.

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