ORIGINAL PAPER

NUMERICAL INVESTIGATION OF SOLUTIONS OF A SYSTEM OF DIFFERENTIAL EQUATIONS USING DTM

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Abstract. *The Differential Transform Method (DTM) is a semi-analytical approach used to approximate solutions to various differential equations (DEs) and systems of DEs, typically yielding solutions in series form. In this paper, DTM is used to approximate solutions of some systems of DEs. To assess the accuracy and precision of the method, the responses obtained by the DTM are compared with those from the exact solutions for selected examples.*

Keywords: *DTM; Differential equations; system of differential equations; approximate solution; exact solution.*

Mathematics Subject Classification: *35A22; 35C10; 41A58; 34A25; 44Axx; 65Lxx.*

1. INTRODUCTION

Throughout the discussion, the abbreviations DE, ODE, PDE, DTM, ADM, RKM, α-PDTM, IVP, IC and DT are used for Differential Equation, Ordinary Differential Equation, Partial Differential Equation, Differential Transform Method, Adomian Decomposition Method, Runge-Kutta Method, Alpha-Parametrized Differential Transform Method, Initial Value Problem, Initial Condition and Differential Transform, respectively. Moreover, the plural of A is written as As, etc. For $p \in Z^+$; the notation $D_t^p(\varphi)$ is used for $\frac{d^p\varphi(t)}{dt^p}$.

DTM is indeed a semi-analytical method, as it combines numerical and analytical techniques to solve ODEs, PDEs, integral equations, fractional ODEs and fractional PDEs, real-world issues in physics and engineering, dynamical systems analysis, transport phenomena modelling, reaction diffusion mechanisms in cell dynamics, electrical circuit analysis and numerical-analytical solutions for various mathematical challenges. DTM uses analytical techniques to transform the original problem into a recursive formula, which is an analytical representation of the solution. The recursive formula is then solved numerically using iterative calculations to obtain solutions at specific points or intervals. The method combines the analytical transformation with numerical computations, making it a semi-analytical approach. DTM provides an approximate solution that can be refined by increasing the number of iterations or by using higher-order transforms. DTM can be more efficient than purely numerical methods because it leverages analytical techniques to reduce computational load.

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Zhou was the first to propose the concept of DTM in 1986, which he subsequently utilized to address linear and nonlinear initial value problems encountered in electric circuit analysis; the DTM was derived from Taylor Series Expansion (TSE) and served as a potent technique addressing systems of differential equations without linearization and discretization, see [1]. Chen and Ho explored two-dimensional DTM and compared the results with other analytical techniques; see [2]. Ayaz [3] studied three-dimensional DTM and compared the results obtained through DTM and ADM. Hassan [4] solved a higher-order initial value problem considering DTM; a contrast analysis was also done against the fourth-order Runge-Kutta method. Erturk and Momani considered the application of DTM to address fractional-order DEs; the results indicated that DTM for fractional order was superior to both the Variational Iteration Method and the ADM; see [5]. Hassan [6] presented solutions to both linear and non-linear systems of ordinary DEs; the results coincided with the exact solution and RKM.

Tari et al. [7] proposed the solution of the Volterra integral equation utilizing DTM; the findings were very close to the exact solution, proving that DTM was a promising approach for solving integral equations. Ni et al. [8] analyzed the vibrations of a pipe carrying fluid; the DTM was utilized to obtain information about the velocities of critical flow and natural frequencies. Benhammouda investigated the use of DTM to solve a pollution model involving a network of three interconnected lakes, see [9]. Ahmad et al. [10] introduced DTM for both SI and SIS epidemic models; the convergence of these models with a constant population was also presented and verified; the results were found to be closely aligned with exact solutions. Gepreel et al. [11] formulated the Reduced DTM for two nonlinear biomathematics models. Mukhtarov et al. [12] introduced a new estimation technique termed α-PDTM, building upon the concept of DTM. Yucel and Mukhtarov [13] considered α-PDTM to address the boundary-value transmission problem associated with DE of order three; an equivalent solution was obtained using DTM, and a graphical comparison was performed between the solutions. Al-Rozbayani and Qasim [14] solved the generalized Grander equation using α-PDTM; a comparison was made with the results obtained using DTM and Exact to ensure the accuracy of the proposed approach.

2. MATERIALS AND METHODS

2.1. THE CLASSICAL DTM

By utilizing the Taylor series about $t = t_0$, an analytic function $\varphi(t)$ can be represented in the form

$$\varphi(t) = \sum_{p=0}^{\infty} \frac{1}{p!} \left[D_t^p (\varphi(t)) \right]_{t=t_0} (t - t_0)^p,$$

where p is a non-negative integer. Taylor series centred at $t = 0$, is stated as

$$\varphi(t) = \sum_{p=0}^{\infty} \frac{1}{p!} \left[D_t^p (\varphi(t)) \right]_{t=0} t^p. \tag{1}$$

The DT of $\varphi(t),$ denoted by $\Phi(p),$ is defined as

$$\Phi(p) = \frac{1}{p!} \left[D_t^p (\varphi(t)) \right]_{t=0}. \tag{2}$$

The inverse DT of $\Phi(p)$ is defined in (1) as

$$\varphi(t) = \sum_{p=0}^{\infty} \Phi(p)\, t^p.$$

In practice, we use

$$\varphi(t) \approx \varphi_n(t) = \sum_{p=0}^{n} \Phi(p)\, t^p$$

for some positive integer n.

2.2. LEIBNIZ RULE

If $\varphi(t)$ and $\psi(t)$ are n-times differentiable functions with respect to t, then for $n, s \in Z^+$ with $s \le n$; according to the Leibniz rule, we have

$$D_t^n(\varphi(t)\psi(t)) = \sum_{s=0}^{n} \overset{n}{\underset{s}{C}}\, D_t^{n-s}(\varphi(t))\, D_t^s(\psi(t)), \tag{3}$$

where $\overset{n}{\underset{s}{C}} = \dfrac{n!}{s!(n-s)!}$ while $n! = \prod_{j=0}^{n-1}(n-j)$.

3. RESULTS AND DISCUSSION

3.1. RESULTS

If $\Phi(p), \Psi(p)$ and $\Omega(p)$ are the DTs of the analytical functions $\varphi(t), \psi(t)$ and $\omega(t)$ with respect to t, respectively, then it is easy to see the following:

Theorem 1. If $\omega(t) = \alpha\varphi(t) \pm \beta\psi(t)$ then $\Omega(p) = \alpha\Phi(p) \pm \beta\Psi(p)$, where α and β are constants.

Theorem 2. If $\omega(t) = D_t^n(\varphi(t))$ then $\Omega(p) = \dfrac{(p+n)!}{p!}\Phi(p+n)$.

Theorem 3. If $\omega(t) = e^{\alpha t}$ then $\Omega(p) = \dfrac{\alpha^p}{p!}$, where α is constant.

Theorem 4. If $\omega(t) = \psi_1(t)\psi_2(t)$ then

$$\Omega(p) = \sum_{r=0}^{p} \Psi_1(r)\Psi_2(p-r) \tag{4}$$

Proof: Since $\omega(t) = \psi_1(t)\psi_2(t)$, applying DT yields

$$\Omega(p) = \frac{1}{p!} D_t^p [\psi_1(t)\psi_2(t)].$$

It, by use of (3), leads to

$$\Omega(p) = \frac{1}{p!} \sum_{r=0}^{p} \overset{p}{\underset{r}{C}} [D_t^r (\psi_1(t))][D_t^{p-r} \ (\psi_2(t))]$$
$$= \sum_{r=0}^{p} \frac{1}{r!(p-r)!} [D_t^r (\psi_1(t))][D_t^{p-r} \ (\psi_2(t))]$$
$$= \sum_{r=0}^{p} [\frac{1}{r!} D_t^r (\psi_1(t))][\frac{1}{(p-r)!} D_t^{p-r} \ (\psi_2(t))].$$

Using (2), the above relation leads to (4).

Theorem 5. If $\varphi(t) = Cos(\alpha t + \beta)$ then $\Phi(p) = \frac{\alpha^p}{p!} Cos(\frac{\pi p}{2} + \beta),$ where α and β are constants

Theorem 6. If $\varphi(t) = Sin(\alpha t + \beta)$ then

$$\Phi(p) = \frac{\alpha^p}{p!} Sin(\frac{\pi p}{2} + \beta), \tag{5}$$

where α and β are constants.

Proof: Since the DT of the function $\varphi(t) = Sin(\alpha t + \beta)$ is

$$\Phi(p) = \frac{1}{p!} [D_t^p Sin(\alpha t + \beta)]_{t=0}.$$

Thus, particularly, we obtain

$$\Phi(0) = Sin\beta, \ \Phi(1) = \alpha Cos\beta, \ \Phi(2) = \frac{-\alpha^2 Sin\beta}{2!}, \ \Phi(3) = \frac{-\alpha^3 Cos\beta}{3!}, \ \ldots$$

Hence, the result (5) follows.

3.2. DISCUSSION

The efficacy of the technique “DTM” is illustrated in the subsequent examples.

Problem 1. Consider the system of Des

$$\left.\begin{aligned} \varphi'(t) &= 2e^{2t} + \sin t, \\ \psi'(t) &= \varphi(t) - e^{4t}, \\ \omega'(t) &= -\theta(t) + 3e^{-3t}, \\ \theta'(t) &= -e^{-t} - \cos t \end{aligned}\right\} \tag{6}$$

with IC: $\left(\varphi(0), \psi(0), \omega(0), \theta(0)\right) = \left(1, -1, 2, 1\right)$.

We want to find the response $\left(\varphi(t), \psi(t), \omega(t), \theta(t)\right)$ of the phenomenon represented by the IVP. The system (6) has the exact solution

$$\begin{cases} \varphi(t) = 1 + e^{2t} - \cos t, \\ \psi(t) = \dfrac{1}{4}(-5 + 2e^{2t} - e^{4t} + 4t - 4\sin t), \\ \omega(t) = -\mathrm{e}^{-3t}(1 - \mathrm{e}^{2t} - 3\mathrm{e}^{3t} + \mathrm{e}^{3t}\cos t), \\ \theta(t) = -\mathrm{e}^{-t}(-1 + \mathrm{e}^{t}\sin t). \end{cases}$$

Now we find the response $\left(\varphi_n(t), \psi_n(t), \omega_n(t), \theta_n(t)\right)$ as approximate values of $\left(\varphi(t), \psi(t), \omega(t), \theta(t)\right)$ as

$$\left.\begin{aligned} \varphi_n(t) &= \sum_{p=0}^{n} \Phi(p) t^p, \\ \psi_n(t) &= \sum_{p=0}^{n} \Psi(p) t^p, \\ \omega_n(t) &= \sum_{p=0}^{n} \Omega(p) t^p, \\ \theta_n(t) &= \sum_{p=0}^{n} \Theta(p) t^p \end{aligned}\right\} \tag{7}$$

where $\Phi(p)$, $\Psi(p)$, $\Omega(p)$, and $\Theta(p)$ are transformed functions of $\varphi(t)$, $\psi(t)$, $\omega(t)$ and $\theta(t)$, respectively. Applying DT on system (6), the following system of recurrence relations arises

$$\left.\begin{aligned} (p+1)\Phi(p+1) &= 2\left(\frac{2^p}{p!}\right) + \frac{1}{p!}\sin\left(\frac{\pi p}{2}\right), \\ (p+1)\Psi(p+1) &= \Phi(p) - \frac{4^p}{p!}, \\ (p+1)\Omega(p+1) &= -\Theta(p) + 3\frac{(-3)^p}{p!}, \\ (p+1)\Theta(p+1) &= -\frac{(-1)^p}{p!} - \frac{1}{p!}\cos\left(\frac{\pi p}{2}\right). \end{aligned}\right\} \tag{8}$$

We find $\Phi(p), \Psi(p), \Omega(p)$ and $\Theta(p)$ for different values of p in terms of $\Phi(0)$, $\Psi(0)$, $\Omega(0)$ and $\Theta(0)$, from the recurrence relation given in (8). Then using the coefficients $\Phi(p)$, $\Psi(p)$, $\Omega(p)$ and $\Theta(p)$ in system (7), the approximated components $\varphi_n(t)$, $\psi_n(t)$, $\omega_n(t)$ and $\theta_n(t)$ of the response (immature solution) are found in terms of $\Phi(0), \Psi(0), \Omega(0)$ and $\Theta(0)$. The values of $\Phi(0), \Psi(0), \Omega(0)$ and $\Theta(0)$ are found by use of ICs in the previously found components of the response, and these values are

$$\begin{cases} \Phi(0) = 1, \\ \Psi(0) = -1, \\ \Omega(0) = 2, \\ \Theta(0) = 1. \end{cases}$$

Then, using these values, the approximate solution $\left(\varphi_n(t), \psi_n(t), \omega_n(t), \theta_n(t)\right)$ is found in the final form from the previously found solution (immature solution). We use $n = 7$, say, and the components of the approximate solution of the system (6) become

$$\begin{cases} \varphi_7(t) = 1 + 2t + \frac{5}{2}t^2 + \frac{4}{3}t^3 + \frac{5}{8}t^4 + \frac{4}{15}t^5 + \frac{13}{144}t^6 + \frac{8}{315}t^7, \\ \psi_7(t) = -1 - t^2 - \frac{11}{6}t^3 - \frac{7}{3}t^4 - \frac{241}{120}t^5 - \frac{62}{45}t^6 - \frac{4031}{5040}t^7, \\ \omega_7(t) = 2 + 2t - \frac{7}{2}t^2 + \frac{13}{3}t^3 - \frac{27}{8}t^4 + \frac{121}{60}t^5 - \frac{727}{720}t^6 + \frac{1093}{2520}t^7, \\ \theta_7(t) = 1 - 2t + \frac{1}{2}t^2 + \frac{1}{24}t^4 - \frac{1}{60}t^5 + \frac{1}{720}t^6. \end{cases}$$

The following table compares the values of $\varphi(t)$, the 1st component of the solution at different points of the domain $[0,1]$; found by the DTM and the exact solution. The accuracy and precision of the first component of the response can be observed from Table 1.

Table 1. Error Analysis of $\varphi(t)$ Exact Solution VS DTM for Problem 1.

t \ Component	Exact Solution $\varphi(t)$	DTM Solution $\Phi_7(t)$	Error
0.0	1	1	0
0.1	1..226398	1..226398	6.4683×10^{-11}
0.2	1.511758	1.511758	1.6942×10^{-8}
0.3	1.866782	1.866782	4.4447×10^{-7}
0.4	2.304479	2.304475	4.5465×10^{-6}
0.5	2.840699	2.840671	2.7763×10^{-5}
0.6	3.494781	3.494659	1.2235×10^{-4}
0.7	4.290357	4.289927	4.3065×10^{-4}
0.8	5.256325	5.255039	1.2858×10^{-3}
0.9	6.428037	6.424651	3.3864×10^{-3}
1.0	7.848754	7.840675	8.0791×10^{-3}

Fig. 1 displays a comparison between the approximate solution found by the DTM and the exact solution for the 1st component $\varphi(t)$ of system (6).

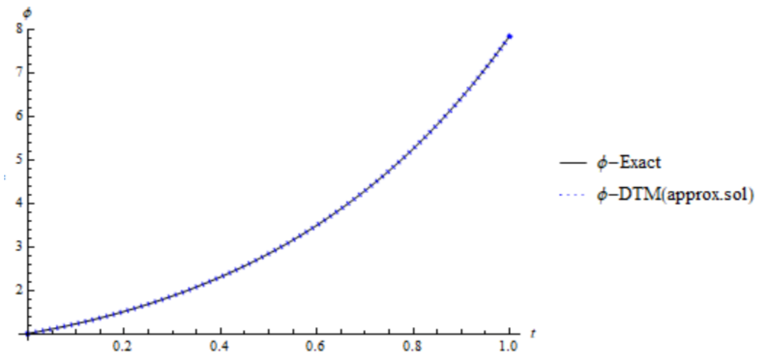

Figure 1. Graphical Error Analysis of $\varphi(t)$ Exact Solution VS DTM for Problem 1.

The following table compares the values of $\psi(t)$, the 2nd component of the response at different points of the domain $[0,1]$; found by the DTM and the exact solution. The accuracy and precision of the second component of the response can be observed from Table 2.

Table 2. Error Analysis of $\psi(t)$ Exact Solution VS DTM for Problem 1.

t \ Component	Exact Solution $\psi(t)$	DTM Solution $\psi_7(t)$	Error
0.0	–1	–1	0
0.1	–1..012088	–1..012088	4.219124×10^{-9}
0.2	–1.0591422	–1.059141	1.13219×10^{-6}
0.3	–1.164490	–1.164459	3.04705×10^{-5}
0.4	–1.364906	–1.364586	3.20211×10^{-4}
0.5	–1.717548	–1.715537	2.01200×10^{-3}
0.6	–2.310378	–2.301239	9.13918×10^{-3}
0.7	–3.27778	–3.244568	3.32109×10^{-2}
0.8	–4.823972	–4.721397	1.02575×10^{-1}
0.9	–7.258062	–6.978051	2.80011×10^{-1}
1.0	–11.046480	–10.352579	6.93901×10^{-1}

The following Fig. 2 displays a comparison between the approximate solution found by the DTM and the exact solution for the 2nd component $\psi(t)$ of system (6).

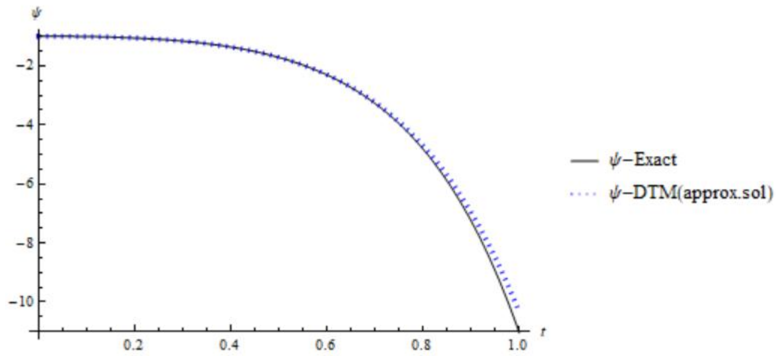

Figure 2. Graphical Error Analysis of $\psi(t)$ Exact Solution VS DTM for Problem 1.

Table 3 compares the values of $\omega(t)$, the 3rd component of the solution at different points of the domain $[0,1]$; found by the DTM and the exact solution. The accuracy and precision of the third component of the response can be observed from Table 3.

Table 3. Error Analysis of $\omega(t)$ Exact Solution VS DTM for Problem 1.

Component / t	Exact Solution $\omega(t)$	DTM Solution $\omega_7(t)$	Error
0.0	2	2	0
0.1	2.169015	2.169015	0
0.2	2.289853	2.289853	0
0.3	2.378922	2.378922	0
0.4	2.448159	2.448159	0
0.5	2.506362	2.506362	0
0.6	2.560448	2.560448	0
0.7	2.616864	2.616864	0
0.8	2.683355	2.683355	0
0.9	2.771328	2.771328	0
1.0	2.899008	2.899008	0

Fig. 3 displays a comparison between the approximate solution found by the DTM and the exact solution for the 3rd component $\omega(t)$ of system (6).

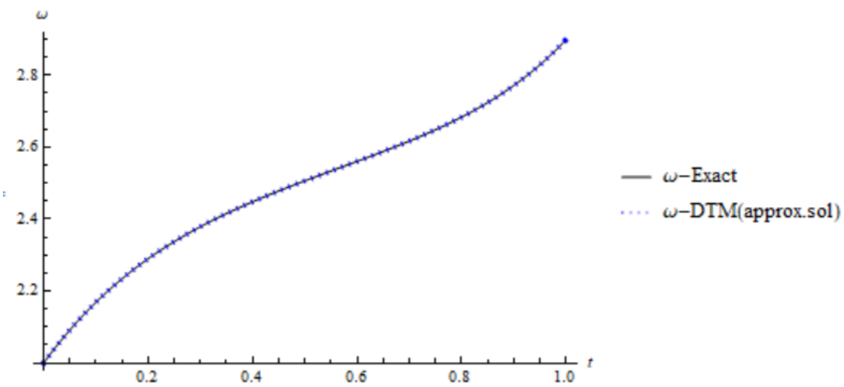

Figure 3. Graphical Error Analysis of $\omega(t)$ Exact Solution VS DTM for Problem 1.

The following table compares the values of $\theta(t)$, the 4th component of the solution at different points of the domain $[0,1]$; found by the DTM and the exact solution. The accuracy and precision of the fourth component of the response can be observed from Table 4.

Table 4. Error Analysis of $\theta(t)$ Exact Solution VS DTM for Problem 1.

t \ Component	Exact Solution $\theta(t)$	DTM Solution $\theta_7(t)$	Error
0.0	1	1	0
0.1	0.805004	0.805004	0
0.2	0.620061	0.620061	0
0.3	0.445298	0.445298	0
0.4	0.280902	0.280902	0
0.5	0.127105	0.127105	0
0.6	–0.015831	–0.015831	0
0.7	–0.147633	–0.147633	0
0.8	–0.2680030	–0.2680030	0
0.9	–0.376766	–0.376766	0
1.0	–0.473611	–0.473611	0

Fig. 4 displays a comparison between the approximate solution found by the DTM and the exact solution for the 4th component $\theta(t)$ of system (6).

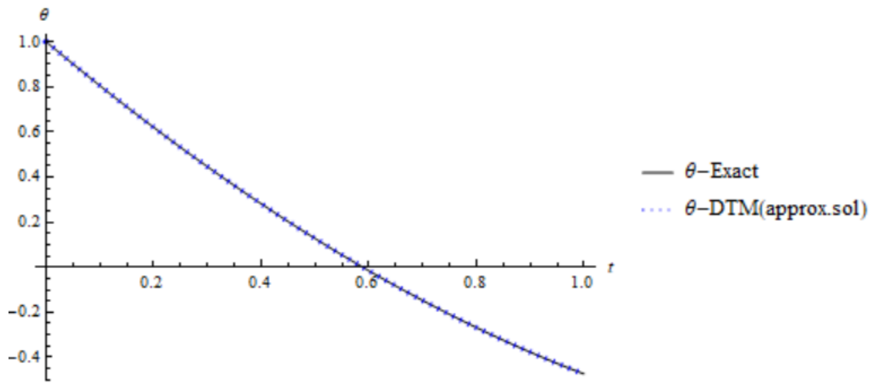

Figure 4. Graphical Error Analysis of $\theta(t)$ Exact Solution VS DTM for Problem 1.

Problem 2. Consider the system of Des

$$\left.\begin{aligned} \varphi'(t) &= 2\psi(t) + 3Cost, \\ \psi'(t) &= 3\varphi(t) - e^{2t}, \\ \omega'(t) &= \varphi(t) - 2\psi(t). \end{aligned}\right\} \tag{9}$$

with IC:

$$\left(\varphi(0), \psi(0), \omega(0)\right) = \left(3, 1, 2\right).$$

We want to find the response $\left(\varphi(t), \psi(t), \omega(t)\right)$ of the phenomenon represented by the IVP. The exact solution is

$$\begin{cases} \varphi(t) = \dfrac{1}{14} e^{-\sqrt{6}t} (14 - 3\sqrt{6} + 14e^{2\sqrt{6}t} + 3\sqrt{6}e^{2\sqrt{6}t} + 14e^{2t+\sqrt{6}t} + 6e^{\sqrt{6}t} Sint), \\ \psi(t) = \dfrac{1}{14} e^{-\sqrt{6}t} (9 - 7\sqrt{6} + 9e^{2\sqrt{6}t} + 7\sqrt{6}e^{2\sqrt{6}t} + 14e^{2t+\sqrt{6}t} - 18e^{\sqrt{6}t} Cost), \\ \omega(t) = -\dfrac{1}{42} e^{-\sqrt{6}t} (33 - 2\sqrt{6} - 189e^{\sqrt{6}t} + 33e^{2\sqrt{6}t} + 2\sqrt{6}e^{2\sqrt{6}t} + 21e^{2t+\sqrt{6}t} + 18e^{\sqrt{6}t} Cost - 108e^{\sqrt{6}t} Sint). \end{cases}$$

Now we find the response $\left(\varphi_n(t),\psi_n(t),\omega_n(t)\right)$ as approximate values of $\left(\varphi(t),\psi(t),\omega(t)\right)$ as

$$\left.\begin{aligned} \varphi_n(t) &= \sum_{p=0}^{n}\Phi(p)t^p, \\ \psi_n(t) &= \sum_{p=0}^{n}\Psi(p)t^p, \\ \omega_n(t) &= \sum_{p=0}^{n}\Omega(p)t^p \end{aligned}\right\} \tag{10}$$

where $\Phi(p)$, $\Psi(p)$ and $\Omega(p)$ are transformed functions of $\varphi(t)$, $\psi(t)$ and $\omega(t)$, respectively. Applying DT on system (9), the following system of recurrence relations arises

$$\left.\begin{aligned} (p+1)\Phi(p+1) &= 2\Psi(p) + 3*\frac{1}{p!}Cos(\frac{p\pi}{2}), \\ (p+1)\Psi(p+1) &= 3\Phi(p) - \frac{2^p}{p!}, \\ (p+1)\Omega(p+1) &= \Phi(p) - 2\Psi(p) \end{aligned}\right\} \tag{11}$$

We find $\Phi(p), \Psi(p)$ and $\Omega(p)$ for different values p of in terms of $\Phi(0)$, $\Psi(0)$ and $\Omega(0)$, from the recurrence relation given in (11). Then, using the coefficients $\Phi(p), \Psi(p)$ and $\Omega(p)$ in system (10), the approximated components $\varphi_n(t)$, $\psi_n(t)$ and $\omega_n(t)$ of the response (immature solution) are found in terms of $\Phi(0)$, $\Psi(0)$ and $\Omega(0)$. The values of $\Phi(0)$, $\Psi(0)$ and $\Omega(0)$ are found by use of ICs in the previously found components of the response and these values are

$$\begin{cases} \Phi(0) = 3, \\ \Psi(0) = 1, \\ \Omega(0) = 2. \end{cases}$$

Then, using these values, the approximate solution $\left(\varphi_n(t),\psi_n(t),\omega_n(t)\right)$ is found in the final form from the previously found solution (immature solution). We use $n = 7$, say, and the components of the approximate solution of the system (9) become

$$\begin{cases} \varphi_7(t) = 3 + 5t + 8t^2 + \frac{23t^3}{6} + \frac{11t^4}{3} + \frac{25t^5}{24} + \frac{31t^6}{45} + \frac{683t^7}{5040}, \\ \psi_7(t) = 1 + 8t + \frac{13t^2}{2} + \frac{22t^3}{3} + \frac{61t^4}{24} + \frac{31t^5}{15} + \frac{343t^6}{720} + \frac{89t^7}{315}, \\ \omega_7(t) = 2 + t - \frac{11t^2}{2} - \frac{5t^3}{3} - \frac{65t^4}{24} - \frac{17t^5}{60} - \frac{371t^6}{720} - \frac{19t^7}{504}. \end{cases}$$

The following table compares the values of $\varphi(t)$, the first component of the solution at different points of the domain $[0,1]$; found by the DTM and the exact solution. The accuracy and precision of the first component of the response can be observed from Table 5.

Table 5. Error Analysis of $\varphi(t)$ Exact Solution VS DTM for Problem 2.

t \ Component	Exact Solution $\varphi(t)$	DTM Solution $\varphi_7(t)$	Error
0.0	3	3	0
0.1	3.584211	3.584211	7.17408×10^{-10}
0.2	4.356913	4.356912	1.86730×10^{-7}
0.3	5.3562678	5.356263	4.87097×10^{-6}
0.4	6.632960	6.632910	4.95746×10^{-5}
0.5	8.253009	8.252708	3.01395×10^{-4}
0.6	10.301458	10.300134	1.32327×10^{-3}
0.7	12.887123	12.882480	4.64255×10^{-3}
0.8	16.148700	16.134874	1.38260×10^{-2}
0.9	20.262556	20.226214	3.63414×10^{-2}
1.0	25.452654	25.366071	8.65821×10^{-2}

Fig. 5 displays a comparison between the approximate solution found by the DTM and the exact solution for the first component $\varphi(t)$ of system (9).

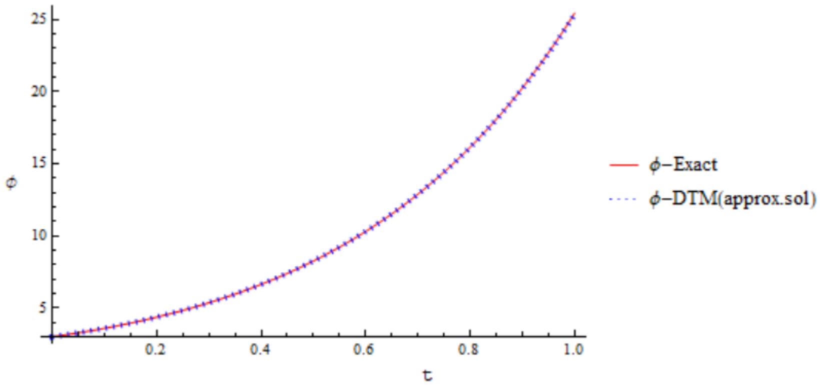

Figure 5. Graphical Error Analysis of $\varphi(t)$ Exact Solution VS DTM for Problem 2.

The following table compares the values of $\psi(t)$, the second component of the response at different points of the domain $[0,1]$; found by the DTM and the exact solution. The accuracy and precision of the second component of the response can be observed from Table 6.

Table 6. Error Analysis of $\psi(t)$ Exact Solution VS DTM for Problem 2.

Component / t	Exact Solution $\psi(t)$	DTM Solution $\psi_7(t)$	Error
0.0	1	1	0
0.1	1.872609	1.872609	4.99594×10^{-10}
0.2	2.923429	2.923428	1.33999×10^{-7}
0.3	4.209022	4.209018	3.59564×10^{-6}
0.4	5.798014	5.797977	3.75834×10^{-5}
0.5	7.774989	7.774755	2.34318×10^{-4}
0.6	10.245293	10.244239	1.05355×10^{-3}
0.7	13.341028	13.337247	3.78049×10^{-2}
0.8	17.228576	17.217074	1.15017×10^{-2}
0.9	22.118094	22.087244	3.08505×10^{-2}
1.0	28.275522	28.200595	7.49269×10^{-2}

Fig. 6 displays a comparison between the approximate solution found by the DTM and the exact solution for the second component $\psi(t)$ of system (9).

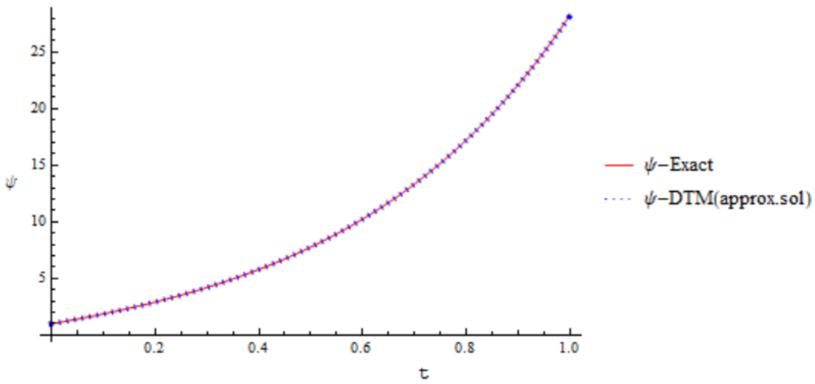

Figure 6. Graphical Error Analysis of $\psi(t)$ Exact Solution VS DTM for Problem 2.

The following table compares the values of $\omega(t)$, the third component of the solution at different points of the domain $[0,1]$; found by the DTM and the exact solution. The accuracy and precision of the third component of the response can be observed from Table 7.

Table 7. Error Analysis of $\omega(t)$ Exact Solution VS DTM for problem 2.

t \ Component	Exact Solution $\omega(t)$	DTM Solution $\omega_7(t)$	Error
0.0	2	2	0
0.1	2.043059	2.043059	5.400462×10^{-10}
0.2	1.962209	1.962209	1.392255×10^{-7}
0.3	1.736986	1.736990	3.597917×10^{-6}
0.4	1.338890	1.338926	3.628421×10^{-5}
0.5	0.729977	0.730196	2.186301×10^{-4}
0.6	–0.139079	–0.1381281	9.515456×10^{-4}
0.7	–1.331594	–1.328284	3.310044×10^{-3}
0.8	–2.928268	–2.918492	9.77604×10^{-3}
0.9	–5.031602	–5.006114	2.54885×10^{-2}
1.0	–7.771557	–7.711309	6.024767×10^{-2}

Fig. 7 displays a comparison between the approximate solution found by the DTM and the exact solution for the third component $\omega(t)$ of system (9).

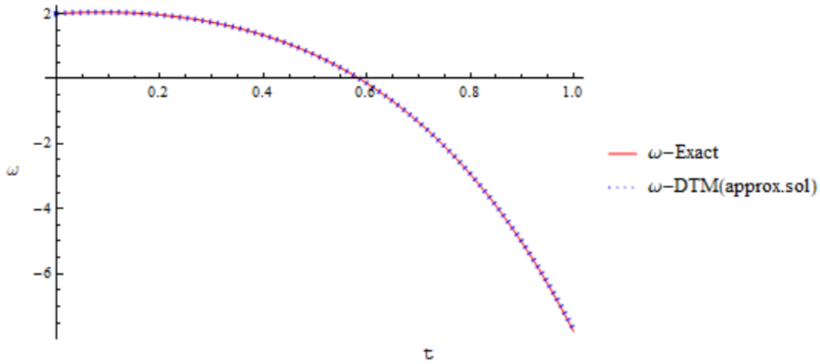

Figure 7. Graphical Error Analysis of $\omega(t)$ Exact Solution VS DTM for Problem 2.

4. CONCLUSION

The Differential Transform Method is applied to some system of ordinary differential equations to find their solutions. The solutions obtained by the classical DTM are compared with the exact solutions. The findings are presented graphically and in tabular form, and it is found that DTM is a much more precise and accurate method, as the errors produced by this method are much closer to zero. Moreover, the accuracy of the method may be increased by using high values of n.

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