ORIGINAL PAPER

A NEW COMPUTATION OF SOLUTIONS FOR ABEL'S FREDHOLM AND ABEL'S VOLTERRA INTEGRAL EQUATIONS

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Abstract. *In this paper, we present numerical solutions of Abel's Fredholm and Abel's Volterra integral equations. This method is based on modifying the cubic spline function to approximate the unknown function; this approximation cancels the singularities in Abel's kernels and transforms the equation into a linear system of algebraic equations. With comparisons to other methods, using illustrative examples, we demonstrate the efficiency of this new method.*

Keywords: *Abel's Fredholm integral equation; Abel's Volterra integral equation; cubic spline function; weakly singular kernels.*

Mathematics Subject Classification: *45D05; 65N20 ; 65M25 ; 65D30.*

1. INTRODUCTION

It is known that Abel's Fredholm and Abel's Volterra integral equations cover many applications, especially in fracture mechanics, transmission lines, high-frequency electromagnetic waves, elasticity, and potential theory. We know that the numerical solution of Abel's Fredholm and Abel's Volterra integral equations leads to the solution of the generalized telegrapher equations. Many methods have been proposed to compute solutions to Abel's Fredholm and Abel's Volterra integral equations. The article [1] presents a numerical method to approximate the solution of Abel's integral equation using Navot's quadrature and Simpson's rule, based on the discrete Gronwall inequality and the Euler-Maclaurin summation formula. The papers [2,3] describe a numerical approach for the solution of weakly singular kernels, Abel's and logarithmic kernels, integral equations based on normalized Euler polynomials, where the so-called Euler operational matrix of integration and the Toeplitz quadrature method are introduced, respectively. In [4], to solve Abel's integral equation, the authors propose a computational method in which the numerical scheme depends on the Daubechies wavelet basis. In [5-9], we find presentations of adapted linear quadrature methods to provide numerical approaches to the singular integral equation, the weakly singular integral equation with logarithmic kernels, and the singular integro-differential equation. The authors in [10] use the Legendre polynomial basis functions with the Galerkin method for Abel's Fredholm integral equations of the second kind and obtain convergence rates for the approximate solution. The authors in [11] use the regularization of Abel's integral equations by the introduction of a new independent variable, so that the singularities of the

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derivatives of the solution disappear completely, and solve the transformed equations by a collocation method.

The goal of this note is to present a new technical approach based on the cubic spline destined to solve the Abel's Fredholm and Abel's Volterra integral equations numerically. This technique converts singular integral equations into a linear system of algebraic equations.

Consider the Abel's Fredholm and Abel's Volterra integral equations $T\varphi(x)$ given by

$$T\varphi(x) = \varphi(x) - \int_\Omega \frac{k(x,t)}{|x-t|^\alpha}\varphi(t)dt = f(x), \quad 0 < \alpha < 1, \tag{1}$$

where $k(x,t)$, $f(x)$ are known functions, continuous on $C([a,b]\times[a,b])$ and $C([a,b])$ respectively, the density function $\varphi(x)$ is an unknown function which could be determined, or still in the operator form

$$T\varphi(x) = (I - A)\varphi(x) = f(x) \tag{2}$$

where I represents the identity and A the compact operator with a weakly singular kernel

$$A\varphi(x) = \int_\Omega \frac{k(x,t)}{|x-t|^\alpha}\varphi(t)dt, \tag{3}$$

Noting that the equation (1) is said to be Abel's Fredholm integral equation if $\Omega = [a,b]$ and Abel's Volterra integral equation in the domain. Sometimes the equation (1) can be written in the form

$$\varphi(x) - b_0(x)\int_\Omega \frac{\varphi(t)}{|x-t|^\alpha}dt - \int_\Omega k_1(x,t)\varphi(t)dt = f(x), \tag{4}$$

Where

$$k_1(x,t) = \frac{k(x,t) - k(x,x)}{|x-t|^\alpha}, \; b_0(x) = k(x,x).$$

or still

$$\varphi(x) - b_0(x)W\varphi(x) + K\varphi(x) = f(x), \tag{5}$$

where the operator W is given as

$$W\varphi(x) = \int_\Omega \frac{\varphi(t)}{|x-t|^\alpha}dt, \tag{6}$$

The operator $B,$ defined by

$$B\varphi(x) = \varphi(x) - b_0(x)W\varphi(x)$$

or still

$$B\varphi(x)=\varphi(x)-b_0(x)\int_\Omega \frac{\varphi(t)}{|x-t|^\alpha}dt, \tag{7}$$

will be called the dominant part of the operator $T,$ whereas the operator is defined as

$$K\varphi(x)=\int_\Omega k_1(x,t)\varphi(t)dt, \tag{8}$$

will be called the regular part of the operator T.

Noting that, the present work has two parts. The first one treats the quadrature formula proposed by the authors for evaluating Abel's Fredholm and Abel's Volterra integral equations. This quadrature formula is based on the modification of the cubic approximation of the density $\varphi(t)$.

The second part presents the convergence of this approximation and the numerical realization of this latter, showing its efficiency and comparing it with other methods.

2. QUADRATURE

Let N be an arbitrary natural number. Generally, we take it large enough and divide the interval $[a,b]$ into N equal subintervals $I_1, I_2,..., I_N$ by the points

$$t_\sigma = a+\sigma\frac{l}{N},\ l=b-a,\ \sigma=0,2,....,N.$$

Further, we fix a natural number $M>1$, and divide each of the segments $[t_\sigma,t_{\sigma+1}]$ by the equidistant points

$$t_{\sigma k}=t_\sigma+k\frac{h}{M},\ \ h=\frac{l}{N},\ \ k=0,1,2,...M.$$

Assuming that, for the indices $\sigma,\nu=0,1,2,....,N,$ the points x and t belong respectively to the subintervals $[t_\sigma,t_{\sigma+1}]$ and $[t_\nu,t_{\nu+1}]$.

For an arbitrary number $\sigma=\ 0,1,2,...,N,$ we define the piecewise cubic Lagrange interpolation polynomial $S_3(\varphi;t,\sigma)$ dependent on φ, t and σ which represents the cubic approximation of the function density $\varphi(t)$ on the subinterval $[t_\sigma,t_{\sigma+1}]$. Noting that, the interval $[t_\sigma,t_{\sigma+1}]$ is divided into subintervals $[t_{\sigma k},t_{\sigma(k+1)}]\cdot$ We interpolate the function density $\varphi(t)$ with respect to the values $\varphi(t_{\sigma 0}),\ \varphi(t_{\sigma 1}),\ \varphi(t_{\sigma 2})$ and $\varphi(t_{\sigma 3})$ at the points $t_{\sigma 0},\ t_{\sigma 1},\ t_{\sigma 2}$ and $t_{\sigma 3}$ respectively with a cubic polynomial, given by the following formula.

For $t_\sigma \le t \le t_{\sigma+1},$

$$S_3(\varphi;t,\sigma)=\sum_{i=0}^{i=3}\left(\prod_{k=0,k\neq i}^{k=3}\frac{t-t_{\sigma k}}{t_{\sigma i}-t_{\sigma k}}\right)\varphi(t_{\sigma i}) \tag{9}$$

This piecewise cubic interpolating polynomial exists and is unique.

For any numbers σ and ν, such that $0 \le \sigma, \nu \le N-1$, we define the continuous functions $U_{\sigma}(\varphi; t, x)$ and $V_{\sigma\nu}(\varphi; t, x)$, dependents on φ, t and x by

$$U_{\sigma}(\varphi; t, x) = \sum_{i=0}^{i=3} \left(\prod_{k=0, k \ne i}^{k=3} \frac{t - t_{\sigma k}}{t_{\sigma i} - t_{\sigma k}} \right) \varphi(t_{\sigma i}) \frac{|x - t_{\sigma i}|^{\alpha}}{|x - t|^{\alpha}} \tag{10}$$

and

$$V_{\sigma\nu}(\varphi; t, x) = \sum_{i=0}^{i=3} \left(\prod_{k=0, k \ne i}^{k=3} \frac{t - t_{\sigma k}}{t_{\sigma i} - t_{\sigma k}} \right) \varphi(t_{\sigma i}) \frac{|x - t_{\sigma i}|^{\alpha}}{|x - t|^{\alpha}} S_3(\varphi; x, \nu) \tag{11}$$

where the functions $U_{\sigma}(\varphi; t, x)$ and $V_{\sigma\nu}(\varphi; t, x)$ represent a modified cubic interpolation of the functions $\varphi(t)$ and $\varphi(x)$ on the subinterval $[t_{\sigma}, t_{\sigma+1}]$ and $[t_{\nu}, t_{\nu+1}]$ respectively. Define the function $\beta_{\sigma\nu}(\varphi; t, x)$ for $t_{\sigma} \le t \le t_{\sigma+1}$

$$\beta_{\sigma\nu}(\varphi; t, x) = \begin{cases} U_{\sigma}(\varphi; t, x) - V_{\sigma\nu}(\varphi; t, x) & \text{for} \quad t \ne x \\ 0 & \text{for} \quad t = x \end{cases}. \tag{12}$$

Denoting by $\psi_{\sigma\nu}(\varphi; t, x)$ the cubic approximation of the density $\varphi(t)$ at the point $t \in [t_{\sigma}, t_{\sigma+1}]$, $x \in [t_{\nu}, t_{\nu+1}]$ and $0 \le \sigma, \nu \le N$ by

$$\psi_{\sigma\nu}(\varphi; t, x) = \varphi(x) + \beta_{\sigma\nu}(\varphi; t, x). \tag{13}$$

Using the cubic spline interpolation of the kernel $k_1(t, x)$ and of the density $\varphi(t)$, of the regular part of the operator T

$$\begin{aligned}
K\varphi(x) &= \frac{1}{\pi i} \int_{\Omega} k_1(t, x) \varphi(t) dt \\
&= \frac{1}{\pi i} \sum_{\sigma=0}^{N-1} \int_{t_{\sigma}}^{t_{\sigma+1}} \frac{(t - t_{\sigma 1})(t - t_{\sigma 2})(t - t_{\sigma 3})}{(t_{\sigma 0} - t_{\sigma 1})(t_{\sigma 0} - t_{\sigma 2})(t_{\sigma 0} - t_{\sigma 3})} k_1(t_{\sigma 0}, x) \varphi(t_{\sigma 0}) \\
&+ \frac{(t - t_{\sigma 0)})(t - t_{\sigma 2})(t - t_{\sigma 3})}{(t_{\sigma 1} - t_{\sigma 0})(t_{\sigma 1} - t_{\sigma 2})(t_{\sigma 1} - t_{\sigma 3})} k_1(t_{\sigma 1}, x) \varphi(t_{\sigma 1}) \\
&+ \frac{(t - t_{\sigma 0})(t - t_{\sigma 1})(t - t_{\sigma 3})}{(t_{\sigma 2} - t_{\sigma 0})(t_{\sigma 2} - t_{\sigma 1})(t_{\sigma 2} - t_{\sigma 3})} k_1(t_{\sigma 2}, x) \varphi(t_{\sigma 2}) \\
&+ \frac{(t - t_{\sigma 0})(t - t_{\sigma 1})(t - t_{\sigma 2})}{(t_{\sigma 3} - t_{\sigma 0})(t_{\sigma 3} - t_{\sigma 1})(t_{\sigma 3} - t_{\sigma 2})} k_1(t_{\sigma 3}, x) \varphi(t_{\sigma 3}) dt. \\
&= \frac{1}{\pi i} \int_{\Omega} k_{1N}(t, x) \varphi_N(t) dt \\
&= K_N \varphi_N(x),
\end{aligned}$$

and the cubic approximation of the density $\varphi(t)$ to the dominant part of the operator T

$$\begin{aligned} B\varphi(x) &= \varphi(x) - b_0(x)W\varphi(x) \\ &= \varphi(x) - \frac{b_0(x)}{\pi i}\int_\Omega \frac{\varphi(t)}{|x-t|^\alpha}dt \\ &= \varphi_N(x) - \frac{b_0(x)}{\pi i}\int_\Omega \frac{\psi_{\sigma\nu}(\varphi;t,x)}{|x-t|^\alpha}dt \\ &= \left(I - b_0(x)W_N\right)\varphi_N(x) \\ &= B_N\varphi_N(x). \end{aligned}$$

Hence, the approximation of Abel's Fredholm and Abel's Volterra integral equations $T_N\varphi(x)$ give the form

$$T_N\varphi(x) = B_N\varphi_N(x) - K_N\varphi_N(x) = f(x), \quad 0 < \alpha < 1, \tag{14}$$

or still

$$T_N\varphi(x) = \varphi_N(x) - b_0(x)W_N\varphi_N(x) - K_N\varphi_N(x) = f(x), \tag{15}$$

3. MAIN RESULTS

Theorem

Suppose that Abel's Fredholm and Volterra integral equations (1) has a unique solution $\varphi(t)$ then the approximate solution $\varphi_N(t)$ of the integral equation (15) converges to the solution $\varphi(t)$ with the following estimation

$$\|\varphi - \varphi_N\| \le \frac{C}{(MN)^{M+2}}, \quad M, N > 1, \tag{16}$$

provided that

$$k(x,t) \in C^{M+1}([a,b]\times[a,b]) \text{ and } \varphi(x) \in C^{M+1}([a,b]) \tag{17}$$

where C is constant, depending on the $(M+1)$ derivative of $k(x,t)$.

Proof: It is easy to show that the operators $\{(W_N + K_N)\}$ are a collectively compacts and pointwise convergent family on $C[a,b]$ to $C[a,b]$. See [4] and for sufficiently large N, the equation (15) is uniquely solvable, and the inverse operators are uniformly bounded for such N. Besides,

$$|\varphi(x) - \varphi_N(x)| \le |b_0(x)||W\varphi(x) - W_N\varphi_N(x)| + |K\varphi(x) - K_N\varphi_N(x)|.$$

It is known that

$$\left|K\varphi(x)-K_N\varphi_N(x)\right|\le\frac{C_1}{(MN)^5},$$

where $C_1=\dfrac{3}{80}\sup\limits_{x\in\Omega}\left|K^{(4)}\varphi(x)\right|$. By the same way and using the relation

$$\frac{\left|x-t_{\sigma i}\right|^{\alpha}}{\left|x-t\right|^{\alpha}}=O(1),\quad i=0,1,2,3, \tag{18}$$

we get

$$\left|b_0(x)\right|\left|W\varphi(x)-W_N\varphi_N(x)\right|\le\frac{C_2}{(MN)^5}$$

where $C_2=\dfrac{3}{80}\sup\limits_{x\in\Omega}\left|b_0(x)\right|\left|W^{(4)}\varphi(x)\right|$, In particular, if we take the expression $(W_N+K_N)\varphi_N(t)$ as a piecewise polynomial interpolation of degree $M>0$, with the condition (17) we obtain directly the relation (16). In our case $M=3$ and so the expression (16) becomes

$$\left\|\varphi-\varphi_N\right\|\le\frac{C}{(3N)^5},\quad N>1, \tag{19}$$

4. NUMERICAL EXPERIMENTS

In this section, we describe some of the numerical experiments performed in solving Abel's Fredholm and Abel's Volterra integral equations (1). In all cases, we chose the right-hand side $f(t)$ in such a way that we know the exact solution. This exact solution is used solely to demonstrate that the numerical solution obtained by our method is correct and efficient.

We know that if the operator $(I-A)^{-1}$ is invertible from a Banach space E into itself, then the operator $(I-A_n)^{-1}$ is also invertible for all sufficiently large n so, in the numerical experiments the matrix of the system of algebraic equations given by our approximation is invertible. Noting that, for each table, φ represents the exact solution of the Abel's Fredholm or Abel's Volterra integral equations and φ_N corresponds to the approximate solution produced by our new technical approximation at interpolation points x_i, the error is given by $\left|\varphi(x_i)-\varphi_N(x_i)\right|$.

Example 1. Consider the Abel-Fredholm integral equation,

$$\varphi(x)-\int_0^1\frac{1}{\sqrt{|x-t|}}\varphi(t)dt=f(x)$$

where the function second member $f(x)$ is given by

$$f(x) = x + \left(-\sqrt{|x-1|}\left(\frac{4}{3}x + \frac{2}{3}\right)\right) + \left(\frac{4}{3}x\sqrt{|x|}\right)$$

and the exact solution $\varphi(x)$ determined as

Table 1. The approximate solution $\varphi_N(x)$ of $\varphi(x)$ is obtained by our modified cubic spline approximation for N=32

Pts of x	Exact sol	Approx sol	**Error**
0.0000e+00	0.0000e+00	-2.1468e-05	2.14e-05
1.2500e-01	1.2500e-01	1.2500e-01	2.64e-07
2.5000e-01	2.5000e-01	2.5000e-01	4.19e-07
5.0000e-01	5.0000e-01	5.0000e-01	2.22e-15
6.2500e-01	6.2500e-01	6.2500e-01	3.01e-07
7.5000e-01	7.5000e-01	7.5000e-01	4.19e-07
1.0000e+00	1.0000e+00	1.0000e+00	2.14e-05

Example 2. Consider the Abel-Fredholm integral equation,

$$\varphi(x) - \frac{2}{3}\int_{-1}^{1}\frac{1}{\sqrt{|x-t|}}\varphi(t)dt = f(x)$$

where the function's second member $f(x)$ is given by

$$f(x) = \left(1 - x^2\right)^{\frac{4}{3}} - \frac{\pi}{2\sqrt{2}}\left(2 - x^2\right)$$

and the exact solution $\varphi(x)$ determined as

$$\varphi(x) = \left(1 - x^2\right)^{\frac{4}{3}}$$

Table 2. The approximate solution $\varphi_N(x)$ of $\varphi(x)$ is obtained by our modified cubic spline approximation for N=32

Pts of x	Exact sol	Approx sol	**Error**	Error [11]
-1.000e+00	0.000e+00	-5.089e-04	5.08e-04	2.65e-04
-7.500e-01	5.379e-01	5.379e-01	3.31e-05	2.65e-04
-5.000e-01	8.059e-01	8.059e-01	1.25e-05	2.65e-04
0.000e+00	1.000e+00	1.000e+00	6.72e-05	2.65e-04
5.000e-01	8.059e-01	8.059e-01	1.25e-05	2.65e-04
7.500e-01	5.379e-01	5.379e-01	3.31e-05	2.65e-04
1.000e+00	0.000e+00	-5.089e-04	5.08e-04	2.65e-04

Example 3. Consider the Abel-Fredholm integral equation,

$$\phi(x) + \frac{1}{\pi}\int_{-1}^{1}\frac{1}{\sqrt{|x-t|}}\phi(t)dt = f\ \ x$$

where the function's second member $f(x)$ is given by

$$f\ \ x\ =1+\frac{2}{\pi}\ \sqrt{|x-1|}-\sqrt{|x+1|}$$

and the exact solution $\phi(x)$ determined as

$$\phi(x)=1$$

Table 3. The approximate solution $\varphi_N(x)$ of $\varphi(x)$ is obtained by our modified cubic spline approximation for N=32.

Pts of x	Exact sol	Approx sol	**Error**	Error [3]
-4.000e-01	1.000e+00	1.000e+00	2.34e-14	5.49e-05
-3.000e-01	1.000e+00	1.000e+00	2.35e-14	1.69e-05
-2.000e-01	1.000e+00	1.000e+00	2.29e-14	3.41e-06
-1.000e-01	1.000e+00	1.000e+00	2.22e-14	2.74e-07
0.000e+00	1.000e+00	1.000e+00	2.20e-14	3.17e-09
1.000e-01	1.000e+00	1.000e+00	2.12e-14	8.01e-08
2.000e-01	1.000e+00	1.000e+00	2.04e-14	1.69e-06
3.000e-01	1.000e+00	1.000e+00	1.48e-14	1.04e-05
4.000e-01	1.000e+00	1.000e+00	4.44e-16	3.81e-05

Example 4. Consider the Abel-Volterra integral equation,

$$\varphi(x)+\frac{1}{4}\int_0^x \frac{1}{\sqrt{x-t}}\varphi(t)dt = f(x)$$

where the function's second member $f(x)$ is given by

$$f(x)=\frac{1}{\sqrt{x+1}}+\frac{\pi}{8}-\frac{1}{4}\arcsin\left(\frac{1-x}{1+x}\right)$$

and the exact solution $\varphi(x)$ determined as

$$\varphi(x)=\frac{1}{\sqrt{x+1}}$$

Table 4. The approximate solution $\varphi_N(x)$ of $\varphi(x)$ is obtained by our modified cubic spline approximation for N=32

Pts of x	Exact sol	Approx sol	**Error**	Error [3]
0.0000e+00	1.000e+00	1.000e+00	0.00e+00	0.00e+00
1.2500e-01	9.428e-01	9.428e-01	2.14e-06	1.07e-03
2.5000e-01	8.944e-01	8.944e-01	6.93e-07	6.83e-04
5.0000e-01	8.164e-01	8.164e-01	7.38e-07	4.17e-04
6.2500e-01	7.844e-01	7.844e-01	1.12e-06	3.53e-04
7.5000e-01	7.559e-01	7.559e-01	1.38e-06	3.53e-04
1.0000e+00	7.071e-01	7.071e-01	1.69e-06	--

Example 5. Consider the Abel-Volterra integral equation,

$$\int_0^x \frac{1}{\sqrt{x-t}}\varphi(t)dt = f(x)$$

where the function's second member $f(x)$ is given by

$$f \quad x = x$$

and the exact solution $\phi(x)$ determined as

$$\varphi(x) = \frac{2}{\pi}\frac{\left(\Gamma(1)\right)^2}{\Gamma(2)}\sqrt{x}$$

Table 5. The approximate solution $\varphi_N(x)$ of $\varphi(x)$ is obtained by our modified cubic spline approximation for N=32

Pts of x	Exact sol	Approx sol	**Error**	Error [2]
0.0000e+00	1.000e+00	0.0000e+00	0.00e+00	3.88e-02
2.0000e-01	2.8470e-01	2.8471e-01	5.28e-06	4.75e-04
4.0000e-01	4.0263e-01	4.0263e-01	2.05e-06	9.07e-04
6.0000e-01	4.9312e-01	4.9312e-01	1.17e-06	9.65e-04
8.0000e-01	5.6941e-01	5.6941e-01	7.89e-07	1.20e-03
1.0000e+00	6.3661e-01	6.3662e-01	5.78e-07	7.71e-03

Example 6. Consider the Abel-Volterra integral equation,

$$\varphi(x) - \int_0^x \frac{1}{\sqrt{x-t}}\varphi(t)dt = f(x)$$

where the function's second member $f(x)$ is given by

$$f(x) = \sqrt{x} - \frac{\pi}{2}x$$

and the exact solution $\phi(x)$ determined as

$$\phi(x) = \sqrt{x}$$

Table 6. The approximate solution $\varphi_N(x)$ of $\varphi(x)$ is obtained by our modified cubic spline approximation for N=32

Pts of x	Exact sol	Approx sol	Error	Error [10]	Error [11]
0.0000e+00	0.0000e+0	0.0000e+0	0.00e+00	5.60e-03	1.79e-03
1.2500e-01	3.5355e-01	3.5368e-01	1.35e-04	5.60e-03	1.79e-03
2.5000e-01	5.0000e-01	5.0010e-01	1.06e-04	5.60e-03	1.79e-03
5.0000e-01	7.0710e-01	7.0718e-01	8.12e-05	5.60e-03	1.79e-03
6.2500e-01	7.9056e-01	7.9064e-01	7.41e-05	5.60e-03	1.79e-03
7.5000e-01	8.6602e-01	8.6609e-01	6.87e-05	5.60e-03	1.79e-03
1.0000e+00	1.0000e+00	1.0000e+00	6.07e-05	5.60e-03	1.79e-03
1.0000e+00	1.0000e+00	1.0000e+00	6.07e-05	5.60e-03	1.79e-03

4. CONCLUSIONS

It is known that the operators of Abel's Fredholm and Volterra integrals are compact operators as weakly singular integrals, so we treat the existence and the uniqueness of the solution of the integral equations of this kind of equations as Fredholm and Volterra integral equations with smooth kernels. We present an efficient technique to approximate the unknown function on the interval; this approximation leads to the cancellation of the singularities in Abel's kernels and the transformation of the equation into a linear system of algebraic equations. This new technique is very convenient for solving Abel's, Fredholm, and Volterra integral equations, and we demonstrate its applicability and consistency. The efficiency of this method is demonstrated by its convergence and comparisons with many other methods.

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