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A NEW STRUCTURE FOR MINIMAL AND MAXIMAL FUNCTION IN INTUITIONISTIC GENERALIZATION OF GENERALIZED STAR OPEN SETS

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Abstract. *In this article, minimal and maximal intuitionistic $(gg)^*$- open sets in intuitionistic topological space is introduced and their basic properties are investigated. Along with it has been developed and categorized minimal and maximal $i(gg)^*$- open set into three categories. According to these categorized sets, these minimal and maximal $i(gg)^*$- open sets are clearly examined and developed within the functions of continuity.*

Keywords: *Min i-open; Max i-open; Min $i(gg)^*$- open; Max $i(gg)^*$- open; Min $i(gg)^*$- continuous; Max $i(gg)^*$- continuous.*

1. INTRODUCTION

In the study of topological spaces, open sets plays a fundamental role in defining the structure of a given space. The minimal and maximal versions of these sets provide the relationship between different open sets within a space, leading to a deeper comprehension of their structural properties. The concept “intuitionistic set” was introduced by coker [1] and it is one of the several ways of introducing vagueness in mathematical objects. Further the concept of “Intuitionistic topological space” was developed by Kim et al. [2]. This concept integrates the intuitionistic fuzzy set introduced by Atanassov (1986) into topological structures, allowing for a more flexible approach to uncertainty in topology [3]. In 2009, it was further investigated and Younis Yaseen and Asmaa Raouf [4] have given some results in intuitionistic generalized closed sets in topological spaces. In 2020, Sivagami et al. developed intuitionistic generalized closed sets in generalized intuitionistic topological space [5]. In 2022, Mathan Kumar and Hari Siva Annam established “The Extension of Generalized Intuitionistic Topological Spaces” [6]. Two years later, the same authors introduced the concept of I-open sets within the framework of generalized intuitionistic topological spaces [7]. This development represents a significant advancement in the study of topological structures. Further, it was introduced intuitionistic generalization of generalized star closed sets in intuitionistic topological space in a proceeding where the $i(gg)^*$ open and closed set is compared with various intuitionistic sets and concluded many results by expanding their properties [8]. The concept of minimal and maximal closed sets was introduced by Yildirim et al.; they significantly contributed to this field by exploring minimal intuitionistic open sets and maximal intuitionistic open sets in their paper “Min i-open and max i-open sets” [9]. This exploration seeks to investigate the characteristics of min and max $i(gg)^*$- open sets (shortly

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$i(gg)^*{}_{MN}0$ ($i(gg)^*{}_{MX}0$). The continuous function is used to further build and discuss our minimal and maximal (shortly $i(gg)^*{}_{MN}c_f$ ($i(gg)^*{}_{MX}c_f$)). Minimal and Maximal irresolute functions (shortly $i(gg)^*{}_{MN}i_r$ ($i(gg)^*{}_{MX}i_r$)) that link the continuous and $i(gg)^*$- O set have been introduced, and results have been found. Composition of functions has distinctive outcomes.

2. PRELIMINARIES

Definition 2.1. Consider a non void set X. The acronym IS stands for intuitionistic set. The object A has the form $L =< X,\ L_1,\ L_2 >$ where L_1 and L_2 are subsets of X that meet the formula $L_1 \cap L_2 = \emptyset_\sim$. While L_2 is referred to as the set of non-members of L, L_1 is referred to as the set of members of L.

Definition 2.2. Consider a non void set X and assume intuitionistic sets L and Q have the following form $L =< X,\ L_1,\ L_2 >,\ Q =< X,\ Q_1,\ Q_2 >$ respectively. Then
(a) $L \subseteq Q$ iff $L_1 \subseteq Q$ and $L_2 \supseteq Q_2$.
(b) $L = Q$ iff $L \subseteq Q$ and $Q \subseteq L$.
(c) $L^c =< X,\ L_2,\ L_1 >$
(d) $[\,]\, L =< X,\ L_1,\ {L_1}^c >$
(e) $L - Q = L \cap \bar{Q}$
(f) $\emptyset_\sim = < X,\ \emptyset,\ X >,\ X_\sim = < X,\ X,\ \emptyset >$
(g) $L \cup Q =< X,\ L_1 \cup Q_1,\ L_2 \cap Q_2 >$
(h) $L \cap Q =< X,\ L_1 \cap Q_1,\ L_2 \cup Q_2 >$.

Definition 2.3. On a non-void set X, an intuitionistic topology (abbreviated IT) is a family τ_I of IS in X that satisfies the following axioms.
(1) $\emptyset_\sim,\ X_\sim \in \tau_I$
(2) $G_1 \cap G_2 \in \tau_I$ for any $G_1, G_2 \in \tau_I$
(3) $\bigcup G_\alpha \in \tau_I$ for any arbitrary family $\{G_\alpha / \alpha \in J\} \subseteq I_\tau$ where (X, τ_I) is called an intuitionistic topological space (for short ITS(X)) and any intuitionistic set in τ_I is called an intuitioistic open set (for short IOS) in X. The complement A^c of an IOS A is called an intuitionistic closed set (for short ICS) in X.

Definition 2.4. Consider the intuitionistic topological space (X, τ_I)(abbreviated ITS(X)) and $L = < X,\ L_1,\ L_2 >$ be an IS in X. Then
(i) $Icl(L) = \bigcap\{K : K$ is in X an ICS and $L \subseteq K\}$
(ii) $Iint(L) = \bigcup\{G : G$ is in X an IOS and $G \subseteq L\}$ are the interior and closure of L.
If $Icl(L) = L$ ($I\,int(L) = L$, hereafter it can be indicatedted as $Icl(L)$ is an ICS, $Iint(L)$ is an IOS in X, and L is an ICS (IOS) in X.

Definition 2.5. Consider an intuitionistic set A of a non - empty set X and $x \in X$.
$x_I = (\{x\}, \{x\}^c)$ is an intuitionistic point (IP) and $x_{IV} = (\emptyset, \{x\}^c)$ is an intuitionistic vanishing point of X.
IP(X) is a collecion of all intuitionistic points or intuitionistic vanishing point in X.

Definition 2.6. Suppose that (X, τ_I) is an ITS(X). It is argued that an intuitionistic set L of X equals

(1) It semi-open and intuitionistic if $L \subseteq Icl(I\ int(L))$.
(2) It is pre-open and intuitionistic if $L \subseteq Iint\big(Icl(L)\big)$.
(3) It is regular-open and intuitionistic if $L = Iint\big(Icl(L)\big)$.
The family of all intuitionistic (semi-open, pre-open, regular-open) of (X, τ_I) are indicated by ISOS, IPOS, IROS respectively.

Definition 2.7. In an intuitionistic topological space (X, τ_I). Given elements $L, Q, S \in IS(X)$ (the set of intuitionistic subsets of X) an element x_I [resp x_{IV}] $\in L$ signifies that x_I [resp x_{IV}] belongs to the intuitionistic set L in the sense of intuitionistic logic.

Definition 2.8. [7] Assume that L is a subset of a topological space (X, τ_I) with intuition is called an intuitionistic generalization of generalized star closed sets ($i\ (gg)^*$- closed) if $ircl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $i\ (gg)$ open in (X, τ_I). The collection of all $i(gg)^*$-closed sets in (X, τ_I) is denoted by $i(gg)^*C(X)$.

Theorem 2.9. Assume that L is a subset of a topological space (X, τ_I) with intuition. Then $x_I \in i(gg)^*cl(L)$ [resp x_{IV}] iff for any $i(gg)^*$- open set U Containing x_I[resp x_{IV}], $L \cap U \neq \emptyset_\sim$.

Definition 2.10. $i(gg)^*$- irresolute if $f^{-1}(L)$ is a $i(gg)^*$- open set in (X, τ_I) for any $i(gg)^*$-open set L in (Y, γ_I) .

3. MINIMAL $i(gg)^*$- OPEN SETS

Definition 3.1. Assume that (X, τ_I) is an intuitionistic topological space. A proper intuitionstic non- void $(gg)^*$ - open set L in (X, τ_I) is said to be a min $i(gg)^*$- open set (shortly $i(gg)^*{}_{MN}0$) if any $i(gg)^*$- O which is included in L is $\emptyset_\sim$ or L.

Example 3.2. Let $X = \{a, b, c\}$ with $\tau_I = \{< X, \emptyset, X >,\ < X, X, \emptyset >, < X, \emptyset, \{a\} >,$ $< X, \{b\}, \{a\} >, < X, \{c\}, \{a\} >, < X, \{b, c\}, \{a\} >\}$ and $i(gg)^*$- open sets are $= \{< X, \emptyset, X >$ $, < X, X, \emptyset >, < X, \emptyset, \{a\} >, < X, \emptyset, \{a, b\} >, < X, \emptyset, \{a, c\} >\}$. Here, $< X, \emptyset, \{a, b\} >$is an $i(gg)^*{}_{MN}$ open set.

Lemma 3.3. Assume that (X, τ_I) is an intuitionistic topological space and $L, Q \in$ IS(X). Then
(i) Let L be an $i(gg)^*{}_{MN}0$ and Q be an $i(gg)^*$- O. Then $L \cap Q = \emptyset_\sim$ or $L \subseteq Q$.
(ii) Let L and Q be a $i(gg)^*{}_{MN}0$. Then $L \cap Q = \emptyset_\sim$ or $L = Q$.

Proof: (i) Let Q be an intuitionistic open set and $L \cap Q \neq \emptyset_\sim$. Since $L \cap Q \subseteq L$ and L is a$i(gg)^*{}_{MN}0$, we get $L \cap Q = L$. That is $L \subseteq Q$.
(ii) Take $L \cap Q \neq \emptyset_\sim$ as assumption. Since L and Q are$i(gg)^*{}_{MN}0$, we have $L \subseteq Q$ and $Q \subseteq L$. Hence $L = Q$.

Theorem 3.4. Assume that (X, τ_I) is an intuitionistic topological space and let L, Q, S are taken as $i(gg)^*{}_{MN}0$ in (X, τ_I) such that $L \neq Q$. If $S \subseteq L \cup Q$, then either $S = L$ or $S = Q$.
Proof : If $S = L$ shows the evidence is evident. Let $S \neq L$ then by lemma (3.3) (ii) we have $S \cap L = \emptyset_\sim$. Hence we obtain $S \cup Q = S \cup (Q \cup \emptyset_\sim) = S \cup (Q \cup (S \cap L)) = (Q \cup L) \cap$

$(Q \cup S) = Q \cup (L \cap S) = Q$. Then $S \subset Q$. Since S and Q are$i(gg)^*{}_{MN}0$, we get $S = Q$. Hence the outcome is proved in a contradiction way.

Theorem 3.5. Assume that (X, τ_I) is an intuitionistic topological space and let L, Q, S are taken as $i(gg)^*{}_{MN}0$ in (X, τ_I) in a way that makes them distinct from one another from each other. Then $L \cup Q \not\subset L \cup S$.

Proof: Suppose that $L \cup Q \subseteq L \cup S$. Then $(L \cup Q) \cap (Q \cup S) \subseteq (L \cup S) \cap (Q \cup S)$. This implies $Q \cup (L \cap S) \subseteq S \cup (L \cap Q)$. (By lemma 3.3) (ii) we obtain $Q \subseteq S$. Hence $Q = S$, Since Q and S are $i(gg)^*{}_{MN}0$. Which is a contradiction. Hence $L \cup Q \not\subset L \cup S$.

Utilizing Lemma (3.2), we can systematically classify the $i(gg)^*{}_{MN}0$ into three distinct categories, providing a structured approach to their development.

Definition 3.6. 1. Let L be the $i(gg)^*{}_{MN}0$ of category -I then $L \cap Q = \emptyset_{\sim}$ or $L \subseteq Q$ for any $Q \in U$.
2. Let L be the $i(gg)^*{}_{MN}0$ of category -II then $(L \cap Q)_T = \emptyset$ or $L \subseteq Q$ for any $Q \in U$.
3. Let L be the $i(gg)^*{}_{MN}0$ of category -III then $(L \cap Q)_F = \emptyset$ or $L \subseteq Q$ for any $Q \in U$.

Definition 3.7. Assume that (X, τ_I) is an intuitionistic topological space, a neighbourhood N of point x_I (resp x_{IV}) and let $N \in \mathrm{IS}(X)$. Then N is called $i(gg)^*$ - neighbourhood of x_I (resp x_{IV}), if there exists a $i(gg)^*$- O set $w \in \tau_I$ such that $x_I \in w \subseteq N$.

Proposition 3.8. Assume that (X, τ_I) is an intuitionistic topological space and $L, Q, S \in \mathrm{IS}(X)$ and
(i) let L be an $i(gg)^*{}_{MN}0$. If $x_I \in L$, then $L \subseteq Q$ for each $Q \in N(x_I)$.
(ii) let L be an $i(gg)^*{}_{MN}0$. If $x_{IV} \in L$, then $L \subseteq S$ for each $S \in N(x_{IV})$.

Theorem 3.9. Let L be a non-void $i(gg)^*$- O. Consequently, the following are comparable.
i) L is an $i(gg)^*{}_{MN}0$.
ii) $L \subseteq i(gg)^*cl(U)$ for any non-void subset of L.
iii) $i(gg)^*cl(L) = i(gg)^*cl(U)$, for any non-void subset U of L.
Proof : (i) ⇒ (ii) Let $x_I \in L$ and U be a non- void subset of L. By proposition (3.8) there is a $i(gg)^*$- O set Q containing x_I such that $L \subseteq Q$. Then we have $U \subseteq L \subseteq Q$ which implies $U \subseteq Q$. Now $U = U \cap L \subseteq U \cap Q$. Since U is non-void, we have $U \cap Q \neq \emptyset_{\sim}$. Since Q is any $i(gg)^*$- O set containing x_I. By (Theorem 2.9) $x_I \in i(gg)^*cl(U)$. That is $x_I \in L$ implies $x_I \in i(gg)^*cl(U)$. Hence $L \subseteq i(gg)^*cl(U)$, for any non-void subset U of L.
(ii) ⇒ (iii) Let U be an non-void subset of L and $L \subseteq i(gg)^*cl(U)$. Then $i(gg)^*cl(U) \subseteq i(gg)^*cl(L)$ and $i(gg)^*cl(L) \subseteq i(gg)^*cl(U)$. Hence $i(gg)^*cl(L) = i(gg)^*cl(U)$ for any non-void subset U of L.
(iii) ⇒ (i) Let $i(gg)^*cl(L) = i(gg)^*cl(U)$, for any non-void subset U of L. Suppose L is not a $i(gg)^*{}_{MN}0$ then there exists a non-void $i(gg)^*$- O set Q such that $Q \subseteq L$ and $Q \neq L$. There exists an element $x_I \in L$ such that $x_I \notin Q$, which implies $x_I \in X - Q$. $\Rightarrow i(gg)^*cl(x_I) \subseteq i(gg)^*cl(X - Q) = X - Q$, is an $i(gg)^*$-C in (X, τ_I). This implies $i(gg)^*cl(x_I) \neq i(gg)^*cl(L)$, which is a contradiction to $i(gg)^*cl(x_I) = i(gg)^*cl(L)$ for non-void subset x_I of L. Thus L is $i(gg)^*{}_{MN}0$.
Theorem 3.10. Let (X, τ_I) be an intuitionistic topological space and $L, Q \in \mathrm{IS}(X)$. If L is a $i(gg)^*{}_{MN}0$ and $\emptyset_{\sim} \neq Q \subseteq L$, then $i(gg)^*cl(L) = i(gg)^*cl(Q)$.

Proof: Let L be a $i(gg)^*{}_{MN}0$ and $\emptyset_\sim \neq Q \subseteq L$. next we have $i(gg)^*cl(Q) \subseteq i(gg)^*cl(L)$. We have to show that $i(gg)^*cl(L) \subseteq i(gg)^*cl(Q)$. Let $x_I \in L$ (resp x_{IV}) by proposition (3.7) $Q = L \cap Q \subseteq Q \cap R$ for each $R \in N(x_I)$. Then $Q \cap R \neq \emptyset_\sim$. By theorem (2.9), $x_I \in i(gg)^*cl(Q)$. Similarly $x_{IV} \in i(gg)^*cl(Q)$ for $x_{IV} \in L$. Therefore $L \subset i(gg)^*cl(Q)$. Thus $i(gg)^*cl(L) \subseteq i(gg)^*cl(Q)$. Hence $i(gg)^*cl(L) = i(gg)^*cl(Q)$.

4. MAXIMAL INTUITIONISTIC $(gg)^*$- OPEN SETS

Definition 4.1. Assume that (X, τ_I) is an intuitionistic topological space. A proper intuitionstic non- void $(gg)^*$ - open set L in (X, τ_I) is said to be a maximal intuitionistic $(gg)^*$- open set (shortly $i(gg)^*{}_{MX}0$) if any $i(gg)^*$- O which contains L is $X_\sim$ or L.

Example 4.2. From example (3.2) here $< X, X, \emptyset >$ is an $i(gg)^*{}_{MX}$ open set.

Lemma 4.3. Assume that (X, τ_I) is an intuitionistic topological space and L,$Q \epsilon$ IS(X). Then
(i) Let L be an $i(gg)^*{}_{MX}0$ and Q be an $i(gg)^*$- O. Then $L \cup Q = X_\sim$ or $Q \subseteq L$.
(ii) Let L and Q be $i(gg)^*{}_{MX}0$. Then $L \cup Q = X_\sim$ or $L = Q$.

Proof: (i) Let Q be an $i(gg)^*$- O and $L \cup Q \neq X_\sim$. Since $L \subseteq L \cup Q$ and L is a $i(gg)^*{}_{MX}0$, we get $L \cup Q = L$. That is $Q \subseteq L$.
(ii) Take $L \cup Q \neq X_\sim$ as assumption. Since L and Q are $i(gg)^*{}_{MX}0$, they are intuitionistic open sets. Then by (i) we have $L \subseteq Q$ and $Q \subseteq L$. Hence $L = Q$.

Theorem 4.4. Assume that (X, τ_I) is an intuitionistic topological space and let L, Q, S be taken as $i(gg)^*{}_{MX}0$ in (X, τ_I) such that $L \neq Q$. If $L \cap Q \subseteq S$, then either $S = L$ or $S = Q$.
Proof: If $S = L$ shows the evidence is evident. Let $S \neq L$ then by theorem (4.3) (ii) we have $L \cup Q = X_\sim$.Then $Q \cap S = Q \cap (S \cap X_\sim) = Q \cap (S \cap (L \cup Q)) = Q \cap (S \cap L) \cup (S \cap Q) = (Q \cap S \cap L) \cup (Q \cap S \cap Q) = (L \cap Q) \cup (Q \cap S) = Q \cap (L \cup S) = Q \cap X_\sim = Q$. This implies $Q \subseteq S$. Since Q and S are $i(gg)^*{}_{MX}0$. We have $Q = S$. Hence proved.

Theorem 4.5. Assume that (X, τ_I) is an intuitionistic topological space and let L, Q, S be taken as $i(gg)^*{}_{MX}0$ in (X, τ_I) in a way that makes them distinct from one another from each other. Then $L \cap Q \not\subset L \cap S$.
Proof: Assume that $L \cap Q \subseteq L \cap S$, then $(L \cap Q) \cup (Q \cap S) \subset (L \cap S) \cup (Q \cap S) \Rightarrow Q \cap (L \cup S) \subseteq S \cap (L \cup Q)$. By theorem (4.3) (ii), $Q \cap X_\sim \subseteq S \cap X_\sim \Rightarrow Q \subseteq S$. Since Q and S are $i(gg)^*{}_{MX}0$, we have $Q = S$. This is a contradiction. Hence $L \cap Q \not\subset L \cap S$.

5. MINIMAL AND MAXIMAL $i(gg)^*$- CONTINUOUS

Definition 5.1. Let (X, τ_I) and (Y, γ_I) be an intuitionistic topological spaces.
A map $f : (X, \tau_I) \to (Y, \gamma_I)$ is called a
(i) minimal $i(gg)^*$- continuous (shortly $i(gg)^*{}_{MN}c_f$) if $f^{-1}(L)$ is an $i(gg)^*$- O in (X, τ_I) for any minimal intuitionistic open L in (Y, γ_I).

(ii) maximal $i(gg)^*$- continuous (shortly $i(gg)^*{}_{MX}c_f$) if $f^{-1}(L)$ is a $i(gg)^*$- Oin (X,τ_I) for any maximal intuitionistic open L in(Y,γ_I).

Theorem 5.2. Every $i(gg)^*c_f$ is a $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$.
Proof: Let L be a min (max) intuitionistic open set in (Y,γ_I) and $f:(X,\tau_I)\to(Y,\gamma_I)$ be a $i(gg)^*c_f$. Hence $f^{-1}(L)$ is a $i(gg)^*$- O in (X,τ_I) as every min (max) intuitionistic open set is an intuitionistic open set. Hence f is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$.

Theorem 5.3. Let (X,τ_I) and (Y,γ_I) be an intuitionistic topological spaces to consider and a map $f:(X,\tau_I)\to(Y,\gamma_I)$ is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$ iff the inverse image of each min (max) closed set in (Y,γ_I) is a $i(gg)^*$-C in(X,τ_I).

Definition 5.4. A mapping $f:(X,\tau_I)\to(Y,\gamma_I)$ is called $i(gg)^*$- irresolute (shortly$i(gg)^*i_r$) if $f^{-1}(L)$ is $i(gg)^*$- O in (X,τ_I) for every $i(gg)^*$- O set L of (Y,γ_I).

Example 5.5. Let $X=\{a,b,c\}$ with $\tau_{I1}=\{<X,\emptyset,X>,\ <X,X,\emptyset>,<X,\emptyset,\{a\}>,$ $<X,\{b\},\{a\}>,<X,\{c\},\{a\}>,<X,\{b,c\},\{a\}>\}$ and $i(gg)^*$- open sets are $\{<X,\emptyset,X>,<X,X,\emptyset>,<X,\emptyset,\{a\}>,<X,\emptyset,\{a,b\}>,<X,\emptyset,\{a,c\}>\}$ and $Y=\{x,y,z\}$ with $\tau_{I2}=\{<X,\emptyset,X>,<X,X,\emptyset>,<X,\{z\},\{a\}>,<X,\{y,z\},\{x\}>,<X,\{y\},\{x,z\}>,$ $<X,\emptyset,\{x,z\}>,<X,\{x,z\},\emptyset>\}$ then $i(gg)^*$- open sets are $\{<X,\emptyset,X>,<X,X,\emptyset>,<X,\emptyset,\{x,z\}>$. Define $f:(X,I_\tau)\to(Y,I_\sigma)$ as $f(a)=x,f(b)=y,f(c)=z$. Hence the function f is $i(gg)^*i_r$.

Theorem 5.6. If $f:(X,\tau_I)\to(Y,\gamma_I)$ and $h:(Y,\gamma_I)\to(Z,\beta_I)$ are $i(gg)^*i_r$ and $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$ respectively then $h\ o\ f:(X,\tau_I)\to(Z,\beta_I)$ is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$
Proof : Let L be a min (max) intuitionistic open set in (Z,β_I). Since h is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$, $h^{-1}(L)$ is $i(gg)^*$- O in (Y,γ_I). Also since f is $i(gg)^*i_r$, $f^{-1}\left(h^{-1}(L)\right)=(h\ o\ f\ )^{-1}(L)$ is $i(gg)^*$-O in (X,τ_I). Hence $h\ o\ f$ is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$

Remark 5.7. Two $i(gg)^*{}_{MN}c_f(i(gg)^*{}_{MX}c_f)$ need not to be a $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$ under composition.

Definition 5.8. A space is called a $i(gg)^*T_{1/2}$ – space if every $i(gg)^*$- O in it is intuitionistic open.

Theorem 5.9. Let $f:(X,\tau_I)\to(Y,\gamma_I)$ is an $i(gg)^*c_f$ with Y is a $i(gg)^*T_{1/2}$ – space and $h:(Y,\gamma_I)\to(Z,\beta_I)$ is a $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$ then $h\ o\ f:(X,\tau_I)\to(Z,\beta_I)$ is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$
Proof: Assign L be a min (max) intuitionistic open set in (Z,β_I). Then $h^{-1}(L)$ is $i(gg)^*$- O in (Y,γ_I) as h is a$i(gg)^*{}_{MN}c_f(i(gg)^*{}_{MX}c_f)$. Since Y is a $i(gg)^*T_{1/2}$ – space, $h^{-1}(L)$ is intuitionistic open in (Y,γ_I). Again since f is a $i(gg)^*c_f$, $f^{-1}\left(h^{-1}(L)\right)=(h\ o\ f\ )^{-1}(L)$ is $i(gg)^*$- O in (X,τ_I). Hence $h\ o\ f$ is $i(gg)^*{}_{MN}c_f(i(gg)^*{}_{MX}c_f)$

Definition 5.10. Let (X,τ_I) and (Y,γ_I) be an intuitionistic topological spaces.
A map $f:(X,\tau_I)\to(Y,\gamma_I)$ is called

(i) minimal $i(gg)^*$- irresolute (shortly $i(gg)^*{}_{MN}i_r$) if $f^{-1}(L)$ is a $i(gg)^*{}_{MN}0$ in (X,τ_I) for any $i(gg)^*{}_{MN}0$ L in (Y,γ_I) .
(ii) maximal $i(gg)^*$- irresolute (shortly $i(gg)^*{}_{MX}i_r$) if $f^{-1}(L)$ is a $i(gg)^*{}_{MX}0$ in (X,τ_I) for any $i(gg)^*{}_{MX}0$ L in (Y,γ_I).

Definition 5.11. The intuitionistic topological space (X,τ_I) is called as
(i) $T_{1/2\,(\min - i(gg)^*)}$ space if every $i(gg)^*$- O in it is min intuitionistic open.
(ii) $T_{1/2\,(\max - i(gg)^*)}$ space if every $i(gg)^*$- O in it is max intuitionistic open.

Theorem 5.12. Let $f:(X,\tau_I)\to(Y,\gamma_I)$ is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$ and Y be a $T_{1/2\,(\min - i(gg)^*)}$ $(T_{1/2\,(\max - i(gg)^*)})$ space. Then f is $i(gg)^*c_f$.

Proof: Let f be a $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$. Let L be an intuitionistic open set in (Y,γ_I). Since every intuitionistic open set is $i(gg)^*$- O then L is $i(gg)^*$- open. Y is $T_{1/2\,(\min - i(gg)^*)}$ $(T_{1/2\,(\max - i(gg)^*)})$ space as by assumption, then L is min (max) intuitionistic open set in (Y,γ_I). Since f is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$, $f^{-1}(L)$ is an $i(gg)^*$- O in (X,τ_I). Hence f is $i(gg)^*c_f$.

Theorem 5.13. Let $f:(X,\tau_I)\to(Y,\gamma_I)$ is $i(gg)^*i_r$ and $h:(Y,\gamma_I)\to(\mathcal{Z},I_\beta)$ is considered as $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$ then $h\ o\ f$ is a $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$.
Proof : Let L be any min (max) open set in (Z,β_I) since h is $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$, $h^{-1}(L)$ is a $i(gg)^*$- O in (Y,γ_I). Since f is $i(gg)^*i_r$, then $f^{-1}\left(h^{-1}(L)\right) = (h\ o\ f\ )^{-1}(L)$ is a $i(gg)^*$- O in (X,τ_I). Therefore $h\ o\ f$ is a $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$.

Result 5.14. Their composition will not be $i(gg)^*{}_{MN}c_f$ $(i(gg)^*{}_{MX}c_f)$if the map utilized here is a $i(gg)^*{}_{MN}i_r$ $(i(gg)^*{}_{MX}i_r$) rather than $i(gg)^*i_r$.

6. CONCLUSIONS

The study of minimal and maximal functions within $i(gg)^*$ open sets provides valuable insights into the structural properties of the set. The minimal function identifies the fundamental boundaries of openness, ensuring the least possible extension while preserving openness properties. Meanwhile, the maximal function characterizes the broadest possible extension without losing essential open set conditions. These concepts are crucial for understanding the behavior of topological structures and their applications in mathematical analysis. Future research can further explore their interactions with other generalized open sets and potential applications in functional analysis and topology.

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